Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/17-1.1.1.6-P-x-a+b-x-^m-c+d-x-^n-e+f-x-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [78]. This is test number [17].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (78)	0.00 (0)
Mathematica	100.00 (78)	0.00 (0)
Maple	100.00 (78)	0.00 (0)
Fricas	82.05 (64)	17.95 (14)
Giac	58.97 (46)	41.03 (32)
Mupad	51.28 (40)	48.72 (38)
Maxima	34.62 (27)	65.38 (51)
Sympy	6.41 (5)	93.59 (73)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

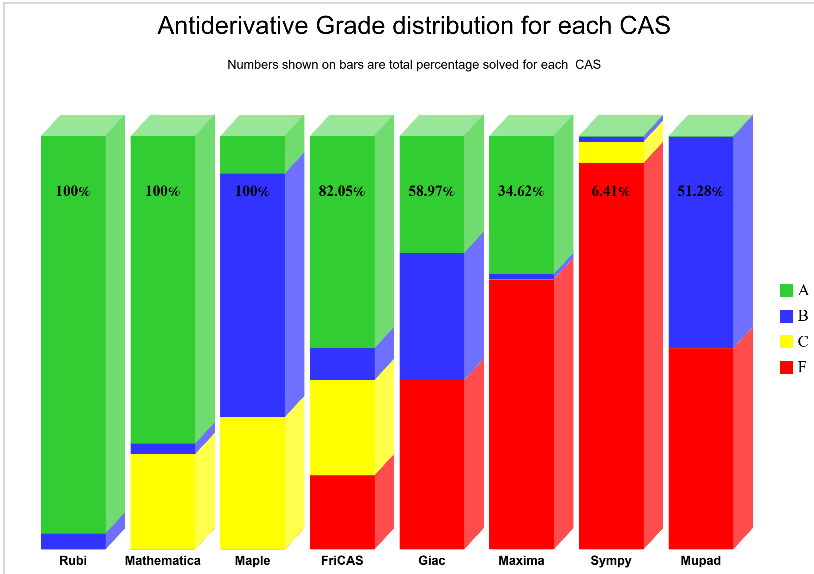
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

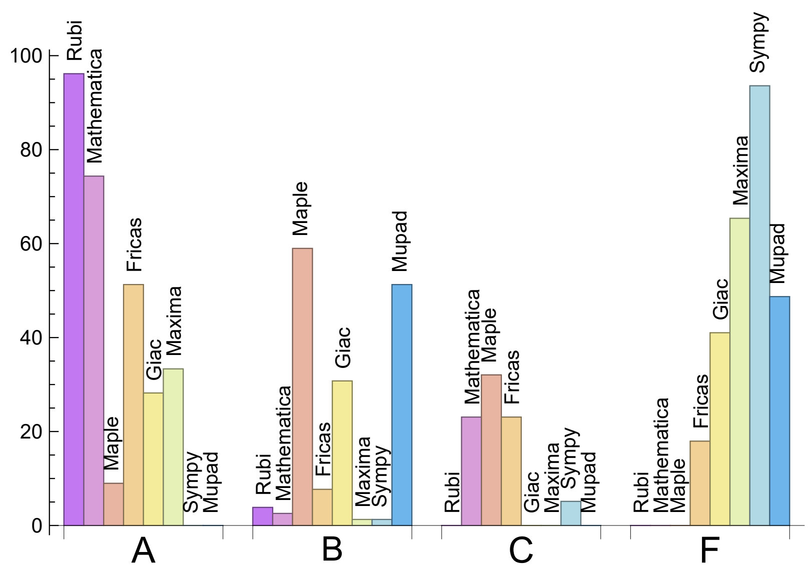
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.15	3.85	0.00	0.00
Mathematica	74.36	2.56	23.08	0.00
Fricas	51.28	7.69	23.08	17.95
Maxima	33.33	1.28	0.00	65.38
Giac	28.21	30.77	0.00	41.03
Maple	8.97	58.97	32.05	0.00
Mupad	N/A	51.28	0.00	48.72
Sympy	0.00	1.28	5.13	93.59

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	14	0.00 %	100.00 %	0.00 %
Giac	32	56.25 %	0.00 %	43.75 %
Maxima	51	35.29 %	0.00 %	64.71 %
Sympy	73	49.32 %	49.32 %	1.37 %
Mupad	38	47.37 %	52.63 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

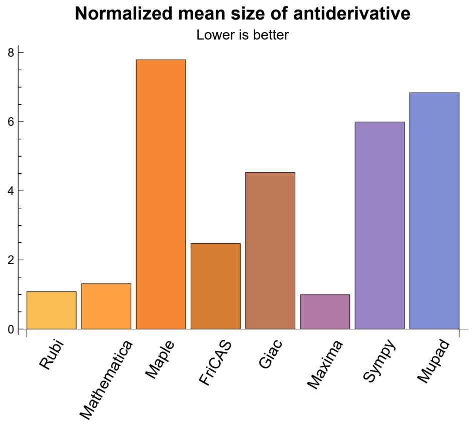
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

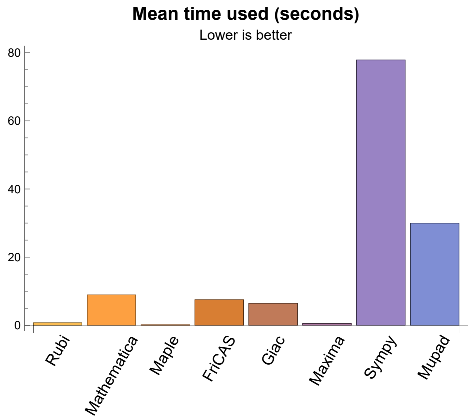
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.69	438.29	1.08	350.50	1.00
Mathematica	8.88	774.13	1.31	283.00	0.99
Maple	0.09	5451.40	7.79	1203.00	3.77
Maxima	0.51	187.52	0.99	100.00	1.02
Fricas	7.45	1224.14	2.48	826.50	1.89
Sympy	77.91	411.80	5.99	240.00	4.60
Giac	6.43	2382.59	4.53	588.00	1.72
Mupad	29.97	1446.72	6.84	1748.50	5.94

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
80
60
40
20
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{34, 35, 36, 37, 38, 39\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

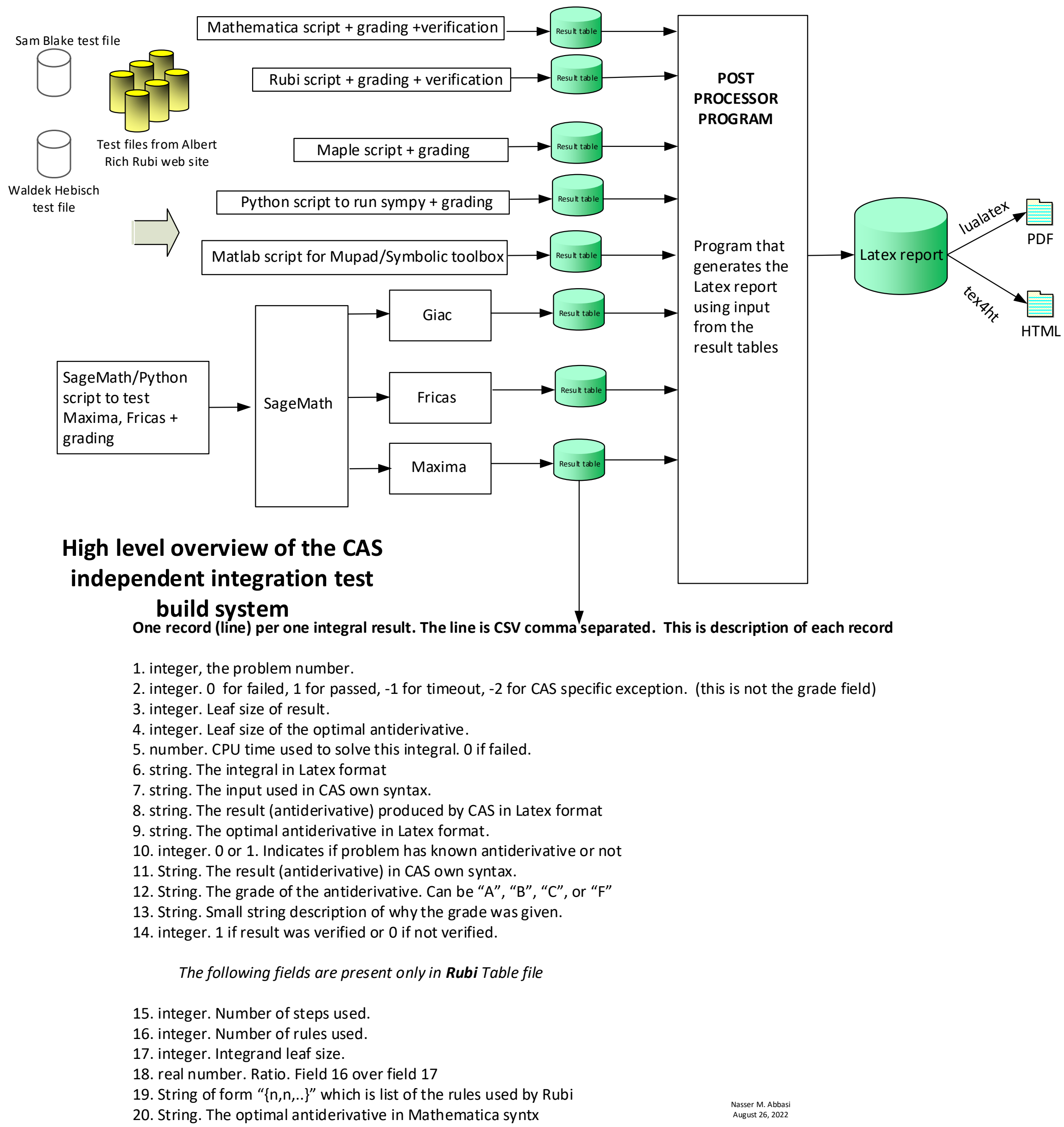
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

B grade: { 35, 36, 37 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

B grade: { 5, 12 }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F grade: { }

2.1.3 Maple

A grade: { 23, 24, 27, 28, 29, 30, 31 }

B grade: { 20, 21, 22, 25, 26, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 34, 35, 36, 37, 38, 39 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 34, 36, 37, 38, 39 }

B grade: { 35 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 24, 25, 26, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 47, 48, 49, 54, 55, 56 }

B grade: { 6, 7, 13, 14, 40, 59 }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F grade: { 24, 25, 31, 32, 44, 45, 46, 50, 51, 52, 53, 57, 58, 60 }

2.1.6 Sympy

A grade: { }

B grade: { 4 }

C grade: { 17, 18, 36, 37 }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.1.7 Giac

A grade: { 8, 9, 10, 11, 15, 16, 25, 27, 28, 29, 30, 32, 34, 35, 36, 37, 47, 48, 49, 54, 55, 56 }

B grade: { 1, 2, 3, 4, 20, 21, 22, 23, 26, 33, 38, 39, 40, 41, 42, 43, 45, 46, 51, 52, 53, 58, 59, 60 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 17, 18, 19, 24, 31, 44, 50, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 49, 55, 56 }

C grade: { }

F grade: { 20, 21, 25, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	387	959	441	405	0	1533	2500
N.S.	1	1.00	0.93	2.31	1.06	0.98	0.00	3.69	6.02
time (sec)	N/A	0.457	1.288	0.118	0.525	1.456	0.000	1.805	47.789

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	279	652	307	280	0	1059	2500
N.S.	1	1.00	0.98	2.28	1.07	0.98	0.00	3.70	8.74
time (sec)	N/A	0.352	0.922	0.112	0.564	1.184	0.000	1.783	36.028

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	170	173	377	180	174	0	637	736
N.S.	1	1.01	1.03	2.24	1.07	1.04	0.00	3.79	4.38
time (sec)	N/A	0.158	0.613	0.109	0.483	0.880	0.000	1.489	12.065

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	185	93	95	1137	284	361
N.S.	1	1.00	1.13	1.95	0.98	1.00	11.97	2.99	3.80
time (sec)	N/A	0.046	0.276	0.107	0.544	1.172	223.065	0.837	7.209

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	559	373	0	457	0	0	2500
N.S.	1	1.00	4.58	3.06	0.00	3.75	0.00	0.00	20.49
time (sec)	N/A	0.201	2.207	0.139	0.000	8.241	0.000	0.000	25.801

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	211	899	0	969	0	0	2500
N.S.	1	1.00	1.29	5.52	0.00	5.94	0.00	0.00	15.34
time (sec)	N/A	0.213	10.293	0.112	0.000	32.090	0.000	0.000	52.173

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	273	1449	0	1502	0	0	2500
N.S.	1	1.00	1.10	5.84	0.00	6.06	0.00	0.00	10.08
time (sec)	N/A	0.233	10.332	0.110	0.000	1.199	0.000	0.000	59.182

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	273	643	353	285	0	373	2500
N.S.	1	1.00	0.80	1.89	1.04	0.84	0.00	1.10	7.35
time (sec)	N/A	0.414	1.008	0.116	0.490	0.769	0.000	1.356	35.295

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	195	423	232	193	0	234	1732
N.S.	1	1.00	0.86	1.86	1.02	0.85	0.00	1.03	7.60
time (sec)	N/A	0.320	0.654	0.118	0.552	1.257	0.000	2.112	33.636

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	133	120	235	135	117	0	117	492
N.S.	1	1.02	0.92	1.81	1.04	0.90	0.00	0.90	3.78
time (sec)	N/A	0.145	0.460	0.118	0.485	1.404	0.000	1.702	12.857

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	82	117	57	67	0	60	232
N.S.	1	1.00	1.30	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.040	0.257	0.115	0.525	0.819	0.000	2.373	7.525

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	559	373	0	457	0	0	2500
N.S.	1	1.00	4.58	3.06	0.00	3.75	0.00	0.00	20.49
time (sec)	N/A	0.179	0.401	0.000	0.000	7.260	0.000	0.000	0.005

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	211	899	0	969	0	0	2500
N.S.	1	1.00	1.29	5.52	0.00	5.94	0.00	0.00	15.34
time (sec)	N/A	0.187	10.289	0.000	0.000	40.515	0.000	0.000	0.008

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	273	1449	0	1502	0	0	2500
N.S.	1	1.00	1.10	5.84	0.00	6.06	0.00	0.00	10.08
time (sec)	N/A	0.208	10.167	0.000	0.000	1.212	0.000	0.000	0.007

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	89	139	87	78	0	76	244
N.S.	1	1.00	1.13	1.76	1.10	0.99	0.00	0.96	3.09
time (sec)	N/A	0.097	0.027	0.001	0.610	0.670	0.000	0.849	7.606

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	82	117	57	67	0	60	232
N.S.	1	1.00	1.30	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.040	0.026	0.000	0.578	0.965	0.000	1.454	7.411

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	93	96	57	81	245	0	122
N.S.	1	1.00	1.94	2.00	1.19	1.69	5.10	0.00	2.54
time (sec)	N/A	0.114	0.035	0.000	0.577	1.373	43.363	0.000	4.331

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	93	97	57	84	221	0	114
N.S.	1	1.00	1.94	2.02	1.19	1.75	4.60	0.00	2.38
time (sec)	N/A	0.108	0.032	0.000	0.481	1.213	41.012	0.000	4.266

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	108	98	65	0	0	312
N.S.	1	1.00	0.97	1.52	1.38	0.92	0.00	0.00	4.39
time (sec)	N/A	0.114	0.021	0.000	0.496	1.101	0.000	0.000	6.304

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	584	402	1370	581	997	0	2665	-1
N.S.	1	0.99	0.68	2.32	0.98	1.69	0.00	4.51	-0.00
time (sec)	N/A	0.979	1.178	0.115	0.513	1.443	0.000	2.399	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	450	286	933	417	703	0	1868	-1
N.S.	1	1.00	0.63	2.07	0.92	1.56	0.00	4.14	-0.00
time (sec)	N/A	0.638	0.810	0.108	0.520	1.446	0.000	1.540	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	297	183	554	254	449	0	1148	1765
N.S.	1	0.99	0.61	1.85	0.85	1.50	0.00	3.83	5.88
time (sec)	N/A	0.285	0.504	0.104	0.555	1.458	0.000	1.060	30.577

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	123	269	140	265	0	527	876
N.S.	1	1.00	0.56	1.22	0.63	1.20	0.00	2.38	3.96
time (sec)	N/A	0.099	0.239	0.113	0.548	0.770	0.000	1.064	16.517

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	178	487	0	0	0	0	2500
N.S.	1	1.00	0.64	1.75	0.00	0.00	0.00	0.00	8.99
time (sec)	N/A	0.320	0.505	0.128	0.000	0.000	0.000	0.000	44.562

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	229	1166	0	0	0	523	-1
N.S.	1	1.00	0.71	3.62	0.00	0.00	0.00	1.62	-0.00
time (sec)	N/A	0.377	0.969	0.108	0.000	0.000	0.000	1.341	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	361	252	1794	0	1347	0	1410	2500
N.S.	1	0.99	0.69	4.94	0.00	3.71	0.00	3.88	6.89
time (sec)	N/A	0.449	1.437	0.106	0.000	61.830	0.000	2.541	86.666

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	496	283	913	469	698	0	571	2500
N.S.	1	0.99	0.56	1.82	0.94	1.39	0.00	1.14	4.99
time (sec)	N/A	0.819	0.801	0.108	0.529	1.582	0.000	1.389	161.428

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	369	200	599	318	484	0	366	2500
N.S.	1	1.00	0.54	1.63	0.86	1.32	0.00	0.99	6.79
time (sec)	N/A	0.569	0.489	0.104	0.505	1.321	0.000	1.010	81.648

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	249	128	343	193	308	0	199	1011
N.S.	1	1.01	0.52	1.39	0.78	1.25	0.00	0.81	4.11
time (sec)	N/A	0.261	0.270	0.109	0.511	0.976	0.000	0.705	30.743

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	90	168	88	196	0	106	489
N.S.	1	1.00	0.51	0.95	0.50	1.11	0.00	0.60	2.76
time (sec)	N/A	0.085	0.139	0.104	0.486	1.584	0.000	0.630	14.952

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	178	487	0	0	0	0	2500
N.S.	1	1.00	0.64	1.75	0.00	0.00	0.00	0.00	8.99
time (sec)	N/A	0.292	0.220	0.000	0.000	0.000	0.000	0.000	0.008

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	229	1166	0	0	0	523	2500
N.S.	1	1.00	0.71	3.62	0.00	0.00	0.00	1.62	7.76
time (sec)	N/A	0.338	0.340	0.000	0.000	0.000	0.000	0.781	19.397

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	361	252	1794	0	1347	0	1410	2500
N.S.	1	0.99	0.69	4.94	0.00	3.71	0.00	3.88	6.89
time (sec)	N/A	0.395	0.467	0.000	0.000	54.947	0.000	0.993	0.008

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	151	74	137	100	73	0	105	318
N.S.	1	1.74	0.85	1.57	1.15	0.84	0.00	1.21	3.66
time (sec)	N/A	0.096	0.080	0.000	0.305	1.142	0.000	0.550	14.762

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	B	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	135	63	120	90	61	0	80	312
N.S.	1	2.60	1.21	2.31	1.73	1.17	0.00	1.54	6.00
time (sec)	N/A	0.049	0.047	0.000	0.270	1.256	0.000	0.552	14.587

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	135	69	95	56	73	240	71	118
N.S.	1	2.45	1.25	1.73	1.02	1.33	4.36	1.29	2.15
time (sec)	N/A	0.119	0.048	0.000	0.489	0.922	41.456	0.676	5.391

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	135	69	96	56	82	216	83	118
N.S.	1	2.45	1.25	1.75	1.02	1.49	3.93	1.51	2.15
time (sec)	N/A	0.115	0.075	0.000	0.506	0.898	40.651	0.548	5.151

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	129	60	103	61	69	0	145	316
N.S.	1	1.55	0.72	1.24	0.73	0.83	0.00	1.75	3.81
time (sec)	N/A	0.120	0.066	0.000	0.509	1.343	0.000	0.828	12.773

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	171	71	123	86	90	0	197	304
N.S.	1	1.47	0.61	1.06	0.74	0.78	0.00	1.70	2.62
time (sec)	N/A	0.143	0.066	0.000	0.503	1.099	0.000	0.740	11.819

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	242	185	1095	0	1137	0	605	2500
N.S.	1	1.22	0.93	5.50	0.00	5.71	0.00	3.04	12.56
time (sec)	N/A	0.228	0.738	0.118	0.000	0.956	0.000	0.873	66.847

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1348	1345	1253	5734	0	3091	0	4708	-1
N.S.	1	1.00	0.93	4.25	0.00	2.29	0.00	3.49	-0.00
time (sec)	N/A	1.517	6.931	0.099	0.000	4.195	0.000	1.220	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	721	719	662	3025	0	1621	0	2643	-1
N.S.	1	1.00	0.92	4.20	0.00	2.25	0.00	3.67	-0.00
time (sec)	N/A	0.615	3.408	0.099	0.000	1.842	0.000	0.959	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	283	1207	0	845	0	1103	-1
N.S.	1	1.00	0.86	3.66	0.00	2.56	0.00	3.34	-0.00
time (sec)	N/A	0.198	0.997	0.098	0.000	1.204	0.000	1.004	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	453	404	3898	0	0	0	0	-1
N.S.	1	1.01	0.90	8.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.913	2.009	0.102	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	358	4680	0	0	0	1585	-1
N.S.	1	1.00	0.69	8.98	0.00	0.00	0.00	3.04	-0.00
time (sec)	N/A	1.137	2.841	0.104	0.000	0.000	0.000	2.274	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	657	536	11204	0	0	0	8347	-1
N.S.	1	1.00	0.81	17.03	0.00	0.00	0.00	12.69	-0.00
time (sec)	N/A	1.753	7.456	0.112	0.000	0.000	0.000	7.150	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1032	1032	888	3958	0	2173	0	1505	-1
N.S.	1	1.00	0.86	3.84	0.00	2.11	0.00	1.46	-0.00
time (sec)	N/A	1.125	7.440	0.113	0.000	2.654	0.000	1.167	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	540	454	2002	0	1117	0	736	-1
N.S.	1	1.00	0.84	3.71	0.00	2.07	0.00	1.36	-0.00
time (sec)	N/A	0.450	2.870	0.102	0.000	1.630	0.000	0.833	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	199	763	0	581	0	315	1832
N.S.	1	1.00	0.81	3.10	0.00	2.36	0.00	1.28	7.45
time (sec)	N/A	0.151	0.890	0.099	0.000	1.771	0.000	0.827	90.550

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	367	1822	0	0	0	0	-1
N.S.	1	1.00	1.27	6.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	10.425	0.105	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	421	3670	0	0	0	1388	-1
N.S.	1	1.00	1.16	10.08	0.00	0.00	0.00	3.81	-0.00
time (sec)	N/A	0.725	10.702	0.105	0.000	0.000	0.000	2.218	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	693	9100	0	0	0	8004	-1
N.S.	1	1.00	1.43	18.80	0.00	0.00	0.00	16.54	-0.00
time (sec)	N/A	1.050	11.946	0.105	0.000	0.000	0.000	26.354	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	847	15990	0	0	0	25485	-1
N.S.	1	1.00	1.24	23.34	0.00	0.00	0.00	37.20	-0.00
time (sec)	N/A	1.219	11.897	0.105	0.000	0.000	0.000	67.226	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	715	632	2528	0	1441	0	951	-1
N.S.	1	1.00	0.88	3.52	0.00	2.01	0.00	1.32	-0.00
time (sec)	N/A	0.864	2.559	0.102	0.000	3.734	0.000	1.328	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	369	314	1199	0	729	0	447	2500
N.S.	1	0.99	0.85	3.23	0.00	1.96	0.00	1.20	6.74
time (sec)	N/A	0.330	1.065	0.106	0.000	2.078	0.000	1.075	105.189

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	141	425	0	391	0	194	833
N.S.	1	1.00	0.86	2.59	0.00	2.38	0.00	1.18	5.08
time (sec)	N/A	0.103	0.398	0.102	0.000	1.110	0.000	0.886	25.888

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	183	746	0	0	0	0	-1
N.S.	1	1.00	0.97	3.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.651	0.124	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	249	2973	0	0	0	1356	-1
N.S.	1	1.00	0.98	11.70	0.00	0.00	0.00	5.34	-0.00
time (sec)	N/A	0.422	1.478	0.104	0.000	0.000	0.000	2.056	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	420	7119	0	4038	0	8019	-1
N.S.	1	1.00	0.99	16.79	0.00	9.52	0.00	18.91	-0.00
time (sec)	N/A	0.625	3.543	0.106	0.000	207.909	0.000	50.496	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	826	826	1036	18802	0	0	0	25778	-1
N.S.	1	1.00	1.25	22.76	0.00	0.00	0.00	31.21	-0.00
time (sec)	N/A	1.603	10.287	0.179	0.000	0.000	0.000	92.839	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1182	1154	1362	14778	0	1910	0	0	-1
N.S.	1	0.98	1.15	12.50	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	2.789	35.442	0.111	0.000	0.568	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	769	917	9543	0	1391	0	0	-1
N.S.	1	0.99	1.18	12.33	0.00	1.80	0.00	0.00	-0.00
time (sec)	N/A	1.421	29.166	0.108	0.000	0.431	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	706	706	633	5787	0	1458	0	0	-1
N.S.	1	1.00	0.90	8.20	0.00	2.07	0.00	0.00	-0.00
time (sec)	N/A	1.241	25.756	0.122	0.000	0.798	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	815	15769	0	2584	0	0	-1
N.S.	1	1.00	1.19	22.95	0.00	3.76	0.00	0.00	-0.00
time (sec)	N/A	1.264	29.814	0.120	0.000	0.373	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	964	964	1444	34620	0	4721	0	0	-1
N.S.	1	1.00	1.50	35.91	0.00	4.90	0.00	0.00	-0.00
time (sec)	N/A	2.016	34.138	0.146	0.000	0.619	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1716	1716	15719	65231	0	9152	0	0	-1
N.S.	1	1.00	9.16	38.01	0.00	5.33	0.00	0.00	-0.00
time (sec)	N/A	4.548	36.521	0.239	0.000	0.614	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1235	1235	12483	15736	0	1925	0	0	-1
N.S.	1	1.00	10.11	12.74	0.00	1.56	0.00	0.00	-0.00
time (sec)	N/A	2.833	35.877	0.111	0.000	0.525	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	766	766	922	10268	0	1388	0	0	-1
N.S.	1	1.00	1.20	13.40	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	1.283	28.927	0.109	0.000	0.500	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	562	6049	0	1036	0	0	-1
N.S.	1	1.00	1.07	11.48	0.00	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.654	26.851	0.108	0.000	0.455	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	540	551	5320	0	1331	0	0	-1
N.S.	1	1.00	1.02	9.85	0.00	2.46	0.00	0.00	-0.00
time (sec)	N/A	0.724	24.774	0.117	0.000	0.540	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	596	724	15372	0	2420	0	0	-1
N.S.	1	1.00	1.21	25.75	0.00	4.05	0.00	0.00	-0.00
time (sec)	N/A	0.923	28.495	0.131	0.000	0.397	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1034	1034	1449	36158	0	4859	0	0	-1
N.S.	1	1.00	1.40	34.97	0.00	4.70	0.00	0.00	-0.00
time (sec)	N/A	2.095	33.759	0.164	0.000	0.520	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	838	831	1000	9580	0	1387	0	0	-1
N.S.	1	0.99	1.19	11.43	0.00	1.66	0.00	0.00	-0.00
time (sec)	N/A	1.374	29.656	0.109	0.000	0.426	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	524	615	5221	0	1036	0	0	-1
N.S.	1	0.99	1.16	9.89	0.00	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.664	25.597	0.122	0.000	0.478	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	384	418	2505	0	808	0	0	-1
N.S.	1	0.99	1.08	6.47	0.00	2.09	0.00	0.00	-0.00
time (sec)	N/A	0.327	24.171	0.107	0.000	0.481	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	477	3763	0	1239	0	0	-1
N.S.	1	1.00	1.13	8.92	0.00	2.94	0.00	0.00	-0.00
time (sec)	N/A	0.445	23.714	0.121	0.000	0.336	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	699	13110	0	2343	0	0	-1
N.S.	1	1.00	1.09	20.42	0.00	3.65	0.00	0.00	-0.00
time (sec)	N/A	0.986	27.866	0.126	0.000	0.392	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1116	1116	1520	32152	0	5104	0	0	-1
N.S.	1	1.00	1.36	28.81	0.00	4.57	0.00	0.00	-0.00
time (sec)	N/A	2.163	33.372	0.214	0.000	0.630	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [20] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	37	0.162
2	A	6	6	1.00	37	0.162
3	A	5	5	1.01	35	0.143
4	A	5	5	1.00	30	0.167
5	A	6	6	1.00	37	0.162
6	A	6	6	1.00	37	0.162
7	A	5	5	1.00	37	0.135
8	A	6	5	1.00	37	0.135
9	A	5	5	1.00	37	0.135
10	A	4	4	1.02	35	0.114
11	A	4	4	1.00	30	0.133
12	A	6	6	1.00	37	0.162
13	A	6	6	1.00	37	0.162
14	A	5	5	1.00	37	0.135
15	A	4	4	1.00	31	0.129
16	A	4	4	1.00	30	0.133
17	A	7	7	1.00	33	0.212
18	A	7	7	1.00	33	0.212
19	A	6	6	1.00	33	0.182
20	A	8	7	0.99	40	0.175
21	A	7	7	1.00	40	0.175
22	A	6	6	0.99	38	0.158
23	A	6	6	1.00	33	0.182
24	A	7	7	1.00	40	0.175
25	A	7	7	1.00	40	0.175

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	5	0.99	40	0.125
27	A	7	6	0.99	40	0.150
28	A	6	6	1.00	40	0.150
29	A	5	5	1.01	38	0.132
30	A	5	5	1.00	33	0.152
31	A	7	7	1.00	40	0.175
32	A	7	7	1.00	40	0.175
33	A	5	5	0.99	40	0.125
34	A	5	5	1.74	30	0.167
35	B	5	5	2.60	29	0.172
36	B	8	8	2.45	32	0.250
37	B	8	8	2.45	32	0.250
38	A	6	6	1.55	32	0.188
39	A	7	7	1.47	32	0.219
40	A	5	5	1.22	32	0.156
41	A	8	7	1.00	36	0.194
42	A	7	6	1.00	34	0.176
43	A	7	6	1.00	29	0.207
44	A	9	8	1.01	36	0.222
45	A	9	8	1.00	36	0.222
46	A	9	9	1.00	36	0.250
47	A	7	7	1.00	36	0.194
48	A	6	6	1.00	34	0.176
49	A	6	6	1.00	29	0.207
50	A	8	8	1.00	36	0.222
51	A	8	8	1.00	36	0.222
52	A	8	8	1.00	36	0.222
53	A	6	6	1.00	36	0.167
54	A	6	6	1.00	36	0.167
55	A	5	5	0.99	34	0.147
56	A	5	5	1.00	29	0.172
57	A	7	7	1.00	36	0.194
58	A	7	7	1.00	36	0.194
59	A	5	5	1.00	36	0.139
60	A	6	5	1.00	36	0.139

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	10	7	0.98	38	0.184
62	A	9	7	0.99	38	0.184
63	A	9	7	1.00	38	0.184
64	A	9	8	1.00	38	0.210
65	A	9	7	1.00	38	0.184
66	A	10	8	1.00	38	0.210
67	A	10	7	1.00	38	0.184
68	A	9	7	1.00	38	0.184
69	A	8	7	1.00	38	0.184
70	A	8	7	1.00	38	0.184
71	A	8	7	1.00	38	0.184
72	A	9	8	1.00	38	0.210
73	A	9	7	0.99	38	0.184
74	A	8	7	0.99	38	0.184
75	A	7	6	0.99	38	0.158
76	A	7	6	1.00	38	0.158
77	A	8	7	1.00	38	0.184
78	A	9	7	1.00	38	0.184

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=415

$$\frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)x\sqrt{1-d^2x^2}}{16d^4}-\frac{\left(7d^2f(Be+2Af)-C(3d^2e^2-8\right.}{70d^4f}$$

```
[Out] -1/70*(7*d^2*f*(2*A*f+B*e)-C*(3*d^2*e^2-8*f^2))*(f*x+e)^2*(-d^2*x^2+1)^(3/2
)/d^4/f+1/42*(-7*B*f+3*C*e)*(f*x+e)^3*(-d^2*x^2+1)^(3/2)/d^2/f-1/7*C*(f*x+e
)^4*(-d^2*x^2+1)^(3/2)/d^2/f+1/840*(8*C*(3*d^4*e^4-30*d^2*e^2*f^2-8*f^4)-56
*d^2*f*(2*A*f*(6*d^2*e^2+f^2)+B*(d^2*e^3+6*e*f^2))+3*d^2*f*(-98*A*d^2*e*f^2
-14*B*d^2*e^2*f+6*C*d^2*e^3-35*B*f^3-41*C*e*f^2)*x)*(-d^2*x^2+1)^(3/2)/d^6/
f+1/16*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d^2*e^2*f+2*C*d^2*e^3+B*f^3+3*C*e*f^2
)*arcsin(d*x)/d^5+1/16*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d^2*e^2*f+2*C*d^2*e^3
+B*f^3+3*C*e*f^2)*x*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A]

time = 0.46, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1623, 1668, 847, 794, 201, 222}

$$\frac{\text{ArcSin}(dx)\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)}{16d^5}-\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2\left(7d^2f(2Af+Be)-C\left(3d^2e^2-8f^2\right)\right)}{70d^4f}+\frac{x\sqrt{1-d^2x^2}\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)}{16d^4}+\frac{\left(1-d^2x^2\right)^{3/2}\left(3d^2fx\left(-98Ad^2ef^2-14Bd^2e^2f-35Bf^3+6Cd^2e^3-41Cef^2\right)+8\left(C\left(3d^4e^4-30d^2e^2f^2-8f^4\right)-7d^2f\left(2Af\left(6d^2e^2+f^2\right)+B\left(d^2e^3+6ef^2\right)\right)\right)\right)}{840d^6f}+\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^3(3Ce-7Bf)}{42d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B
*f^3)*x*Sqrt[1 - d^2*x^2])/(16*d^4) - ((7*d^2*f*(B*e + 2*A*f) - C*(3*d^2*e^
2 - 8*f^2))*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(70*d^4*f) + ((3*C*e - 7*B*f)*
(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(42*d^2*f) - (C*(e + f*x)^4*(1 - d^2*x^2)^
(3/2))/(7*d^2*f) + ((8*(C*(3*d^4*e^4 - 30*d^2*e^2*f^2 - 8*f^4) - 7*d^2*f*(2
*A*f*(6*d^2*e^2 + f^2) + B*(d^2*e^3 + 6*e*f^2))) + 3*d^2*f*(6*C*d^2*e^3 - 1
4*B*d^2*e^2*f - 41*C*e*f^2 - 98*A*d^2*e*f^2 - 35*B*f^3)*x)*(1 - d^2*x^2)^(3
/2))/(840*d^6*f) + ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2
+ 6*A*d^2*e*f^2 + B*f^3)*ArcSin[d*x])/(16*d^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3\left(A+Bx+Cx^2\right)dx = \int (e+fx)^3\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^4\left(1-d^2x^2\right)^{3/2}}{7d^2f} - \frac{\int (e+fx)^3\left(-(4C+ \right.}{}$$

$$= \frac{(3Ce-7Bf)(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4}{7}$$

$$= -\frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2\,(1}{70d^4f}$$

$$= -\frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2\,(1}{70d^4f}$$

$$= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2\right.}{16d^4}$$

$$= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2\right.}{16d^4}$$

Mathematica [**A**]
time = 1.29, size = 387, normalized size = 0.93

$$\frac{\sqrt{1-d^2x^2}\left(14Ad^2\left(-16f^3-d^2f\left(120e^2+45efx+8f^2x^2\right)+6d^4x\left(10e^3+20e^2fx+15ef^2x^2+4f^3x^3\right)\right)+7B\left(-3d^2f^2(32e+5fx)-2d^4\left(40e^3+45e^2fx+24ef^2x^2+5f^3x^3\right)+4d^6x^2\left(20e^3+45e^2fx+36ef^2x^2+10f^3x^3\right)\right)+C\left(-128f^3-d^2f\left(672e^2+315efx+64f^2x^2\right)-6d^4x\left(35e^3+56e^2fx+35ef^2x^2+8f^3x^3\right)+12d^6x^3\left(35e^3+84e^2fx+70ef^2x^2+20f^3x^3\right)\right)\right)+105\sqrt{-d^2}\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{1680d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(14*A*d^2*(-16*f^3 - d^2*f*(120*e^2 + 45*e*f*x + 8*f^2*x
^2) + 6*d^4*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)) + 7*B*(-3*d
^2*f^2*(32*e + 5*f*x) - 2*d^4*(40*e^3 + 45*e^2*f*x + 24*e*f^2*x^2 + 5*f^3*x
^3) + 4*d^6*x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3)) + C*(-12
8*f^3 - d^2*f*(672*e^2 + 315*e*f*x + 64*f^2*x^2) - 6*d^4*x*(35*e^3 + 56*e^2
*f*x + 35*e*f^2*x^2 + 8*f^3*x^3) + 12*d^6*x^3*(35*e^3 + 84*e^2*f*x + 70*e*f
^2*x^2 + 20*f^3*x^3))) + 105*Sqrt[-d^2]*(2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^
2*e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B*f^3)*Log[-(Sqrt[-d^2]*x) + Sqrt[1 -
 d^2*x^2]])/(1680*d^6)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 959, normalized size = 2.31

method	result

risch	$-\frac{(240C\,f^3x^6d^6+280B\,d^6f^3x^5+840C\,d^6e\,f^2x^5+336A\,d^6f^3x^4+1008B\,d^6e\,f^2x^4+1008C\,d^6e^2f\,x^4+1260A\,d^6e\,f^2x^3+1260B\,d^6e^2f\,x^3}{}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-1680A\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4e^2f-64C\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^2f^3x^2-672B\sqrt{-d^2x^2+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/1680*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(-1680*A*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4
*e^2*f-64*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^2*f^3*x^2-672*B*(-d^2*x^2+1)^(1/2)
*csgn(d)*d^2*e*f^2-672*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^2*e^2*f+240*C*csgn(d)
*d^6*f^3*x^6*(-d^2*x^2+1)^(1/2)+280*B*csgn(d)*d^6*f^3*x^5*(-d^2*x^2+1)^(1/2
)+336*A*csgn(d)*d^6*f^3*x^4*(-d^2*x^2+1)^(1/2)+420*C*csgn(d)*d^6*e^3*x^3*(-
d^2*x^2+1)^(1/2)+560*B*csgn(d)*d^6*e^3*x^2*(-d^2*x^2+1)^(1/2)-48*C*(-d^2*x^
2+1)^(1/2)*csgn(d)*d^4*f^3*x^4-70*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*f^3*x^3-
112*A*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*f^3*x^2-105*B*(-d^2*x^2+1)^(1/2)*csgn(
d)*d^2*f^3*x-210*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*e^3*x+840*A*(-d^2*x^2+1)^
(1/2)*csgn(d)*d^6*e^3*x-128*C*(-d^2*x^2+1)^(1/2)*csgn(d)*f^3+840*A*arctan(c
sgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^5*e^3+210*C*arctan(csgn(d)*d*x/(-d^2*x^2+1
)^(1/2))*d^3*e^3+105*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*f^3-560*B*(
-d^2*x^2+1)^(1/2)*csgn(d)*d^4*e^3-224*A*(-d^2*x^2+1)^(1/2)*csgn(d)*d^2*f^3-
630*A*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*e*f^2*x-630*B*(-d^2*x^2+1)^(1/2)*csgn(
d)*d^4*e^2*f*x-315*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^2*e*f^2*x+630*A*arctan(cs
gn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e*f^2+630*B*arctan(csgn(d)*d*x/(-d^2*x^2+
1)^(1/2))*d^3*e^2*f+315*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*e*f^2+12
60*A*csgn(d)*d^6*e*f^2*x^3*(-d^2*x^2+1)^(1/2)+1260*B*csgn(d)*d^6*e^2*f*x^3*
(-d^2*x^2+1)^(1/2)+1680*A*csgn(d)*d^6*e^2*f*x^2*(-d^2*x^2+1)^(1/2)-210*C*(-
d^2*x^2+1)^(1/2)*csgn(d)*d^4*e*f^2*x^3-336*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4
*e*f^2*x^2-336*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*e^2*f*x^2+840*C*csgn(d)*d^6
*e*f^2*x^5*(-d^2*x^2+1)^(1/2)+1008*B*csgn(d)*d^6*e*f^2*x^4*(-d^2*x^2+1)^(1/
2)+1008*C*csgn(d)*d^6*e^2*f*x^4*(-d^2*x^2+1)^(1/2))*csgn(d)/d^6/(-d^2*x^2+1
)^(1/2)
```

Maxima [A]
time = 0.53, size = 441, normalized size = 1.06

$$-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^4}{7d^2}-\frac{4(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^2}{35d^4}-\frac{(Bf^3+3Cf^2e)(-d^2x^2+1)^{\frac{3}{2}}x^3}{6d^2}+\frac{1}{2}\sqrt{-d^2x^2+1}\,Axe^3-\frac{(Af^3+3Bf^2e+3Cfe^2)(-d^2x^2+1)^{\frac{3}{2}}x^2}{5d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Afe^2}{d^2}+\frac{A\arcsin(dx)e^3}{2d}-\frac{(-d^2x^2+1)^{\frac{3}{2}}(3Af^2e+3Bfe^2+Ce^3)x}{4d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Be^3}{3d^2}+\frac{\sqrt{-d^2x^2+1}\,(3Af^2e+3Bfe^2+Ce^3)x}{8d^2}-\frac{8(-d^2x^2+1)^{\frac{3}{2}}Cf^3}{105d^6}-\frac{(Bf^3+3Cf^2e)(-d^2x^2+1)^{\frac{3}{2}}x}{8d^4}+\frac{(3Af^2e+3Bfe^2+Ce^3)\arcsin(dx)}{8d^3}-\frac{2(Af^3+3Bf^2e+3Cfe^2)(-d^2x^2+1)^{\frac{3}{2}}}{15d^4}+\frac{(Bf^3+3Cf^2e)\sqrt{-d^2x^2+1}\,x}{16d^4}+\frac{(Bf^3+3Cf^2e)\arcsin(dx)}{16d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/7*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^4/d^2 - 4/35*(-d^2*x^2 + 1)^(3/2)*C*f^3*x
^2/d^4 - 1/6*(B*f^3 + 3*C*f^2*e)*(-d^2*x^2 + 1)^(3/2)*x^3/d^2 + 1/2*sqrt(-d
^2*x^2 + 1)*A*x*e^3 - 1/5*(A*f^3 + 3*B*f^2*e + 3*C*f*e^2)*(-d^2*x^2 + 1)^(3
/2)*x^2/d^2 - (-d^2*x^2 + 1)^(3/2)*A*f*e^2/d^2 + 1/2*A*arcsin(d*x)*e^3/d -
1/4*(-d^2*x^2 + 1)^(3/2)*(3*A*f^2*e + 3*B*f*e^2 + C*e^3)*x/d^2 - 1/3*(-d^2*
x^2 + 1)^(3/2)*B*e^3/d^2 + 1/8*sqrt(-d^2*x^2 + 1)*(3*A*f^2*e + 3*B*f*e^2 +
C*e^3)*x/d^2 - 8/105*(-d^2*x^2 + 1)^(3/2)*C*f^3/d^6 - 1/8*(B*f^3 + 3*C*f^2*
e)*(-d^2*x^2 + 1)^(3/2)*x/d^4 + 1/8*(3*A*f^2*e + 3*B*f*e^2 + C*e^3)*arcsin(
d*x)/d^3 - 2/15*(A*f^3 + 3*B*f^2*e + 3*C*f*e^2)*(-d^2*x^2 + 1)^(3/2)/d^4 +
1/16*(B*f^3 + 3*C*f^2*e)*sqrt(-d^2*x^2 + 1)*x/d^4 + 1/16*(B*f^3 + 3*C*f^2*e
)*arcsin(d*x)/d^5
```

Fricas [**A**]
time = 1.46, size = 405, normalized size = 0.98

$$\frac{(240\,Cd^6f^3x^6+280\,Bd^6f^3x^5-70\,Bd^4f^3x^3+48\,(7\,Ad^6-Cd^4)f^3x^4-105\,Bd^2f^3x-16\,(7\,Ad^4+4\,Cd^2)f^3x^2-32\,(7\,Ad^2+4\,C)f^3+70\,(6\,Cd^6x^3+8\,Bd^6x^2-8\,Bd^4+3\,(4\,Ad^6-Cd^4)x)e^3+42\,(24\,Cd^6fx^4+30\,Bd^6fx^3-15\,Bd^4fx+8\,(5\,Ad^6-Cd^4)fx^2-8\,(5\,Ad^4+2\,Cd^2)f)e^2+21\,(40\,Cd^6f^2x^5+48\,Bd^6f^2x^4-16\,Bd^4f^2x^2+10\,(6\,Ad^6-Cd^4)f^2x^3-32\,Bd^2f^2-15\,(2\,Ad^4+Cd^2)f^2x)e)\sqrt{dx+1}\,\sqrt{-dx+1}-210\,(6\,Bd^3fe^2+Bdf^3+3\,(2\,Ad^3+Cd)f^2e+2\,(4\,Ad^5+Cd^3)e^3)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{1680\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/1680*((240*C*d^6*f^3*x^6 + 280*B*d^6*f^3*x^5 - 70*B*d^4*f^3*x^3 + 48*(7*A
*d^6 - C*d^4)*f^3*x^4 - 105*B*d^2*f^3*x - 16*(7*A*d^4 + 4*C*d^2)*f^3*x^2 -
32*(7*A*d^2 + 4*C)*f^3 + 70*(6*C*d^6*x^3 + 8*B*d^6*x^2 - 8*B*d^4 + 3*(4*A*d
^6 - C*d^4)*x)*e^3 + 42*(24*C*d^6*f*x^4 + 30*B*d^6*f*x^3 - 15*B*d^4*f*x + 8
*(5*A*d^6 - C*d^4)*f*x^2 - 8*(5*A*d^4 + 2*C*d^2)*f)*e^2 + 21*(40*C*d^6*f^2*
x^5 + 48*B*d^6*f^2*x^4 - 16*B*d^4*f^2*x^2 + 10*(6*A*d^6 - C*d^4)*f^2*x^3 -
32*B*d^2*f^2 - 15*(2*A*d^4 + C*d^2)*f^2*x)*e)*sqrt(d*x + 1)*sqrt(-d*x + 1)
- 210*(6*B*d^3*f*e^2 + B*d*f^3 + 3*(2*A*d^3 + C*d)*f^2*e + 2*(4*A*d^5 + C*d
^3)*e^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^6
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1533 vs. 2(390) = 780.
time = 1.81, size = 1533, normalized size = 3.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] 1/1680*(840*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*
sqrt(d*x + 1)))*A*d^5*e^3 + 1680*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1
/2*sqrt(2)*sqrt(d*x + 1)))*A*d^5*e^3 + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqr
t(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^4*f*e^
2 + 2520*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqr
t(d*x + 1)))*A*d^4*f*e^2 + 210*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1)
- 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*
A*d^3*f^2*e + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*f^2*e + 14*(((2*(3*(4*d*x - 1
7)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d
*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f^3 + 70*(((2*(3*d*x
- 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arc
sin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f^3 + 280*(((2*d*x - 5)*(d*x + 1) + 9
)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^4
*e^3 + 840*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*B*d^4*e^3 + 210*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1)
- 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*
B*d^3*f*e^2 + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^3*f*e^2 + 42*(((2*(3*(4*d*x - 1
7)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d
*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*f^2*e + 210*(((2*(3*d
*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*
arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*f^2*e + 7*(((2*((4*(5*d*x - 26)*(d
*x + 1) + 321)*(d*x + 1) - 451)*(d*x + 1) + 745)*(d*x + 1) - 405)*sqrt(d*x
+ 1)*sqrt(-d*x + 1) - 150*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f^3 + 14*(
((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt
(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f^3 +
70*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*
x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^3*e^3 + 280*(((2*d*x - 5
)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d
*x + 1)))*C*d^3*e^3 + 42*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) -
295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*
sqrt(d*x + 1)))*C*d^2*f*e^2 + 210*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x +
1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)
))*C*d^2*f*e^2 + 21*(((2*((4*(5*d*x - 26)*(d*x + 1) + 321)*(d*x + 1) - 451)
*(d*x + 1) + 745)*(d*x + 1) - 405)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 150*arcsi
n(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*f^2*e + 42*(((2*(3*(4*d*x - 17)*(d*x + 1)
 + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90
*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*f^2*e + (((2*((4*(5*(6*d*x - 37)*(d
*x + 1) + 661)*(d*x + 1) - 4551)*(d*x + 1) + 4781)*(d*x + 1) - 6335)*(d*x +
```

```
 1) + 2835)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 1050*arcsin(1/2*sqrt(2)*sqrt(d*x
 + 1)))*C*f^3 + 7*(((2*((4*(5*d*x - 26)*(d*x + 1) + 321)*(d*x + 1) - 451)*(
d*x + 1) + 745)*(d*x + 1) - 405)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 150*arcsin(
1/2*sqrt(2)*sqrt(d*x + 1)))*C*f^3)/d^6
```

Mupad [B]
time = 47.79, size = 2500, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] - ((((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^
(1/2) - 1)^6 + (((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^22
)/((d*x + 1)^(1/2) - 1)^22 - (((20480*C*f^3)/3 - 448*C*d^2*e^2*f)*((1 - d*x
)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((20480*C*f^3)/3 - 448*C*d^2*e^2
*f)*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (((458752*C*f^3)/1
5 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^
10 + (((458752*C*f^3)/15 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^18)
/((d*x + 1)^(1/2) - 1)^18 - (((1011712*C*f^3)/15 - (13184*C*d^2*e^2*f)/5)*(
(1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 - (((1011712*C*f^3)/15 -
(13184*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 +
 (((9293824*C*f^3)/105 - (15104*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^14)/(
(d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)^(1/2) - 1)^3*((29*C*d^3*e^3)/2 - (41*
C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^25*((29*C*d
^3*e^3)/2 - (41*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^25 - (((1 - d*x)^(1/2)
 - 1)^5*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^5 + (((1
 - d*x)^(1/2) - 1)^23*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/((d*x + 1)^(1/2)
 - 1)^23 - (((1 - d*x)^(1/2) - 1)^7*(209*C*d^3*e^3 + (8755*C*d*e*f^2)/2))/(
(d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^21*(209*C*d^3*e^3 + (8755*C
*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^21 + (((1 - d*x)^(1/2) - 1)^11*((1767*C
*d^3*e^3)/2 - (8267*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^11 - (((1 - d*x)^(
1/2) - 1)^17*((1767*C*d^3*e^3)/2 - (8267*C*d*e*f^2)/4))/((d*x + 1)^(1/2) -
1)^17 + (((1 - d*x)^(1/2) - 1)^13*(646*C*d^3*e^3 - 17527*C*d*e*f^2))/((d*x
+ 1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^15*(646*C*d^3*e^3 - 17527*C*d*e
*f^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^9*((165*C*d^3*e^3)
/2 + (42095*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^9 - (((1 - d*x)^(1/2) - 1)
^19*((165*C*d^3*e^3)/2 + (42095*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^19 - (
d*(2*C*d^2*e^3 + 3*C*e*f^2)*((1 - d*x)^(1/2) - 1))/(4*((d*x + 1)^(1/2) - 1)
) + (d*(2*C*d^2*e^3 + 3*C*e*f^2)*((1 - d*x)^(1/2) - 1)^27)/(4*((d*x + 1)^(1
/2) - 1)^27) + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) -
 1)^4 + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24
)/(d^6 + (14*d^6*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (91*d^6
*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (364*d^6*((1 - d*x)^(1/
```

```
2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (1001*d^6*((1 - d*x)^(1/2) - 1)^8)/((d
*x + 1)^(1/2) - 1)^8 + (2002*d^6*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2)
 - 1)^10 + (3003*d^6*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (
3432*d^6*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (3003*d^6*((1
 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (2002*d^6*((1 - d*x)^(1/2
) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (1001*d^6*((1 - d*x)^(1/2) - 1)^20)/(
(d*x + 1)^(1/2) - 1)^20 + (364*d^6*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/
2) - 1)^22 + (91*d^6*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24 + (
14*d^6*((1 - d*x)^(1/2) - 1)^26)/((d*x + 1)^(1/2) - 1)^26 + (d^6*((1 - d*x)
^(1/2) - 1)^28)/((d*x + 1)^(1/2) - 1)^28) - ((((4928*A*f^3)/3 + 512*A*d^2*e
^2*f)*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((1408*A*f^3)/3 -
 32*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 - (((14
08*A*f^3)/3 - 32*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1
)^6 + (((4928*A*f^3)/3 + 512*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^12)/((d*x +
 1)^(1/2) - 1)^12 - (((11008*A*f^3)/5 - 912*A*d^2*e^2*f)*((1 - d*x)^(1/2) -
 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)*(2*A*d^3*e^3 - (3
*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^19*(2*A*d^3*
e^3 - (3*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^19 - (((1 - d*x)^(1/2) - 1)^3
*(2*A*d^3*e^3 - (99*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^3 + (((1 - d*x)^(1
/2) - 1)^17*(2*A*d^3*e^3 - (99*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^17 - ((
(1 - d*x)^(1/2) - 1)^5*(40*A*d^3*e^3 + 306*A*d*e*f^2))/((d*x + 1)^(1/2) - 1
)^5 + (((1 - d*x)^(1/2) - 1)^15*(40*A*d^3*e^3 + 306*A*d*e*f^2))/((d*x + 1)^
(1/2) - 1)^15 - (((1 - d*x)^(1/2) - 1)^7*(88*A*d^3*e^3 - 306*A*d*e*f^2))/((
d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^13*(88*A*d^3*e^3 - 306*A*d*e
*f^2))/((d*x + 1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^9*(52*A*d^3*e^3 -
663*A*d*e*f^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1)^11*(52*A*d
^3*e^3 - 663*A*d*e*f^2))/((d*x + 1)^(1/2) - 1)^11 + (64*A*f^3*((1 - d*x)^(1
/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (64*A*f^3*((1 - d*x)^(1/2) - 1)^16)/(
(d*x + 1)^(1/2) - 1)^16 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^(1/2
) - 1)^18)/(d^4 + (10*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
+ (45*d^4*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^4*((1 -
 d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (210*d^4*((1 - d*x)^(1/2) - 1
)^8)/((d*x + 1)^(1/2) - 1)^8 + (252*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1
)^(1/2) - 1)^10 + (210*d^4*((1 - d*x)^(1/2) - 1...
```

3.2 $\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^2\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=286

$$\frac{\left(C\left(2d^2e^2+f^2\right)+2d^2\left(2Bef+A\left(4d^2e^2+f^2\right)\right)\right)x\sqrt{1-d^2x^2}}{16d^4}+\frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f}-\frac{C(e+$$

```
[Out] 1/10*(-2*B*f+C*e)*(f*x+e)^2*(-d^2*x^2+1)^(3/2)/d^2/f-1/6*C*(f*x+e)^3*(-d^2*
x^2+1)^(3/2)/d^2/f+1/120*(8*C*(d^2*e^3-4*e*f^2)-16*f*(5*A*d^2*e*f+B*(d^2*e^
2+f^2))-3*f*(5*(2*A*d^2+C)*f^2-2*d^2*e*(-2*B*f+C*e))*x)*(-d^2*x^2+1)^(3/2)/
d^4/f+1/16*(C*(2*d^2*e^2+f^2)+2*d^2*(2*B*e*f+A*(4*d^2*e^2+f^2)))*arcsin(d*x
)/d^5+1/16*(C*(2*d^2*e^2+f^2)+2*d^2*(2*B*e*f+A*(4*d^2*e^2+f^2)))*x*(-d^2*x^
2+1)^(1/2)/d^4
```

Rubi [A]

time = 0.35, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1623, 1668, 847, 794, 201, 222}

$$\frac{\text{ArcSin}(dx)\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)}{16d^5}+\frac{x\sqrt{1-d^2x^2}\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)}{16d^4}+\frac{\left(1-d^2x^2\right)^{3/2}\left(8\left(C\left(d^2e^3-4ef^2\right)-2f\left(5Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-3fx\left(5f^2\left(2Ad^2+C\right)-2d^2e(Ce-2Bf)\right)\right)}{120d^4f}+\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2(Ce-2Bf)}{10d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] ((C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*x*Sqrt[1 - d
^2*x^2])/(16*d^4) + ((C*e - 2*B*f)*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(10*d^2
*f) - (C*(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(6*d^2*f) + ((8*(C*(d^2*e^3 - 4*e
*f^2) - 2*f*(5*A*d^2*e*f + B*(d^2*e^2 + f^2))) - 3*f*(5*(C + 2*A*d^2)*f^2 -
 2*d^2*e*(C*e - 2*B*f))*x)*(1 - d^2*x^2)^(3/2))/(120*d^4*f) + ((C*(2*d^2*e^
2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*ArcSin[d*x])/(16*d^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \int (e+fx)^2\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f} - \frac{\int (e+fx)^2\,(-3(C+$$

$$= \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\,(}{6d}$$

$$= \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\,(}{6d}$$

$$= \frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\,x\sqrt{}}{16d^4}$$

$$= \frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\,x\sqrt{}}{16d^4}$$

Mathematica [A]
time = 0.92, size = 279, normalized size = 0.98

$$\frac{d^2\sqrt{1-d^2x^2}\,(10Ad^2(12d^2e^2x+16ef(-1+d^2x^2)+3f^2x(-1+2d^2x^2))+4B(-8f^2-d^2(20e^2+15efx+4f^2x^2)+2d^4x^2(10e^2+15efx+6f^2x^2))+C(30d^2e^2x(-1+2d^2x^2)+32ef(-2-d^2x^2+3d^4x^4)+5f^2x(-3-2d^2x^2+8d^4x^4)))+15\sqrt{-d^2}\,(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{240d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] (d^2*Sqrt[1 - d^2*x^2]*(10*A*d^2*(12*d^2*e^2*x + 16*e*f*(-1 + d^2*x^2) + 3*
f^2*x*(-1 + 2*d^2*x^2)) + 4*B*(-8*f^2 - d^2*(20*e^2 + 15*e*f*x + 4*f^2*x^2)
 + 2*d^4*x^2*(10*e^2 + 15*e*f*x + 6*f^2*x^2)) + C*(30*d^2*e^2*x*(-1 + 2*d^2
*x^2) + 32*e*f*(-2 - d^2*x^2 + 3*d^4*x^4) + 5*f^2*x*(-3 - 2*d^2*x^2 + 8*d^4
*x^4))) + 15*Sqrt[-d^2]*(C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^
2 + f^2)))*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(240*d^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 652, normalized size = 2.28

method	result
risch	$-\frac{(40C\,f^2x^5d^4+48B\,d^4f^2x^4+96C\,d^4ef\,x^4+60A\,d^4f^2x^3+120B\,d^4ef\,x^3+60C\,d^4e^2x^3+160A\,d^4ef\,x^2+80B\,d^4e^2x^2+120A\,d^4e^2x-10}{24}$

default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-60B\,\mathrm{csgn}(d)d^3\sqrt{-d^2x^2+1}\,efx-80B\,\mathrm{csgn}(d)d^3\sqrt{-d^2x^2+1}\,e^2-32B\,\mathrm{csgn}(d)d\sqrt{-d^2x^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/240*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(-60*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e*
f*x-80*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e^2-32*B*csgn(d)*d*(-d^2*x^2+1)^(1/
2)*f^2-10*C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*f^2*x^3-16*B*csgn(d)*d^3*(-d^2*x
^2+1)^(1/2)*f^2*x^2+60*C*csgn(d)*d^5*e^2*x^3*(-d^2*x^2+1)^(1/2)+80*B*csgn(d
)*d^5*e^2*x^2*(-d^2*x^2+1)^(1/2)-160*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e*f-6
4*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*e*f+40*C*csgn(d)*d^5*f^2*x^5*(-d^2*x^2+1)^
(1/2)+48*B*csgn(d)*d^5*f^2*x^4*(-d^2*x^2+1)^(1/2)+60*A*csgn(d)*d^5*f^2*x^3*
(-d^2*x^2+1)^(1/2)-15*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*f^2*x-30*C*csgn(d)*d^3
*(-d^2*x^2+1)^(1/2)*e^2*x-30*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*f^2*x+120*A*c
sgn(d)*d^5*(-d^2*x^2+1)^(1/2)*e^2*x+120*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(
1/2))*d^4*e^2+30*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*f^2+30*C*arct
an(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2+60*B*arctan(csgn(d)*d*x/(-d^2*x^
2+1)^(1/2))*d^2*e*f+15*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*f^2+160*A*c
sgn(d)*d^5*e*f*x^2*(-d^2*x^2+1)^(1/2)-32*C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e
*f*x^2+96*C*csgn(d)*d^5*e*f*x^4*(-d^2*x^2+1)^(1/2)+120*B*csgn(d)*d^5*e*f*x^
3*(-d^2*x^2+1)^(1/2))*csgn(d)/(-d^2*x^2+1)^(1/2)/d^5
```

Maxima [A]
time = 0.56, size = 307, normalized size = 1.07

$$-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^2x^3}{6\,d^2}+\frac{1}{2}\sqrt{-d^2x^2+1}\,Axe^2-\frac{(-d^2x^2+1)^{\frac{3}{2}}(Bf^2+2\,Cfe)x^2}{5\,d^2}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}Afe}{3\,d^2}+\frac{A\arcsin(dx)\,e^2}{2\,d}-\frac{(-d^2x^2+1)^{\frac{3}{2}}(Af^2+2\,Bfe+Ce^2)x}{4\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^2x}{8\,d^4}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Be^2}{3\,d^2}+\frac{\sqrt{-d^2x^2+1}\,(Af^2+2\,Bfe+Ce^2)x}{8\,d^2}+\frac{\sqrt{-d^2x^2+1}\,Cf^2x}{16\,d^4}+\frac{(Af^2+2\,Bfe+Ce^2)\arcsin(dx)}{8\,d^3}+\frac{Cf^2\arcsin(dx)}{16\,d^5}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}(Bf^2+2\,Cfe)}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/6*(-d^2*x^2 + 1)^(3/2)*C*f^2*x^3/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*x*e^2 -
1/5*(-d^2*x^2 + 1)^(3/2)*(B*f^2 + 2*C*f*e)*x^2/d^2 - 2/3*(-d^2*x^2 + 1)^(3/
2)*A*f*e/d^2 + 1/2*A*arcsin(d*x)*e^2/d - 1/4*(-d^2*x^2 + 1)^(3/2)*(A*f^2 +
2*B*f*e + C*e^2)*x/d^2 - 1/8*(-d^2*x^2 + 1)^(3/2)*C*f^2*x/d^4 - 1/3*(-d^2*x
^2 + 1)^(3/2)*B*e^2/d^2 + 1/8*sqrt(-d^2*x^2 + 1)*(A*f^2 + 2*B*f*e + C*e^2)*
x/d^2 + 1/16*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/8*(A*f^2 + 2*B*f*e + C*e^2)
*arcsin(d*x)/d^3 + 1/16*C*f^2*arcsin(d*x)/d^5 - 2/15*(-d^2*x^2 + 1)^(3/2)*(
B*f^2 + 2*C*f*e)/d^4
```

Fricas [A]
time = 1.18, size = 280, normalized size = 0.98

$$\frac{\left(40\,Cd^5f^2x^5+48\,Bd^5f^2x^4-16\,Bd^3f^2x^2+10\,(6\,Ad^5-Cd^3)f^2x^3-32\,Bdf^2-15\,(2\,Ad^3+Cd)f^2x+10\,(6\,Cd^5x^3+8\,Bd^5x^2-8\,Bd^3+3\,(4\,Ad^5-Cd^3)x)e^2+4\,(24\,Cd^5fx^4+30\,Bd^5fx^3-15\,Bd^3fx+8\,(5\,Ad^5-Cd^3)fx^2-8\,(5\,Ad^3+2\,Cd)f)e\right)\sqrt{dx+1}\,\sqrt{-dx+1}-30\,(4\,Bd^2fe+(2\,Ad^2+C)f^2+2\,(4\,Ad^4+Cd^2)e^2)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{240\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/240*((40*C*d^5*f^2*x^5 + 48*B*d^5*f^2*x^4 - 16*B*d^3*f^2*x^2 + 10*(6*A*d^
5 - C*d^3)*f^2*x^3 - 32*B*d*f^2 - 15*(2*A*d^3 + C*d)*f^2*x + 10*(6*C*d^5*x^
3 + 8*B*d^5*x^2 - 8*B*d^3 + 3*(4*A*d^5 - C*d^3)*x)*e^2 + 4*(24*C*d^5*f*x^4
+ 30*B*d^5*f*x^3 - 15*B*d^3*f*x + 8*(5*A*d^5 - C*d^3)*f*x^2 - 8*(5*A*d^3 +
2*C*d)*f)*e)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(4*B*d^2*f*e + (2*A*d^2 + C)
*f^2 + 2*(4*A*d^4 + C*d^2)*e^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(
d*x)))/d^5
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(271) = 542.
time = 1.78, size = 1059, normalized size = 3.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] 1/240*(120*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*A*d^4*e^2 + 240*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2
*sqrt(2)*sqrt(d*x + 1)))*A*d^4*e^2 + 80*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d
*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*f*e + 2
40*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x
+ 1)))*A*d^3*f*e + 10*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqr
t(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f^2
 + 40*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(
1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f^2 + 40*(((2*d*x - 5)*(d*x + 1) + 9)*sqr
t(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^3*e^2
+ 120*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d
*x + 1)))*B*d^3*e^2 + 20*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*
sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*
```

```
f*e + 80*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*f*e + 2*(((2*(3*(4*d*x - 17)*(d*x + 1)
 + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90
*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f^2 + 10*(((2*(3*d*x - 10)*(d*x + 1
) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2
)*sqrt(d*x + 1)))*B*d*f^2 + 10*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1)
- 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*
C*d^2*e^2 + 40*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) +
6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^2*e^2 + 4*(((2*(3*(4*d*x - 17)*(d*
x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1
) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*f*e + 20*(((2*(3*d*x - 10)*(d
*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*
sqrt(2)*sqrt(d*x + 1)))*C*d*f*e + (((2*((4*(5*d*x - 26)*(d*x + 1) + 321)*(d
*x + 1) - 451)*(d*x + 1) + 745)*(d*x + 1) - 405)*sqrt(d*x + 1)*sqrt(-d*x +
1) - 150*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*f^2 + 2*(((2*(3*(4*d*x - 17)*
(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x
+ 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*f^2)/d^5
```

Mupad [B]
time = 36.03, size = 2500, normalized size = 8.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^8*((4928*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x + 1
)^(1/2) - 1)^8 - (((1 - d*x)^(1/2) - 1)^14*((1408*B*f^2)/3 - (32*B*d^2*e^2)
/3))/((d*x + 1)^(1/2) - 1)^14 - (((1 - d*x)^(1/2) - 1)^6*((1408*B*f^2)/3 -
(32*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^12*((49
28*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^12 - (((1 - d*x)^(1
/2) - 1)^10*((11008*B*f^2)/5 - 304*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^10 + (
64*B*f^2*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (64*B*f^2*((1 -
 d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (8*B*d^2*e^2*((1 - d*x)^(1/
2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^18)
/((d*x + 1)^(1/2) - 1)^18 + (33*B*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)
^(1/2) - 1)^3 - (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)
^5 + (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (442*B
*d*e*f*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(1/2) - 1)^9 - (442*B*d*e*f*((1
- d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 - (204*B*d*e*f*((1 - d*x)^(1
/2) - 1)^13)/((d*x + 1)^(1/2) - 1)^13 + (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^
15)/((d*x + 1)^(1/2) - 1)^15 - (33*B*d*e*f*((1 - d*x)^(1/2) - 1)^17)/((d*x
+ 1)^(1/2) - 1)^17 + (B*d*e*f*((1 - d*x)^(1/2) - 1)^19)/((d*x + 1)^(1/2) -
1)^19 - (B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^4 + (10*d
^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (45*d^4*((1 - d*x)^(1
```

/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^4*((1 - d*x)^(1/2) - 1)^6)/((d
*x + 1)^(1/2) - 1)^6 + (210*d^4*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) -
 1)^8 + (252*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (210*
d^4*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (120*d^4*((1 - d*x
)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (45*d^4*((1 - d*x)^(1/2) - 1)^1
6)/((d*x + 1)^(1/2) - 1)^16 + (10*d^4*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^
(1/2) - 1)^18 + (d^4*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20) -
((((1 - d*x)^(1/2) - 1)^15*((A*f^2)/2 - 2*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)
^15 - (((1 - d*x)^(1/2) - 1)*((A*f^2)/2 - 2*A*d^2*e^2))/((d*x + 1)^(1/2) -
1) + (((1 - d*x)^(1/2) - 1)^3*((35*A*f^2)/2 - 6*A*d^2*e^2))/((d*x + 1)^(1/2
) - 1)^3 - (((1 - d*x)^(1/2) - 1)^13*((35*A*f^2)/2 - 6*A*d^2*e^2))/((d*x +
1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^5*((273*A*f^2)/2 + 30*A*d^2*e^2))
/((d*x + 1)^(1/2) - 1)^5 + (((1 - d*x)^(1/2) - 1)^11*((273*A*f^2)/2 + 30*A*
d^2*e^2))/((d*x + 1)^(1/2) - 1)^11 + (((1 - d*x)^(1/2) - 1)^7*((715*A*f^2)/
2 - 22*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^9*((715
*A*f^2)/2 - 22*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)^9 + (16*A*d*e*f*((1 - d*x)
^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^
4)/((d*x + 1)^(1/2) - 1)^4 + (208*A*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x
 + 1)^(1/2) - 1)^6) + (704*A*d*e*f*((1 - d*x)^(1/2) - 1)^8)/(3*((d*x + 1)^(
1/2) - 1)^8) + (208*A*d*e*f*((1 - d*x)^(1/2) - 1)^10)/(3*((d*x + 1)^(1/2) -
 1)^10) - (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 +
(16*A*d*e*f*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^3 + (8*d
^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^3*((1 - d*x)^(1
/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^3*((1 - d*x)^(1/2) - 1)^6)/((d*
x + 1)^(1/2) - 1)^6 + (70*d^3*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1
)^8 + (56*d^3*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^3*
((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^3*((1 - d*x)^(1/2
) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^3*((1 - d*x)^(1/2) - 1)^16)/((d*x
+ 1)^(1/2) - 1)^16) - ((((1 - d*x)^(1/2) - 1)^23*((C*f^2)/4 + (C*d^2*e^2)/2
))/((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2) - 1)*((C*f^2)/4 + (C*d^2*e^
2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^3*((35*C*f^2)/12 - (3
1*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^3 + (((1 - d*x)^(1/2) - 1)^21*((35*C
*f^2)/12 - (31*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^21 + (((1 - d*x)^(1/2)
- 1)^5*((757*C*f^2)/4 - (139*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^5 - (((1
- d*x)^(1/2) - 1)^19*((757*C*f^2)/4 - (139*C*d^2*e^2)/2))/((d*x + 1)^(1/2)
- 1)^19 - (((1 - d*x)^(1/2) - 1)^7*((7339*C*f^2)/4 + (171*C*d^2*e^2)/2))/((
d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^17*((7339*C*f^2)/4 + (171*C*
d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^17 - (((1 - d*x)^(1/2) - 1)^11*((25661*C
*f^2)/2 - 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^11 + (((1 - d*x)^(1/2) - 1)
^13*((25661*C*f^2)/2 - 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*
x)^(1/2) - 1)^9*((41929*C*f^2)/6 + 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^9
- (((1 - d*x)^(1/2) - 1)^15*((41929*C*f^2)/6 + 323*C*d^2*e^2))/((d*x + 1)^(
1/2) - 1)^15 + (128*C*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^
4 - (2048*C*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x + 1)^(1/2) - 1)^6) + (1
536*C*d*e*f*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (6144*C*d*e*

```
f*((1 - d*x)^(1/2) - 1)^10)/(5*((d*x + 1)^(1/2) - 1)^10) - (33536*C*d*e*f*(
(1 - d*x)^(1/2) - 1)^12)/(15*((d*x + 1)^(1/2) - 1)^12) + (6144*C*d*e*f*((1
- d*x)^(1/2) - 1)^14)/(5*((d*x + 1)^(1/2) - 1)^...
```

3.3 $\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=168

$$\frac{(Ce+4Ad^2e+Bf)\,x\sqrt{1-d^2x^2}}{8d^2}-\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f}-\frac{(4(5d^2f(Be+Af)-C(3d^2e^2-2f^2))-3d^2}{60d^4f}$$

```
[Out] -1/5*C*(f*x+e)^2*(-d^2*x^2+1)^(3/2)/d^2/f-1/60*(20*d^2*f*(A*f+B*e)-4*C*(3*d
^2*e^2-2*f^2)-3*d^2*f*(-5*B*f+3*C*e)*x)*(-d^2*x^2+1)^(3/2)/d^4/f+1/8*(4*A*d
^2*e+B*f+C*e)*arcsin(d*x)/d^3+1/8*(4*A*d^2*e+B*f+C*e)*x*(-d^2*x^2+1)^(1/2)/
d^2
```

Rubi [A]
time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1623, 1668, 794, 201, 222}

$$\frac{\text{ArcSin}(dx)\,(4Ad^2e+Bf+Ce)}{8d^3}+\frac{x\sqrt{1-d^2x^2}\,(4Ad^2e+Bf+Ce)}{8d^2}-\frac{\left(1-d^2x^2\right)^{3/2}\left(4\left(5d^2f(Af+Be)-\frac{1}{4}C(12d^2e^2-8f^2)\right)-3d^2fx(3Ce-5Bf)\right)}{60d^4f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] ((C*e + 4*A*d^2*e + B*f)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (C*(e + f*x)^2*(1 -
 d^2*x^2)^(3/2))/(5*d^2*f) - ((4*(5*d^2*f*(B*e + A*f) - (C*(12*d^2*e^2 - 8*
f^2))/4) - 3*d^2*f*(3*C*e - 5*B*f)*x)*(1 - d^2*x^2)^(3/2))/(60*d^4*f) + ((C
*e + 4*A*d^2*e + B*f)*ArcSin[d*x])/(8*d^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)\left(A+Bx+Cx^2\right)\,dx &= \int (e+fx)\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx \\
&= -\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f}-\frac{\int (e+fx)\left(-(2C+5A\right.}{} \\
&= -\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f}-\frac{\left(4\left(5d^2f(Be+Af)-\frac{1}{4}\right.\right.}{} \\
&= \frac{\left(Ce+4Ad^2e+Bf\right)x\sqrt{1-d^2x^2}}{8d^2}-\frac{C(e+fx)^2\left(1-\right.}{5d^2f} \\
&= \frac{\left(Ce+4Ad^2e+Bf\right)x\sqrt{1-d^2x^2}}{8d^2}-\frac{C(e+fx)^2\left(1-\right.}{5d^2f}
\end{aligned}
$$

Mathematica [A]
time = 0.61, size = 173, normalized size = 1.03

$$\frac{\sqrt{1-d^2x^2}\left(60Ad^4ex+40Ad^2f(-1+d^2x^2)+15Cd^2ex(-1+2d^2x^2)+5Bd^2(-8e-3fx+8d^2ex^2+6d^2fx^3)+8Cf(-2-d^2x^2+3d^4x^4)\right)+15\sqrt{-d^2}\left(Ce+4Ad^2e+Bf\right)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{120d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(60*A*d^4*e*x + 40*A*d^2*f*(-1 + d^2*x^2) + 15*C*d^2*e*x
*(-1 + 2*d^2*x^2) + 5*B*d^2*(-8*e - 3*f*x + 8*d^2*e*x^2 + 6*d^2*f*x^3) + 8*
C*f*(-2 - d^2*x^2 + 3*d^4*x^4)) + 15*Sqrt[-d^2]*(C*e + 4*A*d^2*e + B*f)*Log
[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(120*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 377, normalized size = 2.24

method	result
risch	$-\frac{(24fC\,x^4d^4+30B\,d^4f\,x^3+30C\,d^4e\,x^3+40A\,d^4f\,x^2+40B\,d^4e\,x^2+60A\,d^4ex-8C\,d^2f\,x^2-15B\,d^2fx-15C\,d^2ex-40A\,d^2f-40B\,d^2e}{120d^4\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(24C\,\mathrm{csgn}(d)d^4f\,x^4\sqrt{-d^2x^2+1}+30B\,\mathrm{csgn}(d)d^4f\,x^3\sqrt{-d^2x^2+1}+30C\,\mathrm{csgn}(d)d^4e\,x^3\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/120*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(24*C*csgn(d)*d^4*f*x^4*(-d^2*x^2+1)^(1/
2)+30*B*csgn(d)*d^4*f*x^3*(-d^2*x^2+1)^(1/2)+30*C*csgn(d)*d^4*e*x^3*(-d^2*x
^2+1)^(1/2)+40*A*csgn(d)*d^4*f*x^2*(-d^2*x^2+1)^(1/2)+40*B*csgn(d)*d^4*e*x^
2*(-d^2*x^2+1)^(1/2)+60*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^4*e*x-8*C*csgn(d)*(-
d^2*x^2+1)^(1/2)*d^2*f*x^2-15*B*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f*x-15*C*csg
n(d)*(-d^2*x^2+1)^(1/2)*d^2*e*x-40*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f+60*A*
arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e-40*B*csgn(d)*(-d^2*x^2+1)^(1/2
)*d^2*e+15*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*f-16*C*csgn(d)*(-d^2*
x^2+1)^(1/2)*f+15*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*e)*csgn(d)/d^4
/(-d^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.48, size = 180, normalized size = 1.07

$$-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cfx^2}{5\,d^2}+\frac{1}{2}\,\sqrt{-d^2x^2+1}\,Axe+\frac{A\arcsin(dx)\,e}{2\,d}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Af}{3\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}(Bf+Ce)x}{4\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Be}{3\,d^2}+\frac{\sqrt{-d^2x^2+1}\,(Bf+Ce)x}{8\,d^2}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}Cf}{15\,d^4}+\frac{(Bf+Ce)\arcsin(dx)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/5*(-d^2*x^2 + 1)^(3/2)*C*f*x^2/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*x*e + 1/2*
A*arcsin(d*x)*e/d - 1/3*(-d^2*x^2 + 1)^(3/2)*A*f/d^2 - 1/4*(-d^2*x^2 + 1)^(
```

```
3/2)*(B*f + C*e)*x/d^2 - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e/d^2 + 1/8*sqrt(-d^2*x
^2 + 1)*(B*f + C*e)*x/d^2 - 2/15*(-d^2*x^2 + 1)^(3/2)*C*f/d^4 + 1/8*(B*f +
C*e)*arcsin(d*x)/d^3
```

Fricas [A]
time = 0.88, size = 174, normalized size = 1.04

$$\frac{(24\,Cd^4fx^4+30\,Bd^4fx^3-15\,Bd^2fx+8\,(5\,Ad^4-Cd^2)fx^2-8\,(5\,Ad^2+2\,C)f+5\,(6\,Cd^4x^3+8\,Bd^4x^2-8\,Bd^2+3\,(4\,Ad^4-Cd^2)x)e)\sqrt{dx+1}\,\sqrt{-dx+1}-30\,(Bdf+(4\,Ad^3+Cd)e)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{120\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/120*((24*C*d^4*f*x^4 + 30*B*d^4*f*x^3 - 15*B*d^2*f*x + 8*(5*A*d^4 - C*d^2
)*f*x^2 - 8*(5*A*d^2 + 2*C)*f + 5*(6*C*d^4*x^3 + 8*B*d^4*x^2 - 8*B*d^2 + 3*
(4*A*d^4 - C*d^2)*x)*e)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(B*d*f + (4*A*d^3
 + C*d)*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(157) = 314.
time = 1.49, size = 637, normalized size = 3.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] 1/120*(60*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sq
rt(d*x + 1)))*A*d^3*e + 120*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sq
rt(2)*sqrt(d*x + 1)))*A*d^3*e + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x +
1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f + 60*(sqrt
(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A
*d^2*f + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*a
rcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e + 60*(sqrt(d*x + 1)*(d*x - 2)*sqr
t(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e + 5*(((2*(3*d*x
```

```
- 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arc
sin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqr
t(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f + 5*
(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x +
 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*e + 20*(((2*d*x - 5)*(d*x +
 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))
)*C*d*e + (((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1)
+ 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))
*C*f + 5*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sq
rt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*f)/d^4
```

Mupad [B]
time = 12.06, size = 736, normalized size = 4.38

$$\frac{\frac{Bf\left(\sqrt{1-dx}-1\right)}{2\left(\sqrt{dx+1}-1\right)}-\frac{35Bf\left(\sqrt{1-dx}-1\right)^3}{2\left(\sqrt{dx+1}-1\right)^3}+\frac{273Bf\left(\sqrt{1-dx}-1\right)^5}{2\left(\sqrt{dx+1}-1\right)^5}-\frac{715Bf\left(\sqrt{1-dx}-1\right)^7}{2\left(\sqrt{dx+1}-1\right)^7}+\frac{715Bf\left(\sqrt{1-dx}-1\right)^9}{2\left(\sqrt{dx+1}-1\right)^9}-\frac{273Bf\left(\sqrt{1-dx}-1\right)^{11}}{2\left(\sqrt{dx+1}-1\right)^{11}}+\frac{35Bf\left(\sqrt{1-dx}-1\right)^{13}}{2\left(\sqrt{dx+1}-1\right)^{13}}-\frac{Bf\left(\sqrt{1-dx}-1\right)^{15}}{2\left(\sqrt{dx+1}-1\right)^{15}}}{d^3\left(\frac{\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+1\right)^8}-\sqrt{1-dx}\left(\frac{2Cf\sqrt{dx+1}}{15d^4}-\frac{Cfx^4\sqrt{dx+1}}{5}+\frac{Cfx^2\sqrt{dx+1}}{15d^2}\right)+\frac{\frac{Ce\left(\sqrt{1-dx}-1\right)}{2\left(\sqrt{dx+1}-1\right)}-\frac{35Ce\left(\sqrt{1-dx}-1\right)^3}{2\left(\sqrt{dx+1}-1\right)^3}+\frac{273Ce\left(\sqrt{1-dx}-1\right)^5}{2\left(\sqrt{dx+1}-1\right)^5}-\frac{715Ce\left(\sqrt{1-dx}-1\right)^7}{2\left(\sqrt{dx+1}-1\right)^7}+\frac{715Ce\left(\sqrt{1-dx}-1\right)^9}{2\left(\sqrt{dx+1}-1\right)^9}-\frac{273Ce\left(\sqrt{1-dx}-1\right)^{11}}{2\left(\sqrt{dx+1}-1\right)^{11}}+\frac{35Ce\left(\sqrt{1-dx}-1\right)^{13}}{2\left(\sqrt{dx+1}-1\right)^{13}}-\frac{Ce\left(\sqrt{1-dx}-1\right)^{15}}{2\left(\sqrt{dx+1}-1\right)^{15}}}{d^3\left(\frac{\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+1\right)^8}-\frac{Bf\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{2d^3}-\frac{Ce\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{2d^3}+\frac{Aex\sqrt{1-dx}\sqrt{dx+1}}{2}-\frac{A\sqrt{d}\,e\ln\left(\sqrt{-d}\sqrt{1-dx}\sqrt{dx+1}-d^{3/2}x\right)}{2(-d)^{3/2}}+\frac{Af(d^2x^2-1)\sqrt{1-dx}\sqrt{dx+1}}{3d^2}+\frac{Be(d^2x^2-1)\sqrt{1-dx}\sqrt{dx+1}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((B*f*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) - (35*B*f*((1 - d*x)
^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*B*f*((1 - d*x)^(1/2) - 1)
^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*B*f*((1 - d*x)^(1/2) - 1)^7)/(2*((d*
x + 1)^(1/2) - 1)^7) + (715*B*f*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^(1/2
) - 1)^9) - (273*B*f*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1)^11)
 + (35*B*f*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) - (B*f*((
1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15))/(d^3*(((1 - d*x)^(1/2
) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (1 - d*x)^(1/2)*((2*C*f*(d*x + 1
)^(1/2))/(15*d^4) - (C*f*x^4*(d*x + 1)^(1/2))/5 + (C*f*x^2*(d*x + 1)^(1/2))
/(15*d^2)) + ((C*e*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) - (35*C
*e*((1 - d*x)^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*C*e*((1 - d*
x)^(1/2) - 1)^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*C*e*((1 - d*x)^(1/2) -
1)^7)/(2*((d*x + 1)^(1/2) - 1)^7) + (715*C*e*((1 - d*x)^(1/2) - 1)^9)/(2*((
d*x + 1)^(1/2) - 1)^9) - (273*C*e*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(
1/2) - 1)^11) + (35*C*e*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^
13) - (C*e*((1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15))/(d^3*(((
1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (B*f*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (C*e*atan(((1 - d*x)^(1/2) -
 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) + (A*e*x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2
))/2 - (A*d^(1/2)*e*log((-d)^(1/2)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) - d^(3/2
)*x))/(2*(-d)^(3/2)) + (A*f*(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/
(3*d^2) + (B*e*(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/(3*d^2)
```

3.4 $\int \sqrt{1-dx}\,\sqrt{1+dx}\,(A+Bx+Cx^2)\,dx$

Optimal. Leaf size=95

$$\frac{(C+4Ad^2)\,x\sqrt{1-d^2x^2}}{8d^2}-\frac{B(1-d^2x^2)^{3/2}}{3d^2}-\frac{Cx(1-d^2x^2)^{3/2}}{4d^2}+\frac{(C+4Ad^2)\sin^{-1}(dx)}{8d^3}$$

```
[Out] -1/3*B*(-d^2*x^2+1)^(3/2)/d^2-1/4*C*x*(-d^2*x^2+1)^(3/2)/d^2+1/8*(4*A*d^2+C
)*arcsin(d*x)/d^3+1/8*(4*A*d^2+C)*x*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {913, 1829, 655, 201, 222}

$$\frac{(4Ad^2+C)\,\text{ArcSin}(dx)}{8d^3}+\frac{x\sqrt{1-d^2x^2}\,(4Ad^2+C)}{8d^2}-\frac{B(1-d^2x^2)^{3/2}}{3d^2}-\frac{Cx(1-d^2x^2)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((C + 4*A*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (B*(1 - d^2*x^2)^(3/2))/(3*d^
2) - (C*x*(1 - d^2*x^2)^(3/2))/(4*d^2) + ((C + 4*A*d^2)*ArcSin[d*x])/(8*d^3
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
```

```
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)dx = \int \left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{Cx(1-d^2x^2)^{3/2}}{4d^2} - \frac{\int (-C-4Ad^2-4Bd^2x)\sqrt{1-d^2x^2}\,dx}{4d^2}$$

$$= -\frac{B(1-d^2x^2)^{3/2}}{3d^2} - \frac{Cx(1-d^2x^2)^{3/2}}{4d^2} - \frac{(-C-4Ad^2)\int\sqrt{1-}}{4d^2}$$

$$= \frac{(C+4Ad^2)\,x\sqrt{1-d^2x^2}}{8d^2} - \frac{B(1-d^2x^2)^{3/2}}{3d^2} - \frac{Cx(1-d^2x^2)^{3}}{4d^2}$$

$$= \frac{(C+4Ad^2)\,x\sqrt{1-d^2x^2}}{8d^2} - \frac{B(1-d^2x^2)^{3/2}}{3d^2} - \frac{Cx(1-d^2x^2)^{3}}{4d^2}$$

Mathematica [A]
time = 0.28, size = 107, normalized size = 1.13

$$\frac{\sqrt{1-d^2x^2}\,(-8B-3Cx+12Ad^2x+8Bd^2x^2+6Cd^2x^3)}{24d^2} + \frac{\sqrt{-d^2}\,(C+4Ad^2)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(-8*B - 3*C*x + 12*A*d^2*x + 8*B*d^2*x^2 + 6*C*d^2*x^3))
/(24*d^2) + (Sqrt[-d^2]*(C + 4*A*d^2)*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^
2]])/(8*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 185, normalized size = 1.95

method	result

risch	$-\frac{(6C d^2x^3+8B d^2x^2+12A d^2x-3Cx-8B)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{24d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}} - \frac{\left(-\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)A}{2\sqrt{d^2}} \right.}{}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(6C\,\mathrm{csgn}(d)d^3x^3\sqrt{-d^2x^2+1}+8B\,\mathrm{csgn}(d)d^3x^2\sqrt{-d^2x^2+1}+12A\,\mathrm{csgn}(d)d^3\sqrt{-d^2x^2+\right.}{24}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*C*csgn(d)*d^3*x^3*(-d^2*x^2+1)^(1/2)+8*B*csgn(d)*d^3*x^2*(-d^2*x^2+1)^(1/2)+12*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x-3*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x+12*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2-8*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d+3*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)))*csgn(d)/(-d^2*x^2+1)^(1/2)/d^3

Maxima [A]
time = 0.54, size = 93, normalized size = 0.98

$$\frac{1}{2}\sqrt{-d^2x^2+1}\,Ax-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cx}{4\,d^2}+\frac{A\arcsin(dx)}{2\,d}-\frac{(-d^2x^2+1)^{\frac{3}{2}}B}{3\,d^2}+\frac{\sqrt{-d^2x^2+1}\,Cx}{8\,d^2}+\frac{C\arcsin(dx)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-d^2*x^2 + 1)*A*x - 1/4*(-d^2*x^2 + 1)^(3/2)*C*x/d^2 + 1/2*A*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B/d^2 + 1/8*sqrt(-d^2*x^2 + 1)*C*x/d^2 + 1/8*C*arcsin(d*x)/d^3

Fricas [A]
time = 1.17, size = 95, normalized size = 1.00

$$\frac{(6\,Cd^3x^3+8\,Bd^3x^2-8\,Bd+3\,(4\,Ad^3-Cd)x)\sqrt{dx+1}\,\sqrt{-dx+1}-6\,(4\,Ad^2+C)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/24*((6*C*d^3*x^3 + 8*B*d^3*x^2 - 8*B*d + 3*(4*A*d^3 - C*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(4*A*d^2 + C)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(83) = 166$.
time = 223.06, size = 1137, normalized size = 11.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Piecewise(((2*A*(-Piecewise((sqrt(-d*x + 1)*sqrt(d*x + 1)/2 + asin(sqrt(2)*
sqrt(d*x + 1)/2), (sqrt(d*x + 1) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2))))
+ Piecewise((d*x*sqrt(-d*x + 1)*sqrt(d*x + 1)/4 + asin(sqrt(2)*sqrt(d*x + 1
)/2)/2, (sqrt(d*x + 1) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2))))) + 2*A*Pie
cewise((sqrt(-d*x + 1)*sqrt(d*x + 1)/2 + asin(sqrt(2)*sqrt(d*x + 1)/2), (sq
rt(d*x + 1) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2)))) + 2*B*(-Piecewise((sq
rt(-d*x + 1)*sqrt(d*x + 1)/2 + asin(sqrt(2)*sqrt(d*x + 1)/2), (sqrt(d*x + 1
) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2)))) + Piecewise((d*x*sqrt(-d*x + 1)
*sqrt(d*x + 1)/4 + asin(sqrt(2)*sqrt(d*x + 1)/2)/2, (sqrt(d*x + 1) < sqrt(2
)) & (sqrt(d*x + 1) > -sqrt(2)))))/d + 2*B*(Piecewise((sqrt(-d*x + 1)*sqrt(
d*x + 1)/2 + asin(sqrt(2)*sqrt(d*x + 1)/2), (sqrt(d*x + 1) < sqrt(2)) & (sq
rt(d*x + 1) > -sqrt(2)))) - 2*Piecewise((d*x*sqrt(-d*x + 1)*sqrt(d*x + 1)/4
 + asin(sqrt(2)*sqrt(d*x + 1)/2)/2, (sqrt(d*x + 1) < sqrt(2)) & (sqrt(d*x +
 1) > -sqrt(2)))) + Piecewise((d*x*sqrt(-d*x + 1)*sqrt(d*x + 1)/4 - (-d*x +
 1)**(3/2)*(d*x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(d*x + 1)/2)/2, (sqrt(d*x
+ 1) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2)))))/d + 2*C*(Piecewise((sqrt(-d
*x + 1)*sqrt(d*x + 1)/2 + asin(sqrt(2)*sqrt(d*x + 1)/2), (sqrt(d*x + 1) < s
qrt(2)) & (sqrt(d*x + 1) > -sqrt(2)))) - 2*Piecewise((d*x*sqrt(-d*x + 1)*sq
rt(d*x + 1)/4 + asin(sqrt(2)*sqrt(d*x + 1)/2)/2, (sqrt(d*x + 1) < sqrt(2))
& (sqrt(d*x + 1) > -sqrt(2)))) + Piecewise((d*x*sqrt(-d*x + 1)*sqrt(d*x + 1
)/4 - (-d*x + 1)**(3/2)*(d*x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(d*x + 1)/2)/
2, (sqrt(d*x + 1) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2)))))/d**2 + 2*C*(-P
iecewise((sqrt(-d*x + 1)*sqrt(d*x + 1)/2 + asin(sqrt(2)*sqrt(d*x + 1)/2), (
sqrt(d*x + 1) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2)))) + 3*Piecewise((d*x*
sqrt(-d*x + 1)*sqrt(d*x + 1)/4 + asin(sqrt(2)*sqrt(d*x + 1)/2)/2, (sqrt(d*x
 + 1) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2)))) - 3*Piecewise((d*x*sqrt(-d*
x + 1)*sqrt(d*x + 1)/4 - (-d*x + 1)**(3/2)*(d*x + 1)**(3/2)/6 + asin(sqrt(2
)*sqrt(d*x + 1)/2)/2, (sqrt(d*x + 1) < sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2)
))) + Piecewise((d*x*sqrt(-d*x + 1)*sqrt(d*x + 1)/4 - (-d*x + 1)**(3/2)*(d*
x + 1)**(3/2)/3 - sqrt(-d*x + 1)*sqrt(d*x + 1)*(-5*d*x - 2*(d*x + 1)**3 + 6
*(d*x + 1)**2 - 4)/16 + 5*asin(sqrt(2)*sqrt(d*x + 1)/2)/8, (sqrt(d*x + 1) <
 sqrt(2)) & (sqrt(d*x + 1) > -sqrt(2)))))/d**2)/d, Ne(d, 0)), (A*x + B*x**2
/2 + C*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(81) = 162$.

time = 0.84, size = 284, normalized size = 2.99

$$\frac{12\left(\sqrt{dx+1}\,(dx-2)\sqrt{-dx+1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Ad^2+24\left(\sqrt{dx+1}\sqrt{-dx+1}+2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Ad^2+4\left(((2dx-5)(dx+1)+9)\sqrt{dx+1}\sqrt{-dx+1}+6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Bd+12\left(\sqrt{dx+1}\,(dx-2)\sqrt{-dx+1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Bd+\left(((2(3dx-10)(dx+1)+43)(dx+1)-39)\sqrt{dx+1}\sqrt{-dx+1}-18\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)C+4\left(((2dx-5)(dx+1)+9)\sqrt{dx+1}\sqrt{-dx+1}+6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)C}{24d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(12*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqr
t(d*x + 1)))*A*d^2 + 24*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2
)*sqrt(d*x + 1)))*A*d^2 + 4*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt
(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d + 12*(sqrt(d*x + 1)*(
d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d + (((2*(
3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) -
18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C + 4*(((2*d*x - 5)*(d*x + 1) + 9)*sq
rt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C)/d^3
```

Mupad [B]
time = 7.21, size = 361, normalized size = 3.80

$$\frac{Ax\sqrt{1-dx}\,\sqrt{dx+1}}{2}-\frac{\frac{35C\left(\sqrt{1-dx}-1\right)^3}{2\left(\sqrt{dx+1}-1\right)^3}-\frac{273C\left(\sqrt{1-dx}-1\right)^5}{2\left(\sqrt{dx+1}-1\right)^5}+\frac{715C\left(\sqrt{1-dx}-1\right)^7}{2\left(\sqrt{dx+1}-1\right)^7}-\frac{715C\left(\sqrt{1-dx}-1\right)^9}{2\left(\sqrt{dx+1}-1\right)^9}+\frac{273C\left(\sqrt{1-dx}-1\right)^{11}}{2\left(\sqrt{dx+1}-1\right)^{11}}-\frac{35C\left(\sqrt{1-dx}-1\right)^{13}}{2\left(\sqrt{dx+1}-1\right)^{13}}+\frac{C\left(\sqrt{1-dx}-1\right)^{15}}{2\left(\sqrt{dx+1}-1\right)^{15}}-\frac{C\left(\sqrt{1-dx}-1\right)}{2\left(\sqrt{dx+1}-1\right)}}{d^3\left(\frac{\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+1\right)^8}-\frac{C\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{2d^3}-\frac{A\sqrt{d}\,\ln\left(\sqrt{-d}\,\sqrt{1-dx}\,\sqrt{dx+1}-d^{3/2}x\right)}{2(-d)^{3/2}}+\frac{B\left(d^2x^2-1\right)\sqrt{1-dx}\,\sqrt{dx+1}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] (A*x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/2 - ((35*C*((1 - d*x)^(1/2) - 1)^3)/(
2*((d*x + 1)^(1/2) - 1)^3) - (273*C*((1 - d*x)^(1/2) - 1)^5)/(2*((d*x + 1)^
(1/2) - 1)^5) + (715*C*((1 - d*x)^(1/2) - 1)^7)/(2*((d*x + 1)^(1/2) - 1)^7)
 - (715*C*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^(1/2) - 1)^9) + (273*C*((1
 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1)^11) - (35*C*((1 - d*x)^(1/2
) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) + (C*((1 - d*x)^(1/2) - 1)^15)/(2*(
(d*x + 1)^(1/2) - 1)^15) - (C*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) -
1)))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (C*ata
n(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (A*d^(1/2)*log((-
d)^(1/2)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) - d^(3/2)*x))/(2*(-d)^(3/2)) + (B*
(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/(3*d^2)
```

3.5 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)}\,dx$

Optimal. Leaf size=122

$$-\frac{C\sqrt{1-d^2x^2}}{d^2f}-\frac{(Ce-Bf)\sin^{-1}(dx)}{df^2}+\frac{(Ce^2-Bef+Af^2)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\,\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

```
[Out] -(-B*f+C*e)*arcsin(d*x)/d/f^2+(A*f^2-B*e*f+C*e^2)*arctan((d^2*e*x+f)/(d^2*e
^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(1/2)-C*(-d^2*x^2+1)^(1
/2)/d^2/f
```

Rubi [A]
time = 0.20, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1623, 1668, 858, 222, 739, 210}

$$\frac{(Af^2-Bef+Ce^2)\,\mathrm{ArcTan}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\mathrm{ArcSin}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{\int \frac{-Ad^2f^2+d^2f(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{(Ce^2-Bef+Af^2)\ldots}{\ldots}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} - \frac{(Ce^2-Bef+Af^2)\,\mathrm{Subst}\Big(\ldots}{\ldots}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} + \frac{(Ce^2-Bef+Af^2)\tan^{-1}\Big(\ldots}{f^2\sqrt{d^2e\ldots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 559 vs. 2(122) = 244.
time = 2.21, size = 559, normalized size = 4.58

$$\frac{-\frac{Cf^3\sqrt{1-d^2x^2}}{d^2}+\frac{(Ce^2+f(-Be+Af))\sqrt{2d^2e^2-f^2-2de\sqrt{d^2e^2-f^2}}\left(d^2e^2-f^2+de\sqrt{d^2e^2-f^2}\right)\tan^{-1}\left(\frac{f\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)}{\sqrt{2d^2e^2-f^2-2de\sqrt{d^2e^2-f^2}}}\right)}{(de-f)(de+f)}-\frac{(Ce^2+f(-Be+Af))\left(-d^2e^2+f^2+de\sqrt{d^2e^2-f^2}\right)\sqrt{2d^2e^2-f^2+2de\sqrt{d^2e^2-f^2}}\tan^{-1}\left(\frac{f\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)}{\sqrt{2d^2e^2-f^2+2de\sqrt{d^2e^2-f^2}}}\right)}{(de-f)(de+f)}+\frac{df^2\sqrt{-d^2e^2+f^2}\,(Ce^2+f(-Be+Af))\tan^{-1}\left(\frac{-\sqrt{-d^2}\,f^2x\sqrt{1-d^2x^2}+d^2(e^2-f^2x^2)}{de\sqrt{-d^2e^2+f^2}}\right)}{\sqrt{-d^2}\,(de-f)(de+f)}+\frac{f^2(Ce-Bf)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{\sqrt{-d^2}}}{f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] (-((C*f^3*Sqrt[1 - d^2*x^2])/d^2) + ((C*e^2 + f*(-(B*e) + A*f))*Sqrt[2*d^2*
e^2 - f^2 - 2*d*e*Sqrt[d^2*e^2 - f^2]]*(d^2*e^2 - f^2 + d*e*Sqrt[d^2*e^2 -
f^2])*ArcTan[(f*(Sqrt[-d^2]*x - Sqrt[1 - d^2*x^2]))/Sqrt[2*d^2*e^2 - f^2 -
2*d*e*Sqrt[d^2*e^2 - f^2]]])/((d*e - f)*(d*e + f)) - ((C*e^2 + f*(-(B*e) +
A*f))*(-(d^2*e^2) + f^2 + d*e*Sqrt[d^2*e^2 - f^2])*Sqrt[2*d^2*e^2 - f^2 + 2
*d*e*Sqrt[d^2*e^2 - f^2]]*ArcTan[(f*(Sqrt[-d^2]*x - Sqrt[1 - d^2*x^2]))/Sqr
t[2*d^2*e^2 - f^2 + 2*d*e*Sqrt[d^2*e^2 - f^2]]])/((d*e - f)*(d*e + f)) + (d
*f^2*Sqrt[-(d^2*e^2) + f^2]*(C*e^2 + f*(-(B*e) + A*f))*ArcTan[(-(Sqrt[-d^2]
*f^2*x*Sqrt[1 - d^2*x^2]) + d^2*(e^2 - f^2*x^2))/(d*e*Sqrt[-(d^2*e^2) + f^2
])])/(Sqrt[-d^2]*(d*e - f)*(d*e + f)) + (f^2*(C*e - B*f)*Log[-(Sqrt[-d^2]*x
) + Sqrt[1 - d^2*x^2]])/Sqrt[-d^2])/f^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 373, normalized size = 3.06

method	result
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2f^2+B\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}}{fx+e}\right.\right.}{}$
risch	$\frac{C\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{f\,d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)B}{f\sqrt{d^2}}-\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)Ce}{f^2\sqrt{d^2}}-\ldots\ln\left(\ldots\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*d^2*f^2+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f-C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1
/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2+B*arctan(csgn(d)*d*x/(
-d^2*x^2+1)^(1/2))*d*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*csgn(d)*f^2*(-d^2*x^2
+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*d*e*f*(-(d^2*e^2-f^2)/f^2)^(1/2))*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)/(
-(d^2*e^2-f^2)/f^2)^(1/2)/f^3/(-d^2*x^2+1)^(1/2)/d^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-%e*d>0)', see `assume?` for more
details
```

Fricas [A]
time = 8.24, size = 457, normalized size = 3.75

$$\left[-\frac{(Ad^2f^2-Bd^2fe+Cd^2e^2)\sqrt{-d^2e^2+f^2}\log\left(\frac{d^2fxe-\left(d^2e^2-f^2+\sqrt{-d^2e^2+f^2}\,f\right)\sqrt{dx+1}\,\sqrt{-dx+1}+f^2-(d^2xe+f)\sqrt{-d^2e^2+f^2}}{fx+e}\right)+(Cd^2fe^2-Cf^3)\sqrt{dx+1}\,\sqrt{-dx+1}+2\,(Bd^3fe^2-Bdf^3-Cd^3e^3+Cdf^2e)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{d^4f^2e^2-d^2f^4}, -\frac{2\,(Ad^2f^2-Bd^2fe+Cd^2e^2)\sqrt{d^2e^2-f^2}\arctan\left(-\frac{\sqrt{d^2e^2-f^2}\left(fx-\sqrt{dx+1}\,\sqrt{-dx+1}\,e+e\right)}{d^2xe^2-f^2x}\right)+(Cd^2fe^2-Cf^3)\sqrt{dx+1}\,\sqrt{-dx+1}+2\,(Bd^3fe^2-Bdf^3-Cd^3e^3+Cdf^2e)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{d^4f^2e^2-d^2f^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-((A*d^2*f^2 - B*d^2*f*e + C*d^2*e^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*f*x*e
- (d^2*e^2 - f^2 + sqrt(-d^2*e^2 + f^2)*f)*sqrt(d*x + 1)*sqrt(-d*x + 1) + f
^2 - (d^2*x*e + f)*sqrt(-d^2*e^2 + f^2))/(f*x + e)) + (C*d^2*f*e^2 - C*f^3)
*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(B*d^3*f*e^2 - B*d*f^3 - C*d^3*e^3 + C*d*
f^2*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*f^2*e^2 - d^2
*f^4), -(2*(A*d^2*f^2 - B*d^2*f*e + C*d^2*e^2)*sqrt(d^2*e^2 - f^2)*arctan(-
sqrt(d^2*e^2 - f^2)*(f*x - sqrt(d*x + 1)*sqrt(-d*x + 1)*e + e)/(d^2*x*e^2 -
 f^2*x)) + (C*d^2*f*e^2 - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(B*d^3*f*
e^2 - B*d*f^3 - C*d^3*e^3 + C*d*f^2*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1)
 - 1)/(d*x)))/(d^4*f^2*e^2 - d^2*f^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{-dx+1}\,\sqrt{dx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Undef/Unsigned Inf encountered in lim
itLimit: Max order reached or unable to make series expansion Error: Bad Ar
gument
```

Mupad [B]
time = 25.80, size = 2500, normalized size = 20.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^2
)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) -
1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^
2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))/(((d*x
 + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5
*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(
1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^
3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^3*
f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1/2) - 1)^2*(d
^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*((1 - d
*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1i - d^2*e^2*1i
- (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e^2*((1 -
 d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*e)^(1/2)*(f - d*e
)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))/((d*x
```

+ 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d*e)^(1/2)*(f - d*e)
^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*
e^2*atan(((C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (409
6*((1 - d*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x
+ 1)^(1/2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1
)^(1/2) - 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4)
+ (16384*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d
x + 1)^(1/2) - 1)) + (4096((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 -
144*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*
e^2*((4096*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d
f^4((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^
6))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))
/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^
8 - 9*d^6*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*
(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(
1/2)*(f - d*e)^(1/2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (C*e^2*(
(4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2)
- 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2
) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C
e^2((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 +
9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d
^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*
C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^
2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1
/2) - 1)^2) - (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (163
84*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6)
)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))
/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/
2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))/((131072*C^4*e^7)/(d*f^4) +
(C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d
x)^(1/2) - 1)^2(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/
2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) -
1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384
*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e
^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*e^2*((409
6*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2)
- 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096
*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*
x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^

```
4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d
*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6
*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)
^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))...
```

3.6 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}\,(e+fx)^2}\,dx$

Optimal. Leaf size=163

$$\frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)}+\frac{C\sin^{-1}(dx)}{df^2}-\frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\,\sqrt{1-d^2x^2}}\right)}{f^2\,(d^2e^2-f^2)^{3/2}}$$

```
[Out] C*arcsin(d*x)/d/f^2-(-A*d^2*e*f^2+C*d^2*e^3+B*f^3-2*C*e*f^2)*arctan((d^2*e*
x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(3/2)+(A*f^2
-B*e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)
```

Rubi [A]
time = 0.21, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1623, 1665, 858, 222, 739, 210}

$$-\frac{\text{ArcTan}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2)}{f^2\,(d^2e^2-f^2)^{3/2}}+\frac{\sqrt{1-d^2x^2}\,(Af^2-Bef+Ce^2)}{f\,(d^2e^2-f^2)\,(e+fx)}+\frac{C\,\text{ArcSin}(dx)}{df^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}\,(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)} + \frac{\int \frac{Ce+Ad^2e-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)} + \frac{C\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\Big(2Ce+Ad\ldots}{\ldots}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} - \frac{\Big(2Ce+Ad^2e-\frac{C}{}\ldots}{\ldots}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} + \frac{\Big(2Ce+Ad^2e-\frac{C}{}\ldots}{\ldots}$$

Mathematica [A]

time = 10.29, size = 211, normalized size = 1.29

$$\frac{-\frac{f(Ce^2+f(-Be+Af))\sqrt{1-d^2x^2}}{(-d^2e^2+f^2)(e+fx)}+\frac{C\sin^{-1}(dx)}{d}+\frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\log(e+fx)}{(-d^2e^2+f^2)^{3/2}}-\frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\log\left(f+d^2ex+\sqrt{-d^2e^2+f^2}\,\sqrt{1-d^2x^2}\right)}{(-d^2e^2+f^2)^{3/2}}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] (-((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((-(d^2*e^2) + f^2)*(e
+ f*x))) + (C*ArcSin[d*x])/d + ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^
3)*Log[e + f*x])/(-(d^2*e^2) + f^2)^(3/2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2
*e*f^2 + B*f^3)*Log[f + d^2*e*x + Sqrt[-(d^2*e^2) + f^2]*Sqrt[1 - d^2*x^2]]
)/(-(d^2*e^2) + f^2)^(3/2))/f^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 899, normalized size = 5.52

method	result
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3ef^3x+C\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f}{f^2}}}{fx+e}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*d^3*e*f^3*x+C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^
2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^3*f*x-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn(d)*
ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d
^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2*f^2*x*(-(d^2*e^2-f^
2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)
+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)
/(f*x+e))*d*f^4*x-B*csgn(d)*d*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)
^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)*f+f)/(f*x+e))*d*e*f^3*x+C*csgn(d)*d*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e
^2-f^2)/f^2)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^3*f*(-(d^
2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2
-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+
1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-C*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*f^4*x*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)*(-d*x+1)
```

```
^(1/2)*(d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/d/(d*e+f)/(f*x+e)/(-(d^2*e^
2-f^2)/f^2)^(1/2)/f^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-%e*d>0)', see `assume?` for more
details
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(155) = 310$.
time = 32.09, size = 969, normalized size = 5.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-(A*d*f^6*x - C*d^3*f*e^5 + (B*d*f^4*x*e + C*d^3*f*x*e^4 + C*d^3*e^5 - (A*
d^3 + 2*C*d)*f^2*e^3 + (B*d*f^3 - (A*d^3 + 2*C*d)*f^3*x)*e^2)*sqrt(-d^2*e^2
 + f^2)*log((d^2*f*x*e - (d^2*e^2 - f^2 - sqrt(-d^2*e^2 + f^2)*f)*sqrt(d*x
+ 1)*sqrt(-d*x + 1) + f^2 + (d^2*x*e + f)*sqrt(-d^2*e^2 + f^2))/(f*x + e))
+ (A*d*f^5*e + B*d^3*f^2*e^4 - B*d*f^4*e^2 - C*d^3*f*e^5 - (A*d^3 - C*d)*f^
3*e^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^4*f*x*e^5 - 2*C*d^2*f^3*x*e^3
+ C*f^5*x*e + C*d^4*e^6 - 2*C*d^2*f^2*e^4 + C*f^4*e^2)*arctan((sqrt(d*x + 1
)*sqrt(-d*x + 1) - 1)/(d*x)) - (C*d^3*f^2*x - B*d^3*f^2)*e^4 + (B*d^3*f^3*x
 - (A*d^3 - C*d)*f^3)*e^3 - (B*d*f^4 + (A*d^3 - C*d)*f^4*x)*e^2 - (B*d*f^5*
x - A*d*f^5)*e)/(d^5*f^3*x*e^5 - 2*d^3*f^5*x*e^3 + d*f^7*x*e + d^5*f^2*e^6
- 2*d^3*f^4*e^4 + d*f^6*e^2), -(A*d*f^6*x - C*d^3*f*e^5 - 2*(B*d*f^4*x*e +
C*d^3*f*x*e^4 + C*d^3*e^5 - (A*d^3 + 2*C*d)*f^2*e^3 + (B*d*f^3 - (A*d^3 + 2
*C*d)*f^3*x)*e^2)*sqrt(d^2*e^2 - f^2)*arctan(-sqrt(d^2*e^2 - f^2)*(f*x - sq
rt(d*x + 1)*sqrt(-d*x + 1)*e + e)/(d^2*x*e^2 - f^2*x)) + (A*d*f^5*e + B*d^3
*f^2*e^4 - B*d*f^4*e^2 - C*d^3*f*e^5 - (A*d^3 - C*d)*f^3*e^3)*sqrt(d*x + 1)
*sqrt(-d*x + 1) + 2*(C*d^4*f*x*e^5 - 2*C*d^2*f^3*x*e^3 + C*f^5*x*e + C*d^4*
e^6 - 2*C*d^2*f^2*e^4 + C*f^4*e^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1
)/(d*x)) - (C*d^3*f^2*x - B*d^3*f^2)*e^4 + (B*d^3*f^3*x - (A*d^3 - C*d)*f^3
)*e^3 - (B*d*f^4 + (A*d^3 - C*d)*f^4*x)*e^2 - (B*d*f^5*x - A*d*f^5)*e)/(d^5
```

```
*f^3*x*e^5 - 2*d^3*f^5*x*e^3 + d*f^7*x*e + d^5*f^2*e^6 - 2*d^3*f^4*e^4 + d*
f^6*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{-dx+1}\,\sqrt{dx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Undef/Unsigned Inf encountered in lim
itLimit: Max order reached or unable to make series expansion Error: Bad Ar
gument
```

Mupad [B]
time = 52.17, size = 2500, normalized size = 15.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*
1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1
)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (
2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d
```

```
*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 - d*x)^(1/2)
- 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (A*d^5*e^5*at
an(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)
^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3
*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x
+ 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (A*d^5*e^5*atan(
((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2)
)*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 - (4*A*f^2*((1 - d*x)
^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (A
*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/
2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (
A*d^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 -
(A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
 - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
 d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3
*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 +
 (A*d^4*e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d
^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d^4*
```

```
e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 + (8*A*d*e
*f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
) - 1)^2)/(d^3*e^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - d*e^2*f^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2) - (4*e*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d
*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (4*e*f^3*((1 - d*x)^(1/2) - 1)^3*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (2*d^3*e^4*((1 - d*x)^(
1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((...
```

3.7 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3}\,dx$

Optimal. Leaf size=248

$$\frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f\,(d^2e^2-f^2)\,(e+fx)^2}-\frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f\,(d^2e^2-f^2)^2\,(e+fx)}+\frac{(C(d^2e^2+}{}$$

```
[Out] 1/2*(C*(d^2*e^2+2*f^2)-d^2*(3*B*e*f-A*(2*d^2*e^2+f^2)))*arctan((d^2*e*x+f)/
(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/(d^2*e^2-f^2)^(5/2)+1/2*(A*f^2-B*e*
f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)^2-1/2*(-3*A*d^2*e*f^2+B
*d^2*e^2*f+C*d^2*e^3+2*B*f^3-4*C*e*f^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)^
2/(f*x+e)
```

Rubi [A]
time = 0.23, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1623, 1665, 821, 739, 210}

$$\frac{\operatorname{ArcTan}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)\left(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2))\right)}{2\,(d^2e^2-f^2)^{5/2}}+\frac{\sqrt{1-d^2x^2}\,(Af^2-Bef+Ce^2)}{2f\,(d^2e^2-f^2)\,(e+fx)^2}-\frac{\sqrt{1-d^2x^2}\,(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{2f\,(d^2e^2-f^2)^2\,(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} + \frac{\int \frac{2(Ce+Ad^2e-Bf)+\left(Bd^2e+\frac{Cd^2e^2}{f}-2Cf-A \ldots\right.}{(e+fx)^2\sqrt{1-d^2x^2}}}{2(d^2e^2-f^2)}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2 \ldots}{2f(d^2e^2-f^2)^2 (\ldots}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2 \ldots}{2f(d^2e^2-f^2)^2 (\ldots}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2 \ldots}{2f(d^2e^2-f^2)^2 (\ldots}$$

Mathematica [A]

time = 10.33, size = 273, normalized size = 1.10

$$\frac{1}{2}\left(-\frac{\sqrt{1-d^2x^2}\,(Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)-Ad^2ef(4e+3fx)+Ce(-3ef+d^2e^2x-4f^2x))}{(-d^2e^2+f^2)^2\,(e+fx)^2}+\frac{(C(d^2e^2+2f^2)+d^2(-3Bef+A(2d^2e^2+f^2)))\log(e+fx)}{(-d^2e^2+f^2)^{5/2}}-\frac{(C(d^2e^2+2f^2)+d^2(-3Bef+A(2d^2e^2+f^2)))\log\left(f+d^2ex+\sqrt{-d^2e^2+f^2}\,\sqrt{1-d^2x^2}\right)}{(-d^2e^2+f^2)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] (-((Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x) + B*f^2*(e + 2*f*x) -
A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x - 4*f^2*x)))/((-(d^2*e^2)
 + f^2)^2*(e + f*x)^2)) + ((C*(d^2*e^2 + 2*f^2) + d^2*(-3*B*e*f + A*(2*d^2*
e^2 + f^2)))*Log[e + f*x])/(-(d^2*e^2) + f^2)^(5/2) - ((C*(d^2*e^2 + 2*f^2)
 + d^2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*Log[f + d^2*e*x + Sqrt[-(d^2*e^2)
+ f^2]*Sqrt[1 - d^2*x^2]])/(-(d^2*e^2) + f^2)^(5/2))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 1449, normalized size = 5.84

method	result
default	$-\frac{\left(2B\,f^4x\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}+Be\,f^3\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}-3B\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-}}{fx+e}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f
*x+e))*d^2*f^4*x^2+A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^
(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2
*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f+4*C*ln(2*(d^2*e*x+(-d^2*x^2+1)
^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e*f^3*x+2*B*f^4*x*(-d^2*x^2
+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+B*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-
f^2)/f^2)^(1/2)+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(
1/2)*f+f)/(f*x+e))*d^4*e^2*f^2*x^2+4*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(
d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^4*e^3*f*x-3*B*ln(2*(d^2*e*x+(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f^3*x^2+C*ln(2*(
d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2
*f^2*x^2+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^2*e*f^3*x-6*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2
)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*x+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f*x-4*A*d^2*e^2*f^2*(-d
^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*B*d^2*e^3*f*(-d^2*x^2+1)^(1/2)
*(-(d^2*e^2-f^2)/f^2)^(1/2)-4*C*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/
```

```
f^2)^(1/2)+C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^2*e^4+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^
2)^(1/2)*f+f)/(f*x+e))*f^4*x^2+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*
e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e^2*f^2+A*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e
^2-f^2)/f^2)^(1/2)+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2
)^(1/2)*f+f)/(f*x+e))*d^4*e^4-3*A*d^2*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2
-f^2)/f^2)^(1/2)+B*d^2*e^2*f^2*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)+C*d^2*e^3*f*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-3*C*e^2*f^2
*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)^2*(-d*x+1)^(1/2)*(d
*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/(d*e+f)/(d^2*e^2-f^2)/(f*x+e)^2/(-(d
^2*e^2-f^2)/f^2)^(1/2)/f
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-%e*d>0)', see `assume?` for more
details
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(232) = 464$.
time = 1.20, size = 1502, normalized size = 6.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(A*f^7*x^2 - 2*B*d^4*e^7 - ((A*d^2 + 2*C)*f^4*x^2*e^2 + (2*A*d^4 + C*
d^2)*e^6 - (3*B*d^2*f - 2*(2*A*d^4 + C*d^2)*f*x)*e^5 - (6*B*d^2*f^2*x - (2*
A*d^4 + C*d^2)*f^2*x^2 - (A*d^2 + 2*C)*f^2)*e^4 - (3*B*d^2*f^3*x^2 - 2*(A*d
^2 + 2*C)*f^3*x)*e^3)*sqrt(-d^2*e^2 + f^2)*log((d^2*f*x*e - (d^2*e^2 - f^2
+ sqrt(-d^2*e^2 + f^2)*f)*sqrt(d*x + 1)*sqrt(-d*x + 1) + f^2 - (d^2*x*e + f
)*sqrt(-d^2*e^2 + f^2))/(f*x + e)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*((C*d^4*x
 + 2*B*d^4)*e^7 + (B*d^4*f*x - (4*A*d^4 + 3*C*d^2)*f)*e^6 - (B*d^2*f^2 + (3
*A*d^4 + 5*C*d^2)*f^2*x)*e^5 + (B*d^2*f^3*x + (5*A*d^2 + 3*C)*f^3)*e^4 + ((
3*A*d^2 + 4*C)*f^4*x - B*f^4)*e^3 - (2*B*f^5*x + A*f^5)*e^2) - (4*B*d^4*f*x
 - (4*A*d^4 + 3*C*d^2)*f)*e^6 - (2*B*d^4*f^2*x^2 - B*d^2*f^2 - 2*(4*A*d^4 +
```

```
 3*C*d^2)*f^2*x)*e^5 + (2*B*d^2*f^3*x + (4*A*d^4 + 3*C*d^2)*f^3*x^2 - (5*A*
d^2 + 3*C)*f^3)*e^4 + (B*d^2*f^4*x^2 - 2*(5*A*d^2 + 3*C)*f^4*x + B*f^4)*e^3
 - ((5*A*d^2 + 3*C)*f^5*x^2 - 2*B*f^5*x - A*f^5)*e^2 + (B*f^6*x^2 + 2*A*f^6
*x)*e)/(f^8*x^2*e^2 - 2*d^6*f*x*e^9 + 6*d^4*f^3*x*e^7 - 6*d^2*f^5*x*e^5 + 2
*f^7*x*e^3 - d^6*e^10 - (d^6*f^2*x^2 - 3*d^4*f^2)*e^8 + 3*(d^4*f^4*x^2 - d^
2*f^4)*e^6 - (3*d^2*f^6*x^2 - f^6)*e^4), -1/2*(A*f^7*x^2 - 2*B*d^4*e^7 - 2*
((A*d^2 + 2*C)*f^4*x^2*e^2 + (2*A*d^4 + C*d^2)*e^6 - (3*B*d^2*f - 2*(2*A*d^
4 + C*d^2)*f*x)*e^5 - (6*B*d^2*f^2*x - (2*A*d^4 + C*d^2)*f^2*x^2 - (A*d^2 +
 2*C)*f^2)*e^4 - (3*B*d^2*f^3*x^2 - 2*(A*d^2 + 2*C)*f^3*x)*e^3)*sqrt(d^2*e^
2 - f^2)*arctan(-sqrt(d^2*e^2 - f^2)*(f*x - sqrt(d*x + 1)*sqrt(-d*x + 1)*e
+ e)/(d^2*x*e^2 - f^2*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*((C*d^4*x + 2*B*d^
4)*e^7 + (B*d^4*f*x - (4*A*d^4 + 3*C*d^2)*f)*e^6 - (B*d^2*f^2 + (3*A*d^4 +
5*C*d^2)*f^2*x)*e^5 + (B*d^2*f^3*x + (5*A*d^2 + 3*C)*f^3)*e^4 + ((3*A*d^2 +
 4*C)*f^4*x - B*f^4)*e^3 - (2*B*f^5*x + A*f^5)*e^2) - (4*B*d^4*f*x - (4*A*d
^4 + 3*C*d^2)*f)*e^6 - (2*B*d^4*f^2*x^2 - B*d^2*f^2 - 2*(4*A*d^4 + 3*C*d^2)
*f^2*x)*e^5 + (2*B*d^2*f^3*x + (4*A*d^4 + 3*C*d^2)*f^3*x^2 - (5*A*d^2 + 3*C
)*f^3)*e^4 + (B*d^2*f^4*x^2 - 2*(5*A*d^2 + 3*C)*f^4*x + B*f^4)*e^3 - ((5*A*
d^2 + 3*C)*f^5*x^2 - 2*B*f^5*x - A*f^5)*e^2 + (B*f^6*x^2 + 2*A*f^6*x)*e)/(f
^8*x^2*e^2 - 2*d^6*f*x*e^9 + 6*d^4*f^3*x*e^7 - 6*d^2*f^5*x*e^5 + 2*f^7*x*e^
3 - d^6*e^10 - (d^6*f^2*x^2 - 3*d^4*f^2)*e^8 + 3*(d^4*f^4*x^2 - d^2*f^4)*e^
6 - (3*d^2*f^6*x^2 - f^6)*e^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Undef/Unsigned Inf encountered in lim
itLimit: Max order reached or unable to make series expansion Error: Bad Ar
gument
```

Mupad [B]
time = 59.18, size = 2500, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) - 1
)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 - d*
x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2) -
1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*d^3*e
^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)
) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1
/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) - 1)^5*(7
*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2
*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/(((d*x + 1
)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2)
 - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) -
 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1
)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/
2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*
x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1 - d*x)^(1/2) - 1)^
2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^2
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1/2) - 1)^4*(2*A*f^5 + 4
*A*d^4*e^4*f - 9*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (4*((1 - d*x)^(1/2) - 1)^6*(4*A*d^4*e^4*f - 2*A*f^5 +
 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*
f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^7*(2*A*d*f^3 - 5*A*d^3*e^2*f))/(e*((d*x
+ 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*((1 - d*x)^(1/2)
- 1)^3*(2*A*d*f^3 - 29*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^3*(f^4 + d^4*
e^4 - 2*d^2*e^2*f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^5*(2*A*d*f^3 - 29*A*d^3*
e^2*f))/(e*((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*d*
f*(2*A*f^3 - 5*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/(e*((d*x + 1)^(1/2) - 1)
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2*(16*
f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(16*f^
2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32*f^2
- 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)^8)/(
(d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2)
 - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (8*d*
e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^
(1/2) - 1))/((d*x + 1)^(1/2) - 1)) - ((4*((1 - d*x)^(1/2) - 1)^2*(2*B*f^4 +
```

```
2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4
- 2*d^2*e^2*f^2)) - (8*((1 - d*x)^(1/2) - 1)^4*(2*B*f^4 - 2*B*d^4*e^4 + 3*B
*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (4*((1 - d*x)^(1/2) - 1)^6*(2*B*f^4 + 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*
((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*(11*B*d^3*e
^2 + 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^3)/(((d*x + 1)^(1/2) - 1)^3*(f^4 + d
^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*(11*B*d^3*e^2 + 16*B*d*f^2)*((1 - d*x)^(1/2
) - 1)^5)/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (6*B*
d^3*e^2*f*((1 - d*x)^(1/2) - 1)^7)/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4
- 2*d^2*e^2*f^2)) + (6*B*d^3*e^2*f*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2
*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(
16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32
*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)
^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^
(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 -
(8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + (C*atan(((C*(2*f^2 + d^2*e^2)*((4
*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/
(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4
*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8
+ d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^
2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 -
12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^
4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f
^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*
x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2
*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))
- (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*f^7 + 4*C*d...
```

3.8 $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=340

$$-\frac{(4(4C+5Ad^2)f^2-3d^2e(Ce-5Bf))(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f}+\frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f}-\frac{C(e+fx}{}$$

```
[Out] 1/8*(8*A*d^4*e^3+12*A*d^2*e*f^2+12*B*d^2*e^2*f+4*C*d^2*e^3+3*B*f^3+9*C*e*f^
2)*arcsin(d*x)/d^5-1/60*(4*(5*A*d^2+4*C)*f^2-3*d^2*e*(-5*B*f+C*e))*(f*x+e)^
2*(-d^2*x^2+1)^(1/2)/d^4/f+1/20*(-5*B*f+C*e)*(f*x+e)^3*(-d^2*x^2+1)^(1/2)/d
^2/f-1/5*C*(f*x+e)^4*(-d^2*x^2+1)^(1/2)/d^2/f+1/120*(4*C*(3*d^4*e^4-52*d^2*
e^2*f^2-16*f^4)-20*d^2*f*(4*A*f*(4*d^2*e^2+f^2)+3*B*(d^2*e^3+4*e*f^2))+d^2*
f*(-100*A*d^2*e*f^2-30*B*d^2*e^2*f+6*C*d^2*e^3-45*B*f^3-71*C*e*f^2)*x)*(-d^
2*x^2+1)^(1/2)/d^6/f
```

Rubi [A]
time = 0.41, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1623, 1668, 847, 794, 222}

$$\frac{\text{ArcSin}(dx)\left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)}{8d^5}-\frac{\sqrt{1-d^2x^2}\,(e+fx)^2\left(5f(4Af+3Be)-C\left(3e^2-\frac{16f^2}{d^2}\right)\right)}{60d^2f}+\frac{\sqrt{1-d^2x^2}\,(d^2fx(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Cd^2e^3-71Cef^2)+4(C(3d^4e^4-52d^2e^2f^2-16f^4)-5d^2f(4Af(4d^2e^2+f^2)+3B(d^2e^3+4ef^2))))}{120d^6f}+\frac{\sqrt{1-d^2x^2}\,(e+fx)^3(Ce-5Bf)}{20d^2f}-\frac{C\sqrt{1-d^2x^2}\,(e+fx)^4}{5d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/60*((5*f*(3*B*e + 4*A*f) - C*(3*e^2 - (16*f^2)/d^2))*(e + f*x)^2*Sqrt[1
 - d^2*x^2])/(d^2*f) + ((C*e - 5*B*f)*(e + f*x)^3*Sqrt[1 - d^2*x^2])/(20*d^2
*f) - (C*(e + f*x)^4*Sqrt[1 - d^2*x^2])/(5*d^2*f) + ((4*(C*(3*d^4*e^4 - 52*
d^2*e^2*f^2 - 16*f^4) - 5*d^2*f*(4*A*f*(4*d^2*e^2 + f^2) + 3*B*(d^2*e^3 + 4
*e*f^2))) + d^2*f*(6*C*d^2*e^3 - 30*B*d^2*e^2*f - 71*C*e*f^2 - 100*A*d^2*e*
f^2 - 45*B*f^3)*x)*Sqrt[1 - d^2*x^2])/(120*d^6*f) + ((4*C*d^2*e^3 + 8*A*d^4
*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSin[d*x])/
(8*d^5)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)^3\,(A+Bx+Cx^2)}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx = \int \frac{(e+fx)^3\,(A+Bx+Cx^2)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} - \frac{\int \frac{(e+fx)^3(-(4C+5Ad^2)f^2+d^2f(Ce-5Bf)x)}{\sqrt{1-d^2x^2}}\,dx}{5d^2f^2}$$

$$= \frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f} - \frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} + \frac{\int \frac{(e+fx)^2(}{}}{}$$

$$= -\frac{(4(4C+5Ad^2)f^2-3d^2e(Ce-5Bf))\,(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-}{}$$

$$= -\frac{(4(4C+5Ad^2)f^2-3d^2e(Ce-5Bf))\,(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-}{}$$

$$= -\frac{(4(4C+5Ad^2)f^2-3d^2e(Ce-5Bf))\,(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-}{}$$

Mathematica [A]
time = 1.01, size = 273, normalized size = 0.80

$$\frac{-\sqrt{1-d^2x^2}\,(20Ad^2f(4f^2+d^2(18e^2+9efx+2f^2x^2))+15B(d^2f^2(16e+3fx)+2d^4(4e^3+6e^2fx+4ef^2x^2+f^3x^3))+C(64f^3+d^2f(240e^2+135efx+32f^2x^2)+6d^4x(10e^3+20e^2fx+15ef^2x^2+4f^3x^3)))+15\sqrt{-d^2}\,(4Cd^2e^3+8Ad^4e^3+12Bd^2e^2f+9Cef^2+12Ad^2ef^2+3Bf^3)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{120d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(20*A*d^2*f*(4*f^2 + d^2*(18*e^2 + 9*e*f*x + 2*f^2*x^2
)) + 15*B*(d^2*f^2*(16*e + 3*f*x) + 2*d^4*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2
+ f^3*x^3)) + C*(64*f^3 + d^2*f*(240*e^2 + 135*e*f*x + 32*f^2*x^2) + 6*d^4*
x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 15*Sqrt[-d^2]*(4*C*
d^2*e^3 + 8*A*d^4*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f
^3)*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(120*d^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 643, normalized size = 1.89

method	result
risch	$\frac{(24C\,d^4f^3x^4+30B\,d^4f^3x^3+90C\,d^4e\,f^2x^3+40A\,d^4f^3x^2+120B\,d^4e\,f^2x^2+120C\,d^4e^2f\,x^2+180A\,d^4e\,f^2x+180B\,d^4e^2fx+60C\,d^4e^3x+3}{120d^6\sqrt{-}}$

default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(24C\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4f^3x^4+30B\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4f^3x^3+90C\sqrt{-d^2x^2+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/120*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(24*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*f^
3*x^4+30*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*f^3*x^3+90*C*(-d^2*x^2+1)^(1/2)*c
sgn(d)*d^4*e*f^2*x^3+40*A*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*f^3*x^2+120*B*(-d^
2*x^2+1)^(1/2)*csgn(d)*d^4*e*f^2*x^2+120*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*e
^2*f*x^2+180*A*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*e*f^2*x+180*B*(-d^2*x^2+1)^(1
/2)*csgn(d)*d^4*e^2*f*x+60*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*e^3*x+360*A*(-d
^2*x^2+1)^(1/2)*csgn(d)*d^4*e^2*f-120*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/
2))*d^5*e^3+120*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d^4*e^3+32*C*(-d^2*x^2+1)^(1/2
)*csgn(d)*d^2*f^3*x^2+45*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d^2*f^3*x+135*C*(-d^2
*x^2+1)^(1/2)*csgn(d)*d^2*e*f^2*x+80*A*(-d^2*x^2+1)^(1/2)*csgn(d)*d^2*f^3-1
80*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e*f^2+240*B*(-d^2*x^2+1)^(1
/2)*csgn(d)*d^2*e*f^2-180*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e^2*
f+240*C*(-d^2*x^2+1)^(1/2)*csgn(d)*d^2*e^2*f-60*C*arctan(csgn(d)*d*x/(-d^2*
x^2+1)^(1/2))*d^3*e^3-45*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*f^3+64*
C*(-d^2*x^2+1)^(1/2)*csgn(d)*f^3-135*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*d*e*f^2)*csgn(d)/d^6/(-d^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 353, normalized size = 1.04

$$-\frac{\sqrt{-d^2x^2+1}\,Cf^3x^4}{5\,d^2}-\frac{4\sqrt{-d^2x^2+1}\,Cf^3x^2}{15\,d^4}-\frac{(Bf^3+3\,Cf^2e)\sqrt{-d^2x^2+1}\,x^3}{4\,d^2}+\frac{A\arcsin(dx)\,e^3}{d}-\frac{(Af^3+3\,Bf^2e+3\,Cfe^2)\sqrt{-d^2x^2+1}\,x^2}{3\,d^2}-\frac{3\sqrt{-d^2x^2+1}\,Afe^2}{d^2}-\frac{\sqrt{-d^2x^2+1}\,(3\,Af^2e+3\,Bfe^2+Ce^3)x}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,Be^3}{d^2}+\frac{(3\,Af^2e+3\,Bfe^2+Ce^3)\arcsin(dx)}{2\,d^3}-\frac{8\sqrt{-d^2x^2+1}\,Cf^3}{15\,d^6}-\frac{3\,(Bf^3+3\,Cf^2e)\sqrt{-d^2x^2+1}\,x}{8\,d^4}-\frac{2\,(Af^3+3\,Bf^2e+3\,Cfe^2)\sqrt{-d^2x^2+1}}{3\,d^4}+\frac{3\,(Bf^3+3\,Cf^2e)\arcsin(dx)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/5*sqrt(-d^2*x^2 + 1)*C*f^3*x^4/d^2 - 4/15*sqrt(-d^2*x^2 + 1)*C*f^3*x^2/d
^4 - 1/4*(B*f^3 + 3*C*f^2*e)*sqrt(-d^2*x^2 + 1)*x^3/d^2 + A*arcsin(d*x)*e^3
/d - 1/3*(A*f^3 + 3*B*f^2*e + 3*C*f*e^2)*sqrt(-d^2*x^2 + 1)*x^2/d^2 - 3*sqr
t(-d^2*x^2 + 1)*A*f*e^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(3*A*f^2*e + 3*B*f*e^2
 + C*e^3)*x/d^2 - sqrt(-d^2*x^2 + 1)*B*e^3/d^2 + 1/2*(3*A*f^2*e + 3*B*f*e^2
 + C*e^3)*arcsin(d*x)/d^3 - 8/15*sqrt(-d^2*x^2 + 1)*C*f^3/d^6 - 3/8*(B*f^3
+ 3*C*f^2*e)*sqrt(-d^2*x^2 + 1)*x/d^4 - 2/3*(A*f^3 + 3*B*f^2*e + 3*C*f*e^2)
*sqrt(-d^2*x^2 + 1)/d^4 + 3/8*(B*f^3 + 3*C*f^2*e)*arcsin(d*x)/d^5
```

Fricas [A]
time = 0.77, size = 285, normalized size = 0.84

$$-\frac{(24\,Cd^4f^3x^4+30\,Bd^4f^3x^3+45\,Bd^2f^3x+8\,(5\,Ad^4+4\,Cd^2)f^3x^2+16\,(5\,Ad^2+4\,C)f^3+60\,(Cd^4x+2\,Bd^4)e^3+60\,(2\,Cd^4fx^2+3\,Bd^4fx+2\,(3\,Ad^4+2\,Cd^2)f)e^2+15\,(6\,Cd^4f^2x^3+8\,Bd^4f^2x^2+16\,Bd^2f^2+3\,(4\,Ad^4+3\,Cd^2)f^2x)e)\sqrt{dx+1}\,\sqrt{-dx+1}+30\,(12\,Bd^3fe^2+3\,Bdf^3+3\,(4\,Ad^3+3\,Cd)f^2e+4\,(2\,Ad^5+Cd^3)e^3)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{120\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/120*((24*C*d^4*f^3*x^4 + 30*B*d^4*f^3*x^3 + 45*B*d^2*f^3*x + 8*(5*A*d^4
+ 4*C*d^2)*f^3*x^2 + 16*(5*A*d^2 + 4*C)*f^3 + 60*(C*d^4*x + 2*B*d^4)*e^3 +
60*(2*C*d^4*f*x^2 + 3*B*d^4*f*x + 2*(3*A*d^4 + 2*C*d^2)*f)*e^2 + 15*(6*C*d^
4*f^2*x^3 + 8*B*d^4*f^2*x^2 + 16*B*d^2*f^2 + 3*(4*A*d^4 + 3*C*d^2)*f^2*x)*e
)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(12*B*d^3*f*e^2 + 3*B*d*f^3 + 3*(4*A*d^
3 + 3*C*d)*f^2*e + 4*(2*A*d^5 + C*d^3)*e^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x
 + 1) - 1)/(d*x)))/d^6
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.36, size = 373, normalized size = 1.10

$$-\frac{(360\,Ad^4fe^2-180\,Ad^3f^2e+120\,Ad^2f^3+120\,Bd^4e^3-180\,Bd^3fe^2+360\,Bd^2f^2e-75\,Bdf^3-60\,Cd^3e^3+360\,Cd^2fe^2-225\,Cdf^2e+120\,Cf^3+(180\,Ad^3f^2e-80\,Ad^2f^3+180\,Bd^3fe^2-240\,Bd^2f^2e+135\,Bdf^3+60\,Cd^3e^3-240\,Cd^2fe^2+405\,Cdf^2e-160\,Cf^3+2\,(20\,Ad^2f^3+60\,Bd^2f^2e-45\,Bdf^3+60\,Cd^2fe^2-135\,Cdf^2e+88\,Cf^3+3\,(4\,(dx+1)Cf^3+5\,Bdf^3+15\,Cdf^2e-16\,Cf^3)(dx+1))(dx+1))(dx+1))\sqrt{dx+1}\,\sqrt{-dx+1}-30\,(8\,Ad^5e^3+12\,Ad^3f^2e+12\,Bd^3fe^2+3\,Bdf^3+4\,Cd^3e^3+9\,Cdf^2e)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{120\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] -1/120*((360*A*d^4*f*e^2 - 180*A*d^3*f^2*e + 120*A*d^2*f^3 + 120*B*d^4*e^3
- 180*B*d^3*f*e^2 + 360*B*d^2*f^2*e - 75*B*d*f^3 - 60*C*d^3*e^3 + 360*C*d^2
*f*e^2 - 225*C*d*f^2*e + 120*C*f^3 + (180*A*d^3*f^2*e - 80*A*d^2*f^3 + 180*
B*d^3*f*e^2 - 240*B*d^2*f^2*e + 135*B*d*f^3 + 60*C*d^3*e^3 - 240*C*d^2*f*e^
2 + 405*C*d*f^2*e - 160*C*f^3 + 2*(20*A*d^2*f^3 + 60*B*d^2*f^2*e - 45*B*d*f
^3 + 60*C*d^2*f*e^2 - 135*C*d*f^2*e + 88*C*f^3 + 3*(4*(d*x + 1)*C*f^3 + 5*B
*d*f^3 + 15*C*d*f^2*e - 16*C*f^3)*(d*x + 1))*(d*x + 1))*(d*x + 1))*sqrt(d*x
 + 1)*sqrt(-d*x + 1) - 30*(8*A*d^5*e^3 + 12*A*d^3*f^2*e + 12*B*d^3*f*e^2 +
3*B*d*f^3 + 4*C*d^3*e^3 + 9*C*d*f^2*e)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d
^6
```

Mupad [B]
time = 35.29, size = 2500, normalized size = 7.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^3*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^
(1/2) - 1)^6 + (((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^14
)/((d*x + 1)^(1/2) - 1)^14 - (((4096*C*f^3)/3 - 832*C*d^2*e^2*f)*((1 - d*x)
^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((4096*C*f^3)/3 - 832*C*d^2*e^2*f
)*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (((12288*C*f^3)/5 +
768*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (((1
- d*x)^(1/2) - 1)^3*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)
^3 - (((1 - d*x)^(1/2) - 1)^17*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((d*x + 1)
^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^7*(88*C*d^3*e^3 - 42*C*d*e*f^2))/((
d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^13*(88*C*d^3*e^3 - 42*C*d*e*
f^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*(40*C*d^3*e^3 + 4
26*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d*x)^(1/2) - 1)^15*(40*C*d^
3*e^3 + 426*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^9
*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^9 - (((1 - d*x)^(1/2
) - 1)^11*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^11 - (d*(4*
C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) + (
d*(4*C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1)^19)/(2*((d*x + 1)^(1/2) -
 1)^19) + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4
 + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16)/(d^
6 + (10*d^6*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (45*d^6*((1
- d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^6*((1 - d*x)^(1/2) -
1)^6)/((d*x + 1)^(1/2) - 1)^6 + (210*d^6*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1
)^(1/2) - 1)^8 + (252*d^6*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^1
0 + (210*d^6*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (120*d^6*
((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (45*d^6*((1 - d*x)^(1/
2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (10*d^6*((1 - d*x)^(1/2) - 1)^18)/((
d*x + 1)^(1/2) - 1)^18 + (d^6*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) -
1)^20) - (((64*A*f^3 + 96*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^
(1/2) - 1)^4 + ((64*A*f^3 + 96*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^8)/((d*x
+ 1)^(1/2) - 1)^8 - (((128*A*f^3)/3 - 144*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1
)^6)/((d*x + 1)^(1/2) - 1)^6 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)
^(1/2) - 1)^10 - (6*A*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)
+ (30*A*d*e*f^2*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (36*A*d*
e*f^2*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (36*A*d*e*f^2*((1
- d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (30*A*d*e*f^2*((1 - d*x)^(1/
2) - 1)^9)/((d*x + 1)^(1/2) - 1)^9 + (6*A*d*e*f^2*((1 - d*x)^(1/2) - 1)^11)
/((d*x + 1)^(1/2) - 1)^11)/(d^4 + (6*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1
)^(1/2) - 1)^2 + (15*d^4*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 +
 (20*d^4*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (15*d^4*((1 - d
*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (6*d^4*((1 - d*x)^(1/2) - 1)^10
```

```
)/((d*x + 1)^(1/2) - 1)^10 + (d^4*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2
) - 1)^12) - ((((3*B*f^3)/2 + 6*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^15)/((d*
x + 1)^(1/2) - 1)^15 - (((23*B*f^3)/2 - 18*B*d^2*e^2*f)*((1 - d*x)^(1/2) -
1)^3)/((d*x + 1)^(1/2) - 1)^3 + (((23*B*f^3)/2 - 18*B*d^2*e^2*f)*((1 - d*x)
^(1/2) - 1)^13)/((d*x + 1)^(1/2) - 1)^13 + (((333*B*f^3)/2 + 90*B*d^2*e^2*f
)*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (((333*B*f^3)/2 + 90*B
*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 - (((671*B*f
^3)/2 - 66*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 +
(((671*B*f^3)/2 - 66*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(1/2)
 - 1)^9 + (((1 - d*x)^(1/2) - 1)^4*(48*B*d^3*e^3 + 192*B*d*e*f^2))/((d*x +
1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^12*(48*B*d^3*e^3 + 192*B*d*e*f^2))
/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^8*(160*B*d^3*e^3 + 128*B
*d*e*f^2))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2) - 1)^6*(120*B*d^3*e^
3 + 256*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^10*(12
0*B*d^3*e^3 + 256*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^10 - (((3*B*f^3)/2 + 6*
B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (8*B*d^3*e^3*((
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B*d^3*e^3*((1 - d*x)^(1
/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^5 + (8*d^5*((1 - d*x)^(1/2) - 1)^
2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1
/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (70
*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (56*d^5*((1 - d*x)^
(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*((1 - d*x)^(1/2) - 1)^12)
/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/
2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16) - (3*
B*f*atan((B*f*(f^2 + 4*d^2*e^2)*((1 - d*x)^(1/2) - 1))/((B*f^3 + 4*B*d^2*e^
2*f)*((d*x + 1)^(1/2) - 1)))*(f^2 + 4*d^2*e^2))...
```

3.9 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=228

$$\frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f}-\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f}+\frac{(4(C(d^2e^3-8ef^2)-4f(3Ad^2ef+B(d^2e^2+}{}$$

```
[Out] 1/8*(C*(4*d^2*e^2+3*f^2)+4*d^2*(2*B*e*f+A*(2*d^2*e^2+f^2)))*arcsin(d*x)/d^5
+1/12*(-4*B*f+C*e)*(f*x+e)^2*(-d^2*x^2+1)^(1/2)/d^2/f-1/4*C*(f*x+e)^3*(-d^2
*x^2+1)^(1/2)/d^2/f+1/24*(4*C*(d^2*e^3-8*e*f^2)-16*f*(3*A*d^2*e*f+B*(d^2*e^
2+f^2))-f*(3*(4*A*d^2+3*C)*f^2-2*d^2*e*(-4*B*f+C*e))*x)*(-d^2*x^2+1)^(1/2)/
d^4/f
```

Rubi [A]
time = 0.32, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1623, 1668, 847, 794, 222}

$$\frac{\mathrm{ArcSin}(dx)\left(4d^2(A(2d^2e^2+f^2)+2Bef)+C(4d^2e^2+3f^2)\right)}{8d^5}+\frac{\sqrt{1-d^2x^2}\left(4(C(d^2e^3-8ef^2)-4f(3Ad^2ef+B(d^2e^2+f^2)))-fx(3f^2(4Ad^2+3C)-2d^2e(Ce-4Bf))\right)}{24d^4f}+\frac{\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)}{12d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(12*d^2*f) - (C*(e + f*x)^3*S
qrt[1 - d^2*x^2])/(4*d^2*f) + ((4*(C*(d^2*e^3 - 8*e*f^2) - 4*f*(3*A*d^2*e*f
 + B*(d^2*e^2 + f^2))) - f*(3*(3*C + 4*A*d^2)*f^2 - 2*d^2*e*(C*e - 4*B*f))*
x)*Sqrt[1 - d^2*x^2])/(24*d^4*f) + ((C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f
 + A*(2*d^2*e^2 + f^2)))*ArcSin[d*x])/(8*d^5)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} - \frac{\int \frac{(e+fx)^2\left(-\left(3C+4Ad^2\right)f^2+d^2f(Ce-4Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{4d^2f^2}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{\int \frac{(e+fx)(d}{}}{}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{(4(C(d^2e}{}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{(4(C(d^2e}{}$$

Mathematica [A]

time = 0.65, size = 195, normalized size = 0.86

$$\frac{-d^2\sqrt{1-d^2x^2}\,(12Ad^2f(4e+fx)+C(12d^2e^2x+16ef(2+d^2x^2)+3f^2x(3+2d^2x^2))+8B(2f^2+d^2(3e^2+3efx+f^2x^2)))+3\sqrt{-d^2}\,(C(4d^2e^2+3f^2)+4d^2(2Bef+A(2d^2e^2+f^2)))\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{24d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d^2*Sqrt[1 - d^2*x^2]*(12*A*d^2*f*(4*e + f*x) + C*(12*d^2*e^2*x + 16*e*f
*(2 + d^2*x^2) + 3*f^2*x*(3 + 2*d^2*x^2)) + 8*B*(2*f^2 + d^2*(3*e^2 + 3*e*f
*x + f^2*x^2)))) + 3*Sqrt[-d^2]*(C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f + A
*(2*d^2*e^2 + f^2)))*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(24*d^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 423, normalized size = 1.86

method	result
risch	$\frac{\left(6C\,d^2f^2x^3+8B\,d^2f^2x^2+16C\,d^2ef\,x^2+12A\,d^2f^2x+24B\,d^2efx+12C\,d^2e^2x+48A\,d^2ef+24B\,d^2e^2+9C\,f^2x+16B\,f^2+32Cef\right)\sqrt{dx}}{24d^4\sqrt{-\left(dx+1\right)\left(dx-1\right)}\,\sqrt{-dx+1}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(6C\,\mathrm{csgn}(d)d^3\sqrt{-d^2x^2+1}\,f^2x^3+8B\,\mathrm{csgn}(d)d^3\sqrt{-d^2x^2+1}\,f^2x^2+16C\,\mathrm{csgn}(d)d^3\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*f^2*
x^3+8*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*f^2*x^2+16*C*csgn(d)*d^3*(-d^2*x^2+1
)^(1/2)*e*f*x^2+12*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*f^2*x+24*B*csgn(d)*d^3*
(-d^2*x^2+1)^(1/2)*e*f*x+12*C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e^2*x+48*A*csg
n(d)*d^3*(-d^2*x^2+1)^(1/2)*e*f-24*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))
*d^4*e^2+24*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e^2+9*C*csgn(d)*d*(-d^2*x^2+1)
^(1/2)*f^2*x-12*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*f^2+16*B*csgn(
d)*d*(-d^2*x^2+1)^(1/2)*f^2-24*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2
*e*f+32*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*e*f-12*C*arctan(csgn(d)*d*x/(-d^2*x^
2+1)^(1/2))*d^2*e^2-9*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*f^2)*csgn(d)
/d^5/(-d^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.55, size = 232, normalized size = 1.02

$$-\frac{\sqrt{-d^2x^2+1}\,Cf^2x^3}{4\,d^2}+\frac{A\arcsin\left(dx\right)e^2}{d}-\frac{\sqrt{-d^2x^2+1}\,(Bf^2+2\,Cfe)x^2}{3\,d^2}-\frac{2\,\sqrt{-d^2x^2+1}\,Afe}{d^2}-\frac{\sqrt{-d^2x^2+1}\,(Af^2+2\,Bfe+Ce^2)x}{2\,d^2}-\frac{3\,\sqrt{-d^2x^2+1}\,Cf^2x}{8\,d^4}-\frac{\sqrt{-d^2x^2+1}\,Be^2}{d^2}+\frac{(Af^2+2\,Bfe+Ce^2)\arcsin\left(dx\right)}{2\,d^3}+\frac{3\,Cf^2\arcsin\left(dx\right)}{8\,d^5}-\frac{2\,\sqrt{-d^2x^2+1}\,(Bf^2+2\,Cfe)}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/4*sqrt(-d^2*x^2 + 1)*C*f^2*x^3/d^2 + A*arcsin(d*x)*e^2/d - 1/3*sqrt(-d^2
*x^2 + 1)*(B*f^2 + 2*C*f*e)*x^2/d^2 - 2*sqrt(-d^2*x^2 + 1)*A*f*e/d^2 - 1/2*
sqrt(-d^2*x^2 + 1)*(A*f^2 + 2*B*f*e + C*e^2)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)
*C*f^2*x/d^4 - sqrt(-d^2*x^2 + 1)*B*e^2/d^2 + 1/2*(A*f^2 + 2*B*f*e + C*e^2)
*arcsin(d*x)/d^3 + 3/8*C*f^2*arcsin(d*x)/d^5 - 2/3*sqrt(-d^2*x^2 + 1)*(B*f^
2 + 2*C*f*e)/d^4
```

Fricas [A]
time = 1.26, size = 193, normalized size = 0.85

$$-\frac{(6\,Cd^3f^2x^3 + 8\,Bd^3f^2x^2 + 16\,Bdf^2 + 3\,(4\,Ad^3 + 3\,Cd)f^2x + 12\,(Cd^3x + 2\,Bd^3)e^2 + 8\,(2\,Cd^3fx^2 + 3\,Bd^3fx + 2\,(3\,Ad^3 + 2\,Cd)f)e)\sqrt{dx+1}\,\sqrt{-dx+1} + 6\,(8\,Bd^2fe + (4\,Ad^2 + 3\,C)f^2 + 4\,(2\,Ad^4 + Cd^2)e^2)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{24\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/24*((6*C*d^3*f^2*x^3 + 8*B*d^3*f^2*x^2 + 16*B*d*f^2 + 3*(4*A*d^3 + 3*C*d
)*f^2*x + 12*(C*d^3*x + 2*B*d^3)*e^2 + 8*(2*C*d^3*f*x^2 + 3*B*d^3*f*x + 2*(
3*A*d^3 + 2*C*d)*f)*e)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*B*d^2*f*e + (4*A
*d^2 + 3*C)*f^2 + 4*(2*A*d^4 + C*d^2)*e^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x
+ 1) - 1)/(d*x)))/d^5
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.11, size = 234, normalized size = 1.03

$$-\frac{(48\,Ad^3fe - 12\,Ad^2f^2 + 24\,Bd^3e^2 - 24\,Bd^2fe + 24\,Bdf^2 - 12\,Cd^2e^2 + 48\,Cdfe - 15\,Cf^2 + (12\,Ad^2f^2 + 24\,Bd^2fe - 16\,Bdf^2 + 12\,Cd^2e^2 - 32\,Cdfe + 27\,Cf^2 + 2\,(3\,(dx+1)Cf^2 + 4\,Bdf^2 + 8\,Cdfe - 9\,Cf^2)(dx+1))(dx+1))\sqrt{dx+1}\,\sqrt{-dx+1} - 6\,(8\,Ad^4e^2 + 4\,Ad^2f^2 + 8\,Bd^2fe + 4\,Cd^2e^2 + 3\,Cf^2)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{24\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] -1/24*((48*A*d^3*f*e - 12*A*d^2*f^2 + 24*B*d^3*e^2 - 24*B*d^2*f*e + 24*B*d*
f^2 - 12*C*d^2*e^2 + 48*C*d*f*e - 15*C*f^2 + (12*A*d^2*f^2 + 24*B*d^2*f*e -
 16*B*d*f^2 + 12*C*d^2*e^2 - 32*C*d*f*e + 27*C*f^2 + 2*(3*(d*x + 1)*C*f^2 +
 4*B*d*f^2 + 8*C*d*f*e - 9*C*f^2)*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*sqrt(
-d*x + 1) - 6*(8*A*d^4*e^2 + 4*A*d^2*f^2 + 8*B*d^2*f*e + 4*C*d^2*e^2 + 3*C*
f^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^5
```

Mupad [B]
time = 33.64, size = 1732, normalized size = 7.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^2*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((14*A*f^2*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (2*A*f^2*((
1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*A*f^2*((1 - d*x)^(1/2) - 1
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*A*f^2*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)
^(1/2) - 1)^7 + (16*A*d*e*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^
2 + (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*A*d*
e*f*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6)/(d^3 + (4*d^3*((1 - d
*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((1 - d*x)^(1/2) - 1)^4)
/((d*x + 1)^(1/2) - 1)^4 + (4*d^3*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2)
 - 1)^6 + (d^3*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8) - ((((1 -
d*x)^(1/2) - 1)^4*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (((1
 - d*x)^(1/2) - 1)^8*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^8 - (
((1 - d*x)^(1/2) - 1)^6*((128*B*f^2)/3 - 48*B*d^2*e^2))/((d*x + 1)^(1/2) -
1)^6 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B
*d^2*e^2*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (20*B*d*e*f*(
(1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (24*B*d*e*f*((1 - d*x)^(1
/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (24*B*d*e*f*((1 - d*x)^(1/2) - 1)^7)/
((d*x + 1)^(1/2) - 1)^7 - (20*B*d*e*f*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(
1/2) - 1)^9 + (4*B*d*e*f*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11
 - (4*B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^4 + (6*d^4*(
(1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (15*d^4*((1 - d*x)^(1/2)
- 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (20*d^4*((1 - d*x)^(1/2) - 1)^6)/((d*x +
1)^(1/2) - 1)^6 + (15*d^4*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8
+ (6*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (d^4*((1 - d*
x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12) - ((((1 - d*x)^(1/2) - 1)^15*((
3*C*f^2)/2 + 2*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^15 - (((1 - d*x)^(1/2) - 1
)^3*((23*C*f^2)/2 - 6*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/
2) - 1)*((3*C*f^2)/2 + 2*C*d^2*e^2))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1
/2) - 1)^13*((23*C*f^2)/2 - 6*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 -
 d*x)^(1/2) - 1)^5*((333*C*f^2)/2 + 30*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^5
- (((1 - d*x)^(1/2) - 1)^11*((333*C*f^2)/2 + 30*C*d^2*e^2))/((d*x + 1)^(1/2
```

```
) - 1)^11 - (((1 - d*x)^(1/2) - 1)^7*((671*C*f^2)/2 - 22*C*d^2*e^2))/((d*x
+ 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^9*((671*C*f^2)/2 - 22*C*d^2*e^2)
)/((d*x + 1)^(1/2) - 1)^9 + (128*C*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1
)^(1/2) - 1)^4 + (512*C*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x + 1)^(1/2)
- 1)^6) + (256*C*d*e*f*((1 - d*x)^(1/2) - 1)^8)/(3*((d*x + 1)^(1/2) - 1)^8)
 + (512*C*d*e*f*((1 - d*x)^(1/2) - 1)^10)/(3*((d*x + 1)^(1/2) - 1)^10) + (1
28*C*d*e*f*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12)/(d^5 + (8*d^
5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/
2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x
 + 1)^(1/2) - 1)^6 + (70*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)
^8 + (56*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*(
(1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2)
 - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x +
 1)^(1/2) - 1)^16) - (C*atan((C*((1 - d*x)^(1/2) - 1)*(3*f^2 + 4*d^2*e^2))/
(((d*x + 1)^(1/2) - 1)*(3*C*f^2 + 4*C*d^2*e^2)))*(3*f^2 + 4*d^2*e^2))/(2*d^
5) - (2*A*atan((A*(f^2 + 2*d^2*e^2)*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2
) - 1)*(A*f^2 + 2*A*d^2*e^2)))*(f^2 + 2*d^2*e^2))/d^3 - (4*B*e*f*atan(((1 -
 d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

3.10 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=130

$$-\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f}-\frac{(2(3d^2f(Be+Af)-C(d^2e^2-2f^2))-d^2f(Ce-3Bf)x)\sqrt{1-d^2x^2}}{6d^4f}+\frac{(Ce+2}{}$$

```
[Out] 1/2*(2*A*d^2*e+B*f+C*e)*arcsin(d*x)/d^3-1/3*C*(f*x+e)^2*(-d^2*x^2+1)^(1/2)/
d^2/f-1/6*(6*d^2*f*(A*f+B*e)-2*C*(d^2*e^2-2*f^2)-d^2*f*(-3*B*f+C*e)*x)*(-d^
2*x^2+1)^(1/2)/d^4/f
```

Rubi [A]
time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1623, 1668, 794, 222}

$$\frac{\text{ArcSin}(dx)\,(2Ad^2e+Bf+Ce)}{2d^3}-\frac{\sqrt{1-d^2x^2}\,\left(2\left(3d^2f(Af+Be)-\frac{1}{2}C(2d^2e^2-4f^2)\right)-d^2fx(Ce-3Bf)\right)}{6d^4f}-\frac{C\sqrt{1-d^2x^2}\,(e+fx)^2}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(C*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(d^2*f) - ((2*(3*d^2*f*(B*e + A*f) -
 (C*(2*d^2*e^2 - 4*f^2))/2) - d^2*f*(C*e - 3*B*f)*x)*Sqrt[1 - d^2*x^2])/(6*
d^4*f) + ((C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{\int \frac{(e+fx)\left(-\left(2C+3Ad^2\right)f^2+d^2f(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{3d^2f^2}$$

$$= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{\left(2\left(3d^2f(Be+Af)-\frac{1}{2}C\left(2d^2e^2-4f^2\right)\right)-d^2 \cdots}{6d^4f}$$

$$= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{\left(2\left(3d^2f(Be+Af)-\frac{1}{2}C\left(2d^2e^2-4f^2\right)\right)-d^2 \cdots}{6d^4f}$$

Mathematica [A]
time = 0.46, size = 120, normalized size = 0.92

$$\frac{-\sqrt{1-d^2x^2}\left(6Ad^2f+3Bd^2(2e+fx)+C\left(4f+3d^2ex+2d^2fx^2\right)\right)+3\sqrt{-d^2}\left(Ce+2Ad^2e+Bf\right)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(6*A*d^2*f + 3*B*d^2*(2*e + f*x) + C*(4*f + 3*d^2*e*x
+ 2*d^2*f*x^2))) + 3*Sqrt[-d^2]*(C*e + 2*A*d^2*e + B*f)*Log[-(Sqrt[-d^2]*x)
 + Sqrt[1 - d^2*x^2]])/(6*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 235, normalized size = 1.81

method	result

risch	$\frac{\left(2C\,d^{2}f\,x^{2}+3B\,d^{2}fx+3C\,d^{2}ex+6A\,d^{2}f+6B\,d^{2}e+4fC\right)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)\,(dx+1)}}{6d^{4}\sqrt{-(dx+1)\,(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{-d^{2}}}\right.\right.}{\sqrt{d}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2C\,\mathrm{csgn}(d)\sqrt{-d^{2}x^{2}+1}\,d^{2}f\,x^{2}+3B\,\mathrm{csgn}(d)\sqrt{-d^{2}x^{2}+1}\,d^{2}fx+3C\,\mathrm{csgn}(d)\sqrt{-d^{2}x^{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*C*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f*x^2
+3*B*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f*x+3*C*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*
e*x+6*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f-6*A*arctan(csgn(d)*d*x/(-d^2*x^2+1
)^(1/2))*d^3*e+6*B*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*e-3*B*arctan(csgn(d)*d*x/
(-d^2*x^2+1)^(1/2))*d*f+4*C*csgn(d)*(-d^2*x^2+1)^(1/2)*f-3*C*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*d*e)*csgn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.48, size = 135, normalized size = 1.04

$$-\frac{\sqrt{-d^{2}x^{2}+1}\,Cfx^{2}}{3\,d^{2}}+\frac{A\arcsin\left(dx\right)e}{d}-\frac{\sqrt{-d^{2}x^{2}+1}\,Af}{d^{2}}-\frac{\sqrt{-d^{2}x^{2}+1}\,(Bf+Ce)x}{2\,d^{2}}-\frac{\sqrt{-d^{2}x^{2}+1}\,Be}{d^{2}}+\frac{(Bf+Ce)\arcsin\left(dx\right)}{2\,d^{3}}-\frac{2\sqrt{-d^{2}x^{2}+1}\,Cf}{3\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*C*f*x^2/d^2 + A*arcsin(d*x)*e/d - sqrt(-d^2*x^2 + 1
)*A*f/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(B*f + C*e)*x/d^2 - sqrt(-d^2*x^2 + 1)*B
*e/d^2 + 1/2*(B*f + C*e)*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*C*f/d^4
```

Fricas [A]
time = 1.40, size = 117, normalized size = 0.90

$$-\frac{(2\,Cd^{2}fx^{2}+3\,Bd^{2}fx+2\,(3\,Ad^{2}+2\,C)f+3\,(Cd^{2}x+2\,Bd^{2})e)\sqrt{dx+1}\,\sqrt{-dx+1}+6\,(Bdf+(2\,Ad^{3}+Cd)e)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{6\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/6*((2*C*d^2*f*x^2 + 3*B*d^2*f*x + 2*(3*A*d^2 + 2*C)*f + 3*(C*d^2*x + 2*B
*d^2)*e)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(B*d*f + (2*A*d^3 + C*d)*e)*arcta
n((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.70, size = 117, normalized size = 0.90

$$-\frac{\left(6\,Ad^{2}f+6\,Bd^{2}e-3\,Bdf-3\,Cde+\left(2\,(dx+1)Cf+3\,Bdf+3\,Cde-4\,Cf\right)(dx+1)+6\,Cf\right)\sqrt{dx+1}\,\sqrt{-dx+1}-6\left(2\,Ad^{3}e+Bdf+Cde\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{6\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] -1/6*((6*A*d^2*f + 6*B*d^2*e - 3*B*d*f - 3*C*d*e + (2*(d*x + 1)*C*f + 3*B*d
*f + 3*C*d*e - 4*C*f)*(d*x + 1) + 6*C*f)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(
2*A*d^3*e + B*d*f + C*d*e)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^4
```

Mupad [B]
time = 12.86, size = 492, normalized size = 3.78

$$\frac{\frac{2\,B\,f\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}-\frac{14\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}-\frac{2\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}-\frac{\sqrt{1-d\,x}\,\left(\frac{2\,C\,f}{3\,d^4}+\frac{2\,C\,f\,x}{3\,d^3}+\frac{C\,f\,x^3}{3\,d}+\frac{C\,f\,x^2}{3\,d^2}\right)}{\sqrt{d\,x+1}}+\frac{\frac{2\,C\,e\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}-\frac{14\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}-\frac{2\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}-\frac{\left(\frac{A\,f}{d^2}+\frac{A\,f\,x}{d}\right)\,\sqrt{1-d\,x}}{\sqrt{d\,x+1}}-\frac{\left(\frac{B\,e}{d^2}+\frac{B\,e\,x}{d}\right)\,\sqrt{1-d\,x}}{\sqrt{d\,x+1}}-\frac{4\,A\,e\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,B\,f\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{2\,C\,e\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((2*B*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*B*f*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*B*f*((1 - d*x)^(1/2) - 1)^5)/((d
*x + 1)^(1/2) - 1)^5 - (2*B*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1
)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - ((1 -
d*x)^(1/2)*((2*C*f)/(3*d^4) + (2*C*f*x)/(3*d^3) + (C*f*x^3)/(3*d) + (C*f*x^
2)/(3*d^2)))/(d*x + 1)^(1/2) + ((2*C*e*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1
/2) - 1) - (14*C*e*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*C
*e*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (2*C*e*((1 - d*x)^(1/
2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1
)^(1/2) - 1)^2 + 1)^4) - (((A*f)/d^2 + (A*f*x)/d)*(1 - d*x)^(1/2))/(d*x + 1
)^(1/2) - (((B*e)/d^2 + (B*e*x)/d)*(1 - d*x)^(1/2))/(d*x + 1)^(1/2) - (4*A*
e*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2
)^(1/2) - (2*B*f*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - (
2*C*e*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

3.11 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(C+2Ad^2)\sin^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*(2*A*d^2+C)*arcsin(d*x)/d^3-B*(-d^2*x^2+1)^(1/2)/d^2-1/2*C*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {913, 1829, 655, 222}

$$\frac{(2Ad^2+C)\,\text{ArcSin}(dx)}{2d^3}-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((B*Sqrt[1 - d^2*x^2])/d^2) - (C*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((C + 2*A*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
```

```
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{Cx\sqrt{1-d^2x^2}}{2d^2}-\frac{\int \frac{-C-2Ad^2-2Bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\ &= -\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}-\frac{\left(-C-2Ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\ &= -\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}+\frac{\left(C+2Ad^2\right)\sin^{-1}(dx)}{2d^3}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 82, normalized size = 1.30

$$\frac{(-2B-Cx)\sqrt{1-d^2x^2}}{2d^2}+\frac{\sqrt{-d^2}\left(C+2Ad^2\right)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-2*B - C*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + (Sqrt[-d^2]*(C + 2*A*d^2)*Log[-(
Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(2*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 117, normalized size = 1.86

method	result
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2A\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^2-C\,\mathrm{csgn}(d)d\sqrt{-d^2x^2+1}\,x-2B\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d+\cdots}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(Cx+2B)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)A}{\sqrt{d^2}}+\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)C}{2d^2\sqrt{d^2}}\right)\cdots}{\sqrt{-dx+1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(2*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(
1/2))*d^2-C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x-2*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d
+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)))/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Maxima [**A**]
time = 0.52, size = 57, normalized size = 0.90

$$\frac{A \arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}\, Cx}{2\, d^2} - \frac{\sqrt{-d^2x^2+1}\, B}{d^2} + \frac{C \arcsin(dx)}{2\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*C*x/d^2 - sqrt(-d^2*x^2 + 1)*B/d^2
 + 1/2*C*arcsin(d*x)/d^3
```

Fricas [**A**]
time = 0.82, size = 67, normalized size = 1.06

$$-\frac{(Cdx + 2\, Bd)\sqrt{dx+1}\, \sqrt{-dx+1} + 2\, (2\, Ad^2 + C) \arctan\left(\frac{\sqrt{dx+1}\, \sqrt{-dx+1} - 1}{dx}\right)}{2\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((C*d*x + 2*B*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*A*d^2 + C)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.37, size = 60, normalized size = 0.95

$$-\frac{((dx+1)C + 2\, Bd - C)\sqrt{dx+1}\, \sqrt{-dx+1} - 2\, (2\, Ad^2 + C) \arcsin\left(\frac{1}{2}\, \sqrt{2}\, \sqrt{dx+1}\right)}{2\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(((d*x + 1)*C + 2*B*d - C)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*A*d^2 +
 C)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B]
time = 7.53, size = 232, normalized size = 3.68

$$-\frac{\frac{14\,C\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,C\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,C\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}-\frac{4\,A\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\sqrt{1-d\,x}\,\left(\frac{B}{d^2}+\frac{B\,x}{d}\right)}{\sqrt{d\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((14*C*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*C*((1 - d*x
)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*C*((1 - d*x)^(1/2) - 1)^7)/((d
*x + 1)^(1/2) - 1)^7 - (2*C*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(
d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - (4*A*atan((d
*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) -
 (2*C*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((1 - d*x)^(
1/2)*(B/d^2 + (B*x)/d))/(d*x + 1)^(1/2)
```

3.12 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

Optimal. Leaf size=122

$$-\frac{C\sqrt{1-d^2x^2}}{d^2f}-\frac{(Ce-Bf)\sin^{-1}(dx)}{df^2}+\frac{(Ce^2-Bef+Af^2)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

```
[Out] -(-B*f+C*e)*arcsin(d*x)/d/f^2+(A*f^2-B*e*f+C*e^2)*arctan((d^2*e*x+f)/(d^2*e
^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(1/2)-C*(-d^2*x^2+1)^(1
/2)/d^2/f
```

Rubi [A]

time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1623, 1668, 858, 222, 739, 210}

$$\frac{(Af^2-Bef+Ce^2)\,\mathrm{ArcTan}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\mathrm{ArcSin}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{\int \frac{-Ad^2f^2+d^2f(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{(Ce^2-Bef+Af^2)}{}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} - \frac{(Ce^2-Bef+Af^2)\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} + \frac{(Ce^2-Bef+Af^2)\tan^{-1}\Big(}{f^2\sqrt{d^2e}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 559 vs. 2(122) = 244.
time = 0.40, size = 559, normalized size = 4.58

$$\frac{-\frac{Cf^3\sqrt{1-d^2x^2}}{d^2}+\frac{(Ce^2+f(-Be+Af))\sqrt{2d^2e^2-f^2-2de\sqrt{d^2e^2-f^2}}\left(d^2e^2-f^2+de\sqrt{d^2e^2-f^2}\right)\tan^{-1}\left(\frac{f\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)}{\sqrt{2d^2e^2-f^2-2de\sqrt{d^2e^2-f^2}}}\right)}{(de-f)(de+f)}-\frac{(Ce^2+f(-Be+Af))\left(-d^2e^2+f^2+de\sqrt{d^2e^2-f^2}\right)\sqrt{2d^2e^2-f^2+2de\sqrt{d^2e^2-f^2}}\tan^{-1}\left(\frac{f\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)}{\sqrt{2d^2e^2-f^2+2de\sqrt{d^2e^2-f^2}}}\right)}{(de-f)(de+f)}+\frac{df^2\sqrt{-d^2e^2+f^2}\,(Ce^2+f(-Be+Af))\tan^{-1}\left(\frac{-\sqrt{-d^2}\,f^2x\sqrt{1-d^2x^2}+d^2(e^2-f^2x^2)}{de\sqrt{-d^2e^2+f^2}}\right)}{\sqrt{-d^2}\,(de-f)(de+f)}+\frac{f^2(Ce-Bf)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{\sqrt{-d^2}}}{f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] (-((C*f^3*Sqrt[1 - d^2*x^2])/d^2) + ((C*e^2 + f*(-(B*e) + A*f))*Sqrt[2*d^2*
e^2 - f^2 - 2*d*e*Sqrt[d^2*e^2 - f^2]]*(d^2*e^2 - f^2 + d*e*Sqrt[d^2*e^2 -
f^2])*ArcTan[(f*(Sqrt[-d^2]*x - Sqrt[1 - d^2*x^2]))/Sqrt[2*d^2*e^2 - f^2 -
2*d*e*Sqrt[d^2*e^2 - f^2]]])/((d*e - f)*(d*e + f)) - ((C*e^2 + f*(-(B*e) +
A*f))*(-(d^2*e^2) + f^2 + d*e*Sqrt[d^2*e^2 - f^2])*Sqrt[2*d^2*e^2 - f^2 + 2
*d*e*Sqrt[d^2*e^2 - f^2]]*ArcTan[(f*(Sqrt[-d^2]*x - Sqrt[1 - d^2*x^2]))/Sqr
t[2*d^2*e^2 - f^2 + 2*d*e*Sqrt[d^2*e^2 - f^2]]])/((d*e - f)*(d*e + f)) + (d
*f^2*Sqrt[-(d^2*e^2) + f^2]*(C*e^2 + f*(-(B*e) + A*f))*ArcTan[(-(Sqrt[-d^2]
*f^2*x*Sqrt[1 - d^2*x^2]) + d^2*(e^2 - f^2*x^2))/(d*e*Sqrt[-(d^2*e^2) + f^2
])])/(Sqrt[-d^2]*(d*e - f)*(d*e + f)) + (f^2*(C*e - B*f)*Log[-(Sqrt[-d^2]*x
) + Sqrt[1 - d^2*x^2]])/Sqrt[-d^2])/f^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 373, normalized size = 3.06

method	result
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2f^2+B\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}} \cdots}{fx+e}\right.\right.\cdots}{\cdots}$
risch	$\frac{C\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{f\,d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)B}{f\sqrt{d^2}}-\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)Ce}{f^2\sqrt{d^2}}-\cdots\ln\left(\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*d^2*f^2+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f-C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1
/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2+B*arctan(csgn(d)*d*x/(
-d^2*x^2+1)^(1/2))*d*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*csgn(d)*f^2*(-d^2*x^2
+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*d*e*f*(-(d^2*e^2-f^2)/f^2)^(1/2))*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)/(
-(d^2*e^2-f^2)/f^2)^(1/2)/f^3/(-d^2*x^2+1)^(1/2)/d^2
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-%e*d>0)', see `assume?` for more
details
```

Fricas [A]

time = 7.26, size = 457, normalized size = 3.75

$$\left[-\frac{(Ad^2f^2 - Bd^2fe + Cd^2e^2)\sqrt{-d^2e^2 + f^2}\log\left(\frac{d^2fxe - \left(d^2e^2 - f^2 + \sqrt{-d^2e^2 + f^2}\,f\right)\sqrt{dx+1}\,\sqrt{-dx+1} + f^2 - (d^2xe + f)\sqrt{-d^2e^2 + f^2}}{fx+e}\right) + (Cd^2fe^2 - Cf^3)\sqrt{dx+1}\,\sqrt{-dx+1} + 2\,(Bd^3fe^2 - Bdf^3 - Cd^3e^3 + Cdf^2e)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1} - 1}{dx}\right)}{d^4f^2e^2 - d^2f^4}, -\frac{2\,(Ad^2f^2 - Bd^2fe + Cd^2e^2)\sqrt{d^2e^2 - f^2}\arctan\left(-\frac{\sqrt{d^2e^2 - f^2}\left(fx - \sqrt{dx+1}\,\sqrt{-dx+1}\,e + e\right)}{d^2xe^2 - f^2x}\right) + (Cd^2fe^2 - Cf^3)\sqrt{dx+1}\,\sqrt{-dx+1} + 2\,(Bd^3fe^2 - Bdf^3 - Cd^3e^3 + Cdf^2e)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1} - 1}{dx}\right)}{d^4f^2e^2 - d^2f^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-((A*d^2*f^2 - B*d^2*f*e + C*d^2*e^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*f*x*e
- (d^2*e^2 - f^2 + sqrt(-d^2*e^2 + f^2)*f)*sqrt(d*x + 1)*sqrt(-d*x + 1) + f
^2 - (d^2*x*e + f)*sqrt(-d^2*e^2 + f^2))/(f*x + e)) + (C*d^2*f*e^2 - C*f^3)
*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(B*d^3*f*e^2 - B*d*f^3 - C*d^3*e^3 + C*d*
f^2*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*f^2*e^2 - d^2
*f^4), -(2*(A*d^2*f^2 - B*d^2*f*e + C*d^2*e^2)*sqrt(d^2*e^2 - f^2)*arctan(-
sqrt(d^2*e^2 - f^2)*(f*x - sqrt(d*x + 1)*sqrt(-d*x + 1)*e + e)/(d^2*x*e^2 -
 f^2*x)) + (C*d^2*f*e^2 - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(B*d^3*f*
e^2 - B*d*f^3 - C*d^3*e^3 + C*d*f^2*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1)
 - 1)/(d*x)))/(d^4*f^2*e^2 - d^2*f^4)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(e + fx)\sqrt{-dx + 1}\,\sqrt{dx + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Undef/Unsigned Inf encountered in lim
itLimit: Max order reached or unable to make series expansion Error: Bad Ar
gument
```

Mupad [B]
time = 0.01, size = 2500, normalized size = 20.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^2
)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) -
1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^
2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))/(((d*x
 + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5
*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(
1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^
3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^3*
f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1/2) - 1)^2*(d
^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*((1 - d
*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1i - d^2*e^2*1i
- (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e^2*((1 -
 d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*e)^(1/2)*(f - d*e
)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))/((d*x
```

```
 + 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d*e)^(1/2)*(f - d*e)
^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*
e^2*atan(((C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (409
6*((1 - d*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x
 + 1)^(1/2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1
)^(1/2) - 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4)
 + (16384*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d
*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 -
144*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*
e^2*((4096*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d
*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^
6))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))
/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^
8 - 9*d^6*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*
(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(
1/2)*(f - d*e)^(1/2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (C*e^2*(
(4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2)
 - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2
) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C
*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 +
9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d
^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*
C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^
2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1
/2) - 1)^2) - (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (163
84*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6)
)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))
/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/
2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))/((131072*C^4*e^7)/(d*f^4) +
 (C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d
*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/
2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) -
 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384
*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e
^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*e^2*((409
6*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2)
- 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096
*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*
x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^
```

```
4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d
*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6
*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)
^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))...
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^2}\,dx$

Optimal. Leaf size=163

$$\frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)}+\frac{C\sin^{-1}(dx)}{df^2}-\frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\,\sqrt{1}}\right.}{f^2\left(d^2e^2-f^2\right)^{3/2}}$$

```
[Out] C*arcsin(d*x)/d/f^2-(-A*d^2*e*f^2+C*d^2*e^3+B*f^3-2*C*e*f^2)*arctan((d^2*e*
x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(3/2)+(A*f^2
-B*e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)
```

Rubi [A]
time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1623, 1665, 858, 222, 739, 210}

$$-\frac{\text{ArcTan}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{\sqrt{1-d^2x^2}\,(Af^2-Bef+Ce^2)}{f\left(d^2e^2-f^2\right)(e+fx)}+\frac{C\text{ArcSin}(dx)}{df^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)} + \frac{\int \frac{Ce+Ad^2e-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)} + \frac{C\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(2Ce+A\ldots\right.}{\ldots}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} - \frac{\left(2Ce+Ad^2e-\frac{C\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f\,(d^2e^2-f^2)\,(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} + \frac{\left(2Ce+Ad^2e-\frac{C\ldots}{\ldots}\right.}{\ldots}$$

Mathematica [A]

time = 10.29, size = 211, normalized size = 1.29

$$\frac{-\frac{f(Ce^2+f(-Be+Af))\sqrt{1-d^2x^2}}{(-d^2e^2+f^2)(e+fx)}+\frac{C\sin^{-1}(dx)}{d}+\frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\log(e+fx)}{(-d^2e^2+f^2)^{3/2}}-\frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\log\left(f+d^2ex+\sqrt{-d^2e^2+f^2}\sqrt{1-d^2x^2}\right)}{(-d^2e^2+f^2)^{3/2}}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] (-((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((-(d^2*e^2) + f^2)*(e
+ f*x))) + (C*ArcSin[d*x])/d + ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^
3)*Log[e + f*x])/(-(d^2*e^2) + f^2)^(3/2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2
*e*f^2 + B*f^3)*Log[f + d^2*e*x + Sqrt[-(d^2*e^2) + f^2]*Sqrt[1 - d^2*x^2]]
)/(-(d^2*e^2) + f^2)^(3/2))/f^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 899, normalized size = 5.52

method	result
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e\,f^3x+C\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f}{f^2}}}{fx+e}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*d^3*e*f^3*x+C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^
2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^3*f*x-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn(d)*
ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d
^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2*f^2*x*(-(d^2*e^2-f^
2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)
+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)
/(f*x+e))*d*f^4*x-B*csgn(d)*d*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)
^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)*f+f)/(f*x+e))*d*e*f^3*x+C*csgn(d)*d*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e
^2-f^2)/f^2)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^3*f*(-(d^
2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2
-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+
1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-C*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*f^4*x*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)*(-d*x+1)
```

```
^(1/2)*(d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/d/(d*e+f)/(f*x+e)/(-(d^2*e^
2-f^2)/f^2)^(1/2)/f^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-%e*d>0)', see `assume?` for more
details
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(155) = 310$.
time = 40.52, size = 969, normalized size = 5.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-(A*d*f^6*x - C*d^3*f*e^5 + (B*d*f^4*x*e + C*d^3*f*x*e^4 + C*d^3*e^5 - (A*
d^3 + 2*C*d)*f^2*e^3 + (B*d*f^3 - (A*d^3 + 2*C*d)*f^3*x)*e^2)*sqrt(-d^2*e^2
 + f^2)*log((d^2*f*x*e - (d^2*e^2 - f^2 - sqrt(-d^2*e^2 + f^2)*f)*sqrt(d*x
+ 1)*sqrt(-d*x + 1) + f^2 + (d^2*x*e + f)*sqrt(-d^2*e^2 + f^2))/(f*x + e))
+ (A*d*f^5*e + B*d^3*f^2*e^4 - B*d*f^4*e^2 - C*d^3*f*e^5 - (A*d^3 - C*d)*f^
3*e^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^4*f*x*e^5 - 2*C*d^2*f^3*x*e^3
+ C*f^5*x*e + C*d^4*e^6 - 2*C*d^2*f^2*e^4 + C*f^4*e^2)*arctan((sqrt(d*x + 1
)*sqrt(-d*x + 1) - 1)/(d*x)) - (C*d^3*f^2*x - B*d^3*f^2)*e^4 + (B*d^3*f^3*x
 - (A*d^3 - C*d)*f^3)*e^3 - (B*d*f^4 + (A*d^3 - C*d)*f^4*x)*e^2 - (B*d*f^5*
x - A*d*f^5)*e)/(d^5*f^3*x*e^5 - 2*d^3*f^5*x*e^3 + d*f^7*x*e + d^5*f^2*e^6
- 2*d^3*f^4*e^4 + d*f^6*e^2), -(A*d*f^6*x - C*d^3*f*e^5 - 2*(B*d*f^4*x*e +
C*d^3*f*x*e^4 + C*d^3*e^5 - (A*d^3 + 2*C*d)*f^2*e^3 + (B*d*f^3 - (A*d^3 + 2
*C*d)*f^3*x)*e^2)*sqrt(d^2*e^2 - f^2)*arctan(-sqrt(d^2*e^2 - f^2)*(f*x - sq
rt(d*x + 1)*sqrt(-d*x + 1)*e + e)/(d^2*x*e^2 - f^2*x)) + (A*d*f^5*e + B*d^3
*f^2*e^4 - B*d*f^4*e^2 - C*d^3*f*e^5 - (A*d^3 - C*d)*f^3*e^3)*sqrt(d*x + 1)
*sqrt(-d*x + 1) + 2*(C*d^4*f*x*e^5 - 2*C*d^2*f^3*x*e^3 + C*f^5*x*e + C*d^4*
e^6 - 2*C*d^2*f^2*e^4 + C*f^4*e^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1
)/(d*x)) - (C*d^3*f^2*x - B*d^3*f^2)*e^4 + (B*d^3*f^3*x - (A*d^3 - C*d)*f^3
)*e^3 - (B*d*f^4 + (A*d^3 - C*d)*f^4*x)*e^2 - (B*d*f^5*x - A*d*f^5)*e)/(d^5
```

```
*f^3*x*e^5 - 2*d^3*f^5*x*e^3 + d*f^7*x*e + d^5*f^2*e^6 - 2*d^3*f^4*e^4 + d*
f^6*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(e + fx)^2 \sqrt{-dx + 1}\sqrt{dx + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Undef/Unsigned Inf encountered in lim
itLimit: Max order reached or unable to make series expansion Error: Bad Ar
gument
```

Mupad [B]
time = 0.01, size = 2500, normalized size = 15.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*
1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1
)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (
2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d
```

*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 - d*x)^(1/2)
- 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (A*d^5*e^5*at
an(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)
^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3
*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x
+ 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (A*d^5*e^5*atan(
((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2)
)*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 - (4*A*f^2*((1 - d*x)
^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (A
*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/
2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (
A*d^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 -
(A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
 - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
 d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3
e^3((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 +
 (A*d^4*e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d
^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d^4*

```
e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 + (8*A*d*e
*f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
) - 1)^2)/(d^3*e^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - d*e^2*f^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2) - (4*e*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d
*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (4*e*f^3*((1 - d*x)^(1/2) - 1)^3*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (2*d^3*e^4*((1 - d*x)^(
1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((...
```

3.14 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3}\,dx$

Optimal. Leaf size=248

$$\frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2}-\frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)}+\frac{(C(d^2e^2+$$

```
[Out] 1/2*(C*(d^2*e^2+2*f^2)-d^2*(3*B*e*f-A*(2*d^2*e^2+f^2)))*arctan((d^2*e*x+f)/
(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/(d^2*e^2-f^2)^(5/2)+1/2*(A*f^2-B*e*
f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)^2-1/2*(-3*A*d^2*e*f^2+B
*d^2*e^2*f+C*d^2*e^3+2*B*f^3-4*C*e*f^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)^
2/(f*x+e)
```

Rubi [A]

time = 0.21, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1623, 1665, 821, 739, 210}

$$\frac{\text{ArcTan}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))}{2(d^2e^2-f^2)^{5/2}}+\frac{\sqrt{1-d^2x^2}\,(Af^2-Bef+Ce^2)}{2f(d^2e^2-f^2)(e+fx)^2}-\frac{\sqrt{1-d^2x^2}\,(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{2f(d^2e^2-f^2)^2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} + \frac{\int \frac{2\left(Ce+Ad^2e-Bf\right)+\left(Bd^2e+\frac{Cd^2e^2}{f}-2Cf-A\right.}{(e+fx)^2\sqrt{1-d^2x^2}}}{2\left(d^2e^2-f^2\right)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2\right.}{2f\left(d^2e^2-f^2\right)^2(}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2\right.}{2f\left(d^2e^2-f^2\right)^2(}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2\right.}{2f\left(d^2e^2-f^2\right)^2(}$$

Mathematica [A]

time = 10.17, size = 273, normalized size = 1.10

$$\frac{1}{2}\left(-\frac{\sqrt{1-d^2x^2}\,(Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)-Ad^2ef(4e+3fx)+Ce(-3ef+d^2e^2x-4f^2x))}{(-d^2e^2+f^2)^2\,(e+fx)^2}+\frac{(C(d^2e^2+2f^2)+d^2(-3Bef+A(2d^2e^2+f^2)))\log(e+fx)}{(-d^2e^2+f^2)^{5/2}}-\frac{(C(d^2e^2+2f^2)+d^2(-3Bef+A(2d^2e^2+f^2)))\log\left(f+d^2ex+\sqrt{-d^2e^2+f^2}\,\sqrt{1-d^2x^2}\right)}{(-d^2e^2+f^2)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] (-((Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x) + B*f^2*(e + 2*f*x) -
A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x - 4*f^2*x)))/((-(d^2*e^2)
 + f^2)^2*(e + f*x)^2)) + ((C*(d^2*e^2 + 2*f^2) + d^2*(-3*B*e*f + A*(2*d^2*
e^2 + f^2)))*Log[e + f*x])/(-(d^2*e^2) + f^2)^(5/2) - ((C*(d^2*e^2 + 2*f^2)
 + d^2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*Log[f + d^2*e*x + Sqrt[-(d^2*e^2)
+ f^2]*Sqrt[1 - d^2*x^2]])/(-(d^2*e^2) + f^2)^(5/2))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 1449, normalized size = 5.84

method	result
default	$-\frac{\left(2B\,f^4x\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}+Be\,f^3\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}-3B\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-}}{fx+e}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f
*x+e))*d^2*f^4*x^2+A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^
(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2
*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f+4*C*ln(2*(d^2*e*x+(-d^2*x^2+1)
^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e*f^3*x+2*B*f^4*x*(-d^2*x^2
+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+B*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-
f^2)/f^2)^(1/2)+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(
1/2)*f+f)/(f*x+e))*d^4*e^2*f^2*x^2+4*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(
d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^4*e^3*f*x-3*B*ln(2*(d^2*e*x+(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f^3*x^2+C*ln(2*(
d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2
*f^2*x^2+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^2*e*f^3*x-6*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2
)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*x+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f*x-4*A*d^2*e^2*f^2*(-d
^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*B*d^2*e^3*f*(-d^2*x^2+1)^(1/2)
*(-(d^2*e^2-f^2)/f^2)^(1/2)-4*C*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/
```

```
f^2)^(1/2)+C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^2*e^4+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^
2)^(1/2)*f+f)/(f*x+e))*f^4*x^2+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*
e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e^2*f^2+A*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e
^2-f^2)/f^2)^(1/2)+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2
)^(1/2)*f+f)/(f*x+e))*d^4*e^4-3*A*d^2*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2
-f^2)/f^2)^(1/2)+B*d^2*e^2*f^2*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)+C*d^2*e^3*f*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-3*C*e^2*f^2
*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)^2*(-d*x+1)^(1/2)*(d
*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/(d*e+f)/(d^2*e^2-f^2)/(f*x+e)^2/(-(d
^2*e^2-f^2)/f^2)^(1/2)/f
```

<u>**Maxima**</u> **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-%e*d>0)', see `assume?` for more
details
```

<u>**Fricas**</u> **[B]** Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(232) = 464$.
time = 1.21, size = 1502, normalized size = 6.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(A*f^7*x^2 - 2*B*d^4*e^7 - ((A*d^2 + 2*C)*f^4*x^2*e^2 + (2*A*d^4 + C*
d^2)*e^6 - (3*B*d^2*f - 2*(2*A*d^4 + C*d^2)*f*x)*e^5 - (6*B*d^2*f^2*x - (2*
A*d^4 + C*d^2)*f^2*x^2 - (A*d^2 + 2*C)*f^2)*e^4 - (3*B*d^2*f^3*x^2 - 2*(A*d
^2 + 2*C)*f^3*x)*e^3)*sqrt(-d^2*e^2 + f^2)*log((d^2*f*x*e - (d^2*e^2 - f^2
+ sqrt(-d^2*e^2 + f^2)*f)*sqrt(d*x + 1)*sqrt(-d*x + 1) + f^2 - (d^2*x*e + f
)*sqrt(-d^2*e^2 + f^2))/(f*x + e)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*((C*d^4*x
 + 2*B*d^4)*e^7 + (B*d^4*f*x - (4*A*d^4 + 3*C*d^2)*f)*e^6 - (B*d^2*f^2 + (3
*A*d^4 + 5*C*d^2)*f^2*x)*e^5 + (B*d^2*f^3*x + (5*A*d^2 + 3*C)*f^3)*e^4 + ((
3*A*d^2 + 4*C)*f^4*x - B*f^4)*e^3 - (2*B*f^5*x + A*f^5)*e^2) - (4*B*d^4*f*x
 - (4*A*d^4 + 3*C*d^2)*f)*e^6 - (2*B*d^4*f^2*x^2 - B*d^2*f^2 - 2*(4*A*d^4 +
```

```
 3*C*d^2)*f^2*x)*e^5 + (2*B*d^2*f^3*x + (4*A*d^4 + 3*C*d^2)*f^3*x^2 - (5*A*
d^2 + 3*C)*f^3)*e^4 + (B*d^2*f^4*x^2 - 2*(5*A*d^2 + 3*C)*f^4*x + B*f^4)*e^3
 - ((5*A*d^2 + 3*C)*f^5*x^2 - 2*B*f^5*x - A*f^5)*e^2 + (B*f^6*x^2 + 2*A*f^6
*x)*e)/(f^8*x^2*e^2 - 2*d^6*f*x*e^9 + 6*d^4*f^3*x*e^7 - 6*d^2*f^5*x*e^5 + 2
*f^7*x*e^3 - d^6*e^10 - (d^6*f^2*x^2 - 3*d^4*f^2)*e^8 + 3*(d^4*f^4*x^2 - d^
2*f^4)*e^6 - (3*d^2*f^6*x^2 - f^6)*e^4), -1/2*(A*f^7*x^2 - 2*B*d^4*e^7 - 2*
((A*d^2 + 2*C)*f^4*x^2*e^2 + (2*A*d^4 + C*d^2)*e^6 - (3*B*d^2*f - 2*(2*A*d^
4 + C*d^2)*f*x)*e^5 - (6*B*d^2*f^2*x - (2*A*d^4 + C*d^2)*f^2*x^2 - (A*d^2 +
 2*C)*f^2)*e^4 - (3*B*d^2*f^3*x^2 - 2*(A*d^2 + 2*C)*f^3*x)*e^3)*sqrt(d^2*e^
2 - f^2)*arctan(-sqrt(d^2*e^2 - f^2)*(f*x - sqrt(d*x + 1)*sqrt(-d*x + 1)*e
+ e)/(d^2*x*e^2 - f^2*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*((C*d^4*x + 2*B*d^
4)*e^7 + (B*d^4*f*x - (4*A*d^4 + 3*C*d^2)*f)*e^6 - (B*d^2*f^2 + (3*A*d^4 +
5*C*d^2)*f^2*x)*e^5 + (B*d^2*f^3*x + (5*A*d^2 + 3*C)*f^3)*e^4 + ((3*A*d^2 +
 4*C)*f^4*x - B*f^4)*e^3 - (2*B*f^5*x + A*f^5)*e^2) - (4*B*d^4*f*x - (4*A*d
^4 + 3*C*d^2)*f)*e^6 - (2*B*d^4*f^2*x^2 - B*d^2*f^2 - 2*(4*A*d^4 + 3*C*d^2)
*f^2*x)*e^5 + (2*B*d^2*f^3*x + (4*A*d^4 + 3*C*d^2)*f^3*x^2 - (5*A*d^2 + 3*C
)*f^3)*e^4 + (B*d^2*f^4*x^2 - 2*(5*A*d^2 + 3*C)*f^4*x + B*f^4)*e^3 - ((5*A*
d^2 + 3*C)*f^5*x^2 - 2*B*f^5*x - A*f^5)*e^2 + (B*f^6*x^2 + 2*A*f^6*x)*e)/(f
^8*x^2*e^2 - 2*d^6*f*x*e^9 + 6*d^4*f^3*x*e^7 - 6*d^2*f^5*x*e^5 + 2*f^7*x*e^
3 - d^6*e^10 - (d^6*f^2*x^2 - 3*d^4*f^2)*e^8 + 3*(d^4*f^4*x^2 - d^2*f^4)*e^
6 - (3*d^2*f^6*x^2 - f^6)*e^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Undef/Unsigned Inf encountered in lim
itLimit: Max order reached or unable to make series expansion Error: Bad Ar
gument
```

Mupad [B]
time = 0.01, size = 2500, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) - 1
)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 - d*
x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2) -
1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*d^3*e
^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)
) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1
/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) - 1)^5*(7
*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2
*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/(((d*x + 1
)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2)
 - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) -
 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1
)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/
2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*
x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1 - d*x)^(1/2) - 1)^
2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^2
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1/2) - 1)^4*(2*A*f^5 + 4
*A*d^4*e^4*f - 9*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (4*((1 - d*x)^(1/2) - 1)^6*(4*A*d^4*e^4*f - 2*A*f^5 +
 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*
f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^7*(2*A*d*f^3 - 5*A*d^3*e^2*f))/(e*((d*x
+ 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*((1 - d*x)^(1/2)
- 1)^3*(2*A*d*f^3 - 29*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^3*(f^4 + d^4*
e^4 - 2*d^2*e^2*f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^5*(2*A*d*f^3 - 29*A*d^3*
e^2*f))/(e*((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*d*
f*(2*A*f^3 - 5*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/(e*((d*x + 1)^(1/2) - 1)
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2*(16*
f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(16*f^
2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32*f^2
- 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)^8)/(
(d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2)
 - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (8*d*
e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^
(1/2) - 1))/((d*x + 1)^(1/2) - 1)) - ((4*((1 - d*x)^(1/2) - 1)^2*(2*B*f^4 +
```

```
 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4
- 2*d^2*e^2*f^2)) - (8*((1 - d*x)^(1/2) - 1)^4*(2*B*f^4 - 2*B*d^4*e^4 + 3*B
*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (4*((1 - d*x)^(1/2) - 1)^6*(2*B*f^4 + 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*
((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*(11*B*d^3*e
^2 + 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^3)/(((d*x + 1)^(1/2) - 1)^3*(f^4 + d
^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*(11*B*d^3*e^2 + 16*B*d*f^2)*((1 - d*x)^(1/2
) - 1)^5)/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (6*B*
d^3*e^2*f*((1 - d*x)^(1/2) - 1)^7)/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4
- 2*d^2*e^2*f^2)) + (6*B*d^3*e^2*f*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2
*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(
16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32
*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)
^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^
(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 -
(8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + (C*atan(((C*(2*f^2 + d^2*e^2)*((4
*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/
(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4
*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8
+ d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^
2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 -
12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^
4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f
^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*
x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2
*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))
- (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*f^7 + 4*C*d...
```

3.15 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=79

$$-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*b*arcsin(d*x)/d^3-1/3*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/6*(3*b*d^2*x+6*a*d
^2+4*c)*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1623, 1823, 794, 222}

$$-\frac{\sqrt{1-d^2x^2}\,(2(3ad^2+2c)+3bd^2x)}{6d^4}+\frac{b\text{ArcSin}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(c*x^2*Sqrt[1 - d^2*x^2])/d^2 - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{x(a+bx+cx^2)}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{\int \frac{x(-2c-3ad^2-3bd^2x)}{\sqrt{1-d^2x^2}}\,dx}{3d^2}\\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4}+\frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 1.13

$$\frac{-\sqrt{1-d^2x^2}\,(3d^2(2a+bx)+2c(2+d^2x^2))+3b\sqrt{-d^2}\,\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (-(Sqrt[1 - d^2*x^2]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))) + 3*b*Sqrt[-d
^2]*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(6*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 139, normalized size = 1.76

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2\,\mathrm{csgn}(d)c\,d^2x^2\sqrt{-d^2x^2+1}+3\,\mathrm{csgn}(d)\sqrt{-d^2x^2+1}\,b\,d^2x+6\,\mathrm{csgn}(d)\sqrt{-d^2x^2+1}\right.}{6d^4\sqrt{-d^2x^2+1}}$

risch	$\frac{(2c x^2 d^2+3b d^2 x+6a d^2+4c)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{6d^4\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}} + \frac{b\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)}}{2d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{dx+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*c*d^2*x^2*(-d^2*x^2+1)^(1/2)+3
*csgn(d)*(-d^2*x^2+1)^(1/2)*b*d^2*x+6*csgn(d)*(-d^2*x^2+1)^(1/2)*a*d^2+4*cs
gn(d)*(-d^2*x^2+1)^(1/2)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)*cs
gn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.61, size = 87, normalized size = 1.10

$$-\frac{\sqrt{-d^2x^2+1}\,cx^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}\,bx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,a}{d^2}+\frac{b\arcsin\left(dx\right)}{2\,d^3}-\frac{2\,\sqrt{-d^2x^2+1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

Fricas [A]
time = 0.67, size = 78, normalized size = 0.99

$$-\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)+\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\,\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.85, size = 76, normalized size = 0.96

$$\frac{6\,bd\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right) - \left(6\,ad^2 + \left(2\,(dx+1)c + 3\,bd - 4\,c\right)(dx+1) - 3\,bd + 6\,c\right)\sqrt{dx+1}\,\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*d*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - (6*a*d^2 + (2*(d*x + 1)*c +
3*b*d - 4*c)*(d*x + 1) - 3*b*d + 6*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Mupad [B]
time = 7.61, size = 244, normalized size = 3.09

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}-\frac{\sqrt{1-d\,x}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x
 + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d
) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

3.16 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A]

time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {913, 1829, 655, 222}

$$\frac{(2ad^2+c)\,\text{ArcSin}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
```

```
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{(-c-2ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 1.30

$$
\frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2} + \frac{\sqrt{-d^2}\,(c+2ad^2)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{2d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + (Sqrt[-d^2]*(c + 2*a*d^2)*Log[-(
Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(2*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 117, normalized size = 1.86

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)dcx-2\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)a\,d^2+2\,\mathrm{csgn}(d)d\sqrt{-d^2x^2+1}\right.}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(cx+2b)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)\,(dx+1)}}{2d^2\sqrt{-\,(dx+1)\,(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)a}{\sqrt{d^2}}+\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)c}{2d^2\sqrt{d^2}}\right)}{\sqrt{-dx+1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*((-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Maxima [A]
time = 0.58, size = 57, normalized size = 0.90

$$\frac{a \arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}\,cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}\,b}{d^2} + \frac{c \arcsin(dx)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

Fricas [A]
time = 0.97, size = 67, normalized size = 1.06

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.45, size = 60, normalized size = 0.95

$$-\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\,\sqrt{-dx+1}-2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2 +
 c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B]
time = 7.41, size = 232, normalized size = 3.68

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

3.17 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] b*arcsin(d*x)/d-a*arctanh((-d^2*x^2+1)^(1/2))-c*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {1623, 1823, 858, 222, 272, 65, 214}

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\text{ArcSin}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}-\frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2}\\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx+b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right)\\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-\frac{a\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right)}{d^2}\\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A]

time = 0.04, size = 93, normalized size = 1.94

$$-\frac{c\sqrt{1-d^2x^2}}{d^2}+2a\tanh^{-1}\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)-\frac{b\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + 2*a*ArcTanh[Sqrt[-d^2]*x - Sqrt[1 - d^2*x^2]
] - (b*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/Sqrt[-d^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 96, normalized size = 2.00

method	result
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-\mathrm{csgn}(d)\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^2-\mathrm{csgn}(d)\sqrt{-d^2x^2+1}\,c+\arctan\left(\frac{\mathrm{csgn}(d)}{\sqrt{-(dx+1)}}\right.\right.}{d^2\sqrt{-d^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^2*(-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*
d^2-csgn(d)*(-d^2*x^2+1)^(1/2)*c+arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2
))*b*d)*csgn(d)/(-d^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.58, size = 57, normalized size = 1.19

$$-a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*c/d^2
```

Fricas [A]
time = 1.37, size = 81, normalized size = 1.69

$$\frac{ad^2\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\,\sqrt{-dx+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Sympy [C] Result contains complex when optimal does not.
time = 43.36, size = 245, normalized size = 5.10

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [42,56
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[-9,-13]1/sageVA
```

Mupad [B]
time = 4.33, size = 122, normalized size = 2.54

$$a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\sqrt{1-d\,x}\,\left(\frac{c}{d^2}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d))/
(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(d^2)^(1/2))))/(d^2)^(1/2)
```

3.18 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\sin^{-1}(dx)}{d}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] c*arcsin(d*x)/d-b*arctanh((-d^2*x^2+1)^(1/2))-a*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {1623, 1821, 858, 222, 272, 65, 214}

$$-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\text{ArcSin}(dx)}{d}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2}\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 93, normalized size = 1.94

$$-\frac{a\sqrt{1-d^2x^2}}{x}+2b\tanh^{-1}\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)-\frac{c\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + 2*b*ArcTanh[Sqrt[-d^2]*x - Sqrt[1 - d^2*x^2]]
- (c*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/Sqrt[-d^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 97, normalized size = 2.02

method	result
default	$\frac{\left(-\mathrm{csgn}(d)d\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)bx-\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)da+\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)cx\right)\sqrt{-dx+1}\,\sqrt{d}}{\sqrt{-d^2x^2+1}\,xd}$
risch	$\frac{a\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{x\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{c\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}-b\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\right)\sqrt{(-}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(d)*d*arctanh(1/(-d^2*x^2+1)^(1/2))*b*x-(-d^2*x^2+1)^(1/2)*csgn(d)*d*
a+arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c*x)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*
csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

Maxima [A]
time = 0.48, size = 57, normalized size = 1.19

$$-b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^2x^2+1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*a/x
```

Fricas [A]
time = 1.21, size = 84, normalized size = 1.75

$$\frac{bdx\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-\sqrt{dx+1}\,\sqrt{-dx+1}\,ad-2\,cx\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 41.01, size = 221, normalized size = 4.60

$$\frac{iadG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{adG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [42,56
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[-9,-13](-2*sage
```

Mupad [B]
time = 4.27, size = 114, normalized size = 2.38

$$b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1))/
(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(d*x
 + 1)^(1/2))/x
```

3.19 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=71

$$-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] -1/2*(a*d^2+2*c)*arctanh((-d^2*x^2+1)^(1/2))-1/2*a*(-d^2*x^2+1)^(1/2)/x^2-b
*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1623, 1821, 821, 272, 65, 214}

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*(a*Sqrt[1 - d^2*x^2])/x^2 - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2b-(2c+ad^2)\,x}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(-2c-ad^2\right)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{4}\left(-2c-ad^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x \right. \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(a+\frac{2c}{d^2}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-} \right. \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 69, normalized size = 0.97

$$
\frac{(-a-2bx)\sqrt{1-d^2x^2}}{2x^2}+\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-a - 2*b*x)*Sqrt[1 - d^2*x^2])/(2*x^2) + (2*c + a*d^2)*ArcTanh[Sqrt[-d^2]
*x - Sqrt[1 - d^2*x^2]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 108, normalized size = 1.52

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)^{2}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)a\,d^{2}x^{2}+2\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)c\,x^{2}+2\sqrt{-d^{2}x^{2}}\right.}{2\sqrt{-d^{2}x^{2}+1}\,x^{2}}$
risch	$\frac{\sqrt{dx+1}\,(dx-1)(2bx+a)\sqrt{(-dx+1)\,(dx+1)}}{2x^{2}\sqrt{-\,(dx+1)\,(dx-1)}\,\sqrt{-dx+1}}-\frac{\left(c+\frac{a\,d^{2}}{2}\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)\sqrt{(-dx+1)\,(dx+}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
a*d^2*x^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*c*x^2+2*(-d^2*x^2+1)^(1/2)*b*x+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

Maxima [A]
time = 0.50, size = 98, normalized size = 1.38

$$-\frac{1}{2}\,ad^{2}\log\left(\frac{2\sqrt{-d^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-c\log\left(\frac{2\sqrt{-d^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-d^{2}x^{2}+1}\,b}{x}-\frac{\sqrt{-d^{2}x^{2}+1}\,a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

Fricas [A]
time = 1.10, size = 65, normalized size = 0.92

$$\frac{(ad^{2}+2\,c)x^{2}\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\,\sqrt{-dx+1}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [42,56
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[-9,-13](-1/2*(s
```

Mupad [**B**]
time = 6.30, size = 312, normalized size = 4.39

$$c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2}{2}+\frac{15\,a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^4}{2\,\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\,\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\,\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}+\frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)}{2}-\frac{a\,d^2\,\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2}-\frac{b\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{32\,\left(\sqrt{d\,x+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] c*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2*((
d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 - d*x)^
(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log(((1 - d*x)^(1/2) - 1)^2
/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log(((1 - d*x)^(1/2) - 1)/((d*x +
 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2*((1 - d
*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.20 $\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=591

$$\frac{\left(A\left(8b^4e^3+6a^2b^2ef^2\right)+a^2\left(a^2f^2(3Ce+Bf)+2b^2e^2(Ce+3Bf)\right)\right)x\sqrt{a+bx}\,\sqrt{ac-bcx}}{16b^4}-\frac{\left(8a^2Cf^2-b^2(3C\right.}{}$$

```
[Out] 1/16*(A*(6*a^2*b^2*e*f^2+8*b^4*e^3)+a^2*(a^2*f^2*(B*f+3*C*e)+2*b^2*e^2*(3*B
*f+C*e)))*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4-1/70*(8*a^2*C*f^2-b^2*(3*C
*e^2-7*f*(2*A*f+B*e)))*(f*x+e)^2*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^
(1/2)/b^4/f+1/42*(-7*B*f+3*C*e)*(f*x+e)^3*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*
c*x+a*c)^(1/2)/b^2/f-1/7*C*(f*x+e)^4*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a
*c)^(1/2)/b^2/f-1/840*(64*a^4*C*f^4+16*a^2*b^2*f^2*(15*C*e^2+7*f*(A*f+3*B*e
))-8*b^4*e^2*(3*C*e^2-7*f*(12*A*f+B*e))+3*b^2*f*(a^2*f^2*(35*B*f+41*C*e)-2*
b^2*e*(3*C*e^2-7*f*(7*A*f+B*e)))*x)*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*
c)^(1/2)/b^6/f+1/16*a^2*(A*(6*a^2*b^2*e*f^2+8*b^4*e^3)+a^2*(a^2*f^2*(B*f+3*
C*e)+2*b^2*e^2*(3*B*f+C*e)))*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*c
^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^5/(-b^2*c*x^2+a^2*c)^(1/2)
```

Rubi [A]
time = 0.98, antiderivative size = 584, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1624, 1668, 847, 794, 201, 223, 209}

$$\frac{\sqrt{a+bx}\,(a^2-b^2x^2)\,(e+fx)^2\sqrt{ac-bcx}\left(-\frac{8a^2Cf^2}{b^2}-7f(2Af+Be)+3Ce^2\right)}{70b^2f}+\frac{\sqrt{a+bx}\,(a^2-b^2x^2)\,(e+fx)^3\sqrt{ac-bcx}\,(3Ce-7Bf)}{42b^2f}-\frac{C\sqrt{a+bx}\,(a^2-b^2x^2)\,(e+fx)^4\sqrt{ac-bcx}}{7b^2f}+\frac{a^2\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^4f^2(Bf+3Ce)+A(6a^2b^2ef^2+8b^4e^3)+2a^2b^2e^2(3Bf+Ce)\right)}{16b^5\sqrt{a^2c-b^2cx^2}}+\frac{x\sqrt{a+bx}\,\sqrt{ac-bcx}\left(a^4f^2(Bf+3Ce)+A(6a^2b^2ef^2+8b^4e^3)+2a^2b^2e^2(3Bf+Ce)\right)}{16b^4}-\frac{\sqrt{a+bx}\,(a^2-b^2x^2)\sqrt{ac-bcx}\left(3b^2fx\left(a^2f^2(35Bf+41Ce)-b^2(6Ce^3-14ef(7Af+Be))\right)+8\left(8a^4Cf^4+2a^2b^2f^2(7f(Af+3Be)+15Ce^2)-\left(b^4(3Ce^4-7e^2f(12Af+Be))\right)\right)\right)}{840b^6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] ((a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^
2*b^2*e*f^2))*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((3*C*e^2 - (8*
a^2*C*f^2)/b^2 - 7*f*(B*e + 2*A*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*
x)^2*(a^2 - b^2*x^2))/(70*b^2*f) + ((3*C*e - 7*B*f)*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(42*b^2*f) - (C*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]*(e + f*x)^4*(a^2 - b^2*x^2))/(7*b^2*f) - (Sqrt[a + b*x]*Sqrt[a*c
 - b*c*x]*(8*(8*a^4*C*f^4 + 2*a^2*b^2*f^2*(15*C*e^2 + 7*f*(3*B*e + A*f)) -
b^4*(3*C*e^4 - 7*e^2*f*(B*e + 12*A*f))) + 3*b^2*f*(a^2*f^2*(41*C*e + 35*B*f
) - b^2*(6*C*e^3 - 14*e*f*(B*e + 7*A*f)))*x)*(a^2 - b^2*x^2))/(840*b^6*f) +
 (a^2*Sqrt[c]*(a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b
^4*e^3 + 6*a^2*b^2*e*f^2))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c
]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3\left(A+Bx+Cx^2\right)dx = \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\right)\int (e+fx)^3\sqrt{a^2c-b^2cx^2}\,(\ldots}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f} - \ldots$$

$$= \frac{(3Ce-7Bf)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3\left(a^2-\ldots\right.}{42b^2f}$$

$$= -\frac{\left(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af))\right)\sqrt{a+bx}\,\ldots}{70b^4f}$$

$$= -\frac{\left(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af))\right)\sqrt{a+bx}\,\ldots}{70b^4f}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3\ldots\right.}{16b^4}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3\ldots\right.}{16b^4}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3\ldots\right.}{16b^4}$$

Mathematica [A]
time = 1.18, size = 402, normalized size = 0.68

$$\frac{\sqrt{c(a-bx)}\left(-\sqrt{a-bx}\,\sqrt{a+bx}\,\left(128a^6Cf^3+a^4b^2f(7f(96Be+32Af+15Bfx)+C(672e^2+315efx+64f^2x^2))+2a^2b^4(7Af(120e^2+45efx+8f^2x^2)+7B(40e^3+45e^2fx+24ef^2x^2+5f^3x^3)+3Cx(35e^3+56e^2fx+35ef^2x^2+8f^3x^3))-4b^6x(21A(10e^3+20e^2fx+15ef^2x^2+4f^3x^3)+x(7B(20e^3+45e^2fx+36ef^2x^2+10f^3x^3)+3Cx(35e^3+84e^2fx+70ef^2x^2+20f^3x^3)))\right)+210a^2b\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2)\right)\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{1680b^6\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(-(Sqrt[a - b*x]*Sqrt[a + b*x]*(128*a^6*C*f^3 + a^4*b^2*
f*(7*f*(96*B*e + 32*A*f + 15*B*f*x) + C*(672*e^2 + 315*e*f*x + 64*f^2*x^2))
 + 2*a^2*b^4*(7*A*f*(120*e^2 + 45*e*f*x + 8*f^2*x^2) + 7*B*(40*e^3 + 45*e^2
*f*x + 24*e*f^2*x^2 + 5*f^3*x^3) + 3*C*x*(35*e^3 + 56*e^2*f*x + 35*e*f^2*x^
2 + 8*f^3*x^3)) - 4*b^6*x*(21*A*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3
*x^3) + x*(7*B*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3) + 3*C*x*(3
5*e^3 + 84*e^2*f*x + 70*e*f^2*x^2 + 20*f^3*x^3))))) + 210*a^2*b*(a^4*f^2*(3
*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^2*b^2*e*f^2)
)*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(1680*b^6*Sqrt[a - b*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. $2(544) = 1088$.
time = 0.12, size = 1370, normalized size = 2.32 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/1680*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)*(-336*B*a^2*b^4*e*f^2*x^2*(c*(-b^2*
x^2+a^2))^(1/2)*(b^2*c)^(1/2)-336*C*a^2*b^4*e^2*f*x^2*(c*(-b^2*x^2+a^2))^(1
/2)*(b^2*c)^(1/2)-210*C*a^2*b^4*e*f^2*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^
(1/2)+840*A*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*b^6*c*e^3+
105*B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^6*b^2*c*f^3+420*C*
b^6*e^3*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+560*B*b^6*e^3*x^2*(c*(-b
^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-224*A*a^4*b^2*f^3*(c*(-b^2*x^2+a^2))^(1/2)
*(b^2*c)^(1/2)-560*B*a^2*b^4*e^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+336
*A*b^6*f^3*x^4*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+210*C*arctan((b^2*c)^
(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*b^4*c*e^3+840*A*(c*(-b^2*x^2+a^2))^(1
/2)*(b^2*c)^(1/2)*b^6*e^3*x+240*C*b^6*f^3*x^6*(c*(-b^2*x^2+a^2))^(1/2)*(b^2
*c)^(1/2)+280*B*b^6*f^3*x^5*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-128*C*a^
6*f^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-630*A*(c*(-b^2*x^2+a^2))^(1/2)
*(b^2*c)^(1/2)*a^2*b^4*e*f^2*x+1260*A*b^6*e*f^2*x^3*(c*(-b^2*x^2+a^2))^(1/2
)*(b^2*c)^(1/2)-630*B*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)*a^2*b^4*e^2*f*
x-315*C*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)*a^4*b^2*e*f^2*x+630*A*arctan
((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*b^4*c*e*f^2+630*B*arctan((b^
2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*b^4*c*e^2*f+315*C*arctan((b^2*c)
^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^6*b^2*c*e*f^2-210*C*(c*(-b^2*x^2+a^2))
^(1/2)*(b^2*c)^(1/2)*a^2*b^4*e^3*x-105*B*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(
1/2)*a^4*b^2*f^3*x-70*B*a^2*b^4*f^3*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1
/2)+1260*B*b^6*e^2*f*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-112*A*a^2*b
^4*f^3*x^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+1680*A*b^6*e^2*f*x^2*(c*(
-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-64*C*a^4*b^2*f^3*x^2*(c*(-b^2*x^2+a^2))^
(1/2)*(b^2*c)^(1/2)+840*C*b^6*e*f^2*x^5*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1
/2)+1008*B*b^6*e*f^2*x^4*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-48*C*a^2*b^
4*f^3*x^4*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+1008*C*b^6*e^2*f*x^4*(c*(-
b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-1680*A*a^2*b^4*e^2*f*(c*(-b^2*x^2+a^2))^(
```

```
1/2)*(b^2*c)^(1/2)-672*B*a^4*b^2*e*f^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/
2)-672*C*a^4*b^2*e^2*f*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2))/(c*(-b^2*x^2
+a^2))^(1/2)/b^6/(b^2*c)^(1/2)
```

Maxima [A]
time = 0.51, size = 581, normalized size = 0.98

$$-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cf^3x^4}{7\,b^2c}-\frac{4\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f^3x^2}{35\,b^4c}+\frac{Aa^2\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)e^3}{2\,b}+\frac{(Bf^3+3\,Cf^2e)a^6\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{16\,b^5}+\frac{(3\,Af^2e+3\,Bfe^2+Ce^3)a^4\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}\,Axe^3-\frac{8\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^4f^3}{105\,b^6c}+\frac{\sqrt{-b^2cx^2+a^2c}\,(Bf^3+3\,Cf^2e)a^4x}{16\,b^4}+\frac{\sqrt{-b^2cx^2+a^2c}\,(3\,Af^2e+3\,Bfe^2+Ce^3)a^2x}{8\,b^2}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Bf^3+3\,Cf^2e)x^3}{6\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Af^3+3\,Bf^2e+3\,Cfe^2)x^2}{5\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Afe^2}{b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Bf^3+3\,Cf^2e)a^2x}{8\,b^4c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Af^2e+3\,Bfe^2+Ce^3)x}{4\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be^3}{3\,b^2c}-\frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}(Af^3+3\,Bf^2e+3\,Cfe^2)a^2}{15\,b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/7*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^3*x^4/(b^2*c) - 4/35*(-b^2*c*x^2 + a^2*
c)^(3/2)*C*a^2*f^3*x^2/(b^4*c) + 1/2*A*a^2*sqrt(c)*arcsin(b*x/a)*e^3/b + 1/
16*(B*f^3 + 3*C*f^2*e)*a^6*sqrt(c)*arcsin(b*x/a)/b^5 + 1/8*(3*A*f^2*e + 3*B
*f*e^2 + C*e^3)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/2*sqrt(-b^2*c*x^2 + a^2*c
)*A*x*e^3 - 8/105*(-b^2*c*x^2 + a^2*c)^(3/2)*C*a^4*f^3/(b^6*c) + 1/16*sqrt(
-b^2*c*x^2 + a^2*c)*(B*f^3 + 3*C*f^2*e)*a^4*x/b^4 + 1/8*sqrt(-b^2*c*x^2 + a
^2*c)*(3*A*f^2*e + 3*B*f*e^2 + C*e^3)*a^2*x/b^2 - 1/6*(-b^2*c*x^2 + a^2*c)^
(3/2)*(B*f^3 + 3*C*f^2*e)*x^3/(b^2*c) - 1/5*(-b^2*c*x^2 + a^2*c)^(3/2)*(A*f
^3 + 3*B*f^2*e + 3*C*f*e^2)*x^2/(b^2*c) - (-b^2*c*x^2 + a^2*c)^(3/2)*A*f*e^
2/(b^2*c) - 1/8*(-b^2*c*x^2 + a^2*c)^(3/2)*(B*f^3 + 3*C*f^2*e)*a^2*x/(b^4*c
) - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*(3*A*f^2*e + 3*B*f*e^2 + C*e^3)*x/(b^2*c
) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^3/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c
)^(3/2)*(A*f^3 + 3*B*f^2*e + 3*C*f*e^2)*a^2/(b^4*c)
```

Fricas [A]
time = 1.44, size = 997, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/3360*(105*(B*a^6*b*f^3 + 6*B*a^4*b^3*f*e^2 + 3*(C*a^6*b + 2*A*a^4*b^3)*f
^2*e + 2*(C*a^4*b^3 + 4*A*a^2*b^5)*e^3)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(240*C*b^6*f^3*x^6 + 2
80*B*b^6*f^3*x^5 - 70*B*a^2*b^4*f^3*x^3 - 105*B*a^4*b^2*f^3*x - 48*(C*a^2*b
^4 - 7*A*b^6)*f^3*x^4 - 16*(4*C*a^4*b^2 + 7*A*a^2*b^4)*f^3*x^2 - 32*(4*C*a^
6 + 7*A*a^4*b^2)*f^3 + 70*(6*C*b^6*x^3 + 8*B*b^6*x^2 - 8*B*a^2*b^4 - 3*(C*a
^2*b^4 - 4*A*b^6)*x)*e^3 + 42*(24*C*b^6*f*x^4 + 30*B*b^6*f*x^3 - 15*B*a^2*b
^4*f*x - 8*(C*a^2*b^4 - 5*A*b^6)*f*x^2 - 8*(2*C*a^4*b^2 + 5*A*a^2*b^4)*f)*e
^2 + 21*(40*C*b^6*f^2*x^5 + 48*B*b^6*f^2*x^4 - 16*B*a^2*b^4*f^2*x^2 - 32*B*
a^4*b^2*f^2 - 10*(C*a^2*b^4 - 6*A*b^6)*f^2*x^3 - 15*(C*a^4*b^2 + 2*A*a^2*b^
```

```
4)*f^2*x)*e)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^6, -1/1680*(105*(B*a^6*b*f
^3 + 6*B*a^4*b^3*f*e^2 + 3*(C*a^6*b + 2*A*a^4*b^3)*f^2*e + 2*(C*a^4*b^3 + 4
*A*a^2*b^5)*e^3)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*
x/(b^2*c*x^2 - a^2*c)) - (240*C*b^6*f^3*x^6 + 280*B*b^6*f^3*x^5 - 70*B*a^2*
b^4*f^3*x^3 - 105*B*a^4*b^2*f^3*x - 48*(C*a^2*b^4 - 7*A*b^6)*f^3*x^4 - 16*(
4*C*a^4*b^2 + 7*A*a^2*b^4)*f^3*x^2 - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70*(6
*C*b^6*x^3 + 8*B*b^6*x^2 - 8*B*a^2*b^4 - 3*(C*a^2*b^4 - 4*A*b^6)*x)*e^3 + 4
2*(24*C*b^6*f*x^4 + 30*B*b^6*f*x^3 - 15*B*a^2*b^4*f*x - 8*(C*a^2*b^4 - 5*A*
b^6)*f*x^2 - 8*(2*C*a^4*b^2 + 5*A*a^2*b^4)*f)*e^2 + 21*(40*C*b^6*f^2*x^5 +
48*B*b^6*f^2*x^4 - 16*B*a^2*b^4*f^2*x^2 - 32*B*a^4*b^2*f^2 - 10*(C*a^2*b^4
- 6*A*b^6)*f^2*x^3 - 15*(C*a^4*b^2 + 2*A*a^2*b^4)*f^2*x)*e)*sqrt(-b*c*x + a
*c)*sqrt(b*x + a))/b^6]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-a+bx\right)}\,\sqrt{a+bx}\,\left(e+fx\right)^{3}\left(A+Bx+Cx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**3*(A + B*x + C*x**2),
 x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2665 vs. 2(556) = 1112.
time = 2.40, size = 2665, normalized size = 4.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/1680*(1680*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
2*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^5*e^3 -
 2520*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)
))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*a*b^4
*f*e^2 + 840*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c +
2*a*c)*sqrt(b*x + a))*A*a*b^3*f^2*e - 70*(18*a^4*c*log(abs(-sqrt(b*x + a)*s
qrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*
a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))
*A*a*b^2*f^3 - 840*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x +
a)*c + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x -
```

```
2*a))*B*a*b^4*e^3 - 840*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b
*x + a)*c + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b
*x - 2*a))*A*b^5*e^3 + 840*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(
-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-
(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*a*b^3*f*e^2 + 840*(6*a^3*c*log(abs(-s
qrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5
*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*b^4*f*e^
2 - 210*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a
*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))
*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*a*b^2*f^2*e - 210*(18*a^4*c*lo
g(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39
*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c +
 2*a*c)*sqrt(b*x + a))*A*b^3*f^2*e + 14*(90*a^5*c*log(abs(-sqrt(b*x + a)*sq
rt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3
*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*
c + 2*a*c)*sqrt(b*x + a))*B*a*b*f^3 + 14*(90*a^5*c*log(abs(-sqrt(b*x + a)*s
qrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(
3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)
*c + 2*a*c)*sqrt(b*x + a))*A*b^2*f^3 + 280*(6*a^3*c*log(abs(-sqrt(b*x + a)*
sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a)
 + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b^3*e^3 + 280*(6*a^
3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
 - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x +
a))*B*b^4*e^3 - 210*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
+ a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2
)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b^2*f*e^2 - 210*
(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sq
rt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(
b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^3*f*e^2 + 42*(90*a^5*c*log(abs(-sqrt
(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (29
5*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt
(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b*f^2*e + 42*(90*a^5*c*log(abs(-s
qrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 -
(295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*s
qrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^2*f^2*e - 7*(150*a^6*c*log(abs
(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (405*a^5
 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x + a) + 321*a^2)*(b*x + a)
)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*f^3 -
 7*(150*a^6*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c))
)/sqrt(-c) - (405*a^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x + a)
 + 321*a^2)*(b*x + a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqr
t(b*x + a))*B*b*f^3 - 70*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-
(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 4
3*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*b^3*e^3 + 42*
(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sq
```

```
rt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*
x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*b^2*f*e^2 -
21*(150*a^6*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c))
)/sqrt(-c) - (405*a^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x + a)
 + 321*a^2)*(b*x + a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqr
t(b*x + a))*C*b*f^2*e + (1050*a^7*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(
-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (2835*a^6 - (6335*a^5 - 2*(4781*a^4 - (4
551*a^3 - 4*(5*(6*b*x - 37*a)*(b*x + a) + 661*a^2)*(b*x + a))*(b*x + a))*(b
*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*f^3)/b^6
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] \text{Hanged}
```

3.21 $\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=451

$$\frac{\left(2A\left(4b^4e^2+a^2b^2f^2\right)+a^2\left(a^2Cf^2+2b^2e(Ce+2Bf)\right)\right)x\sqrt{a+bx}\,\sqrt{ac-bcx}}{16b^4}+\frac{(Ce-2Bf)\sqrt{a+bx}\,\sqrt{ac-b}}{10b^2f}$$

```
[Out] 1/16*(2*A*(a^2*b^2*f^2+4*b^4*e^2)+a^2*(a^2*C*f^2+2*b^2*e*(2*B*f+C*e)))*x*(b
*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4+1/10*(-2*B*f+C*e)*(f*x+e)^2*(-b^2*x^2+a^
2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/6*C*(f*x+e)^3*(-b^2*x^2+a^2)*(b
*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/120*(16*a^2*f^2*(B*f+2*C*e)-8*b^2*e*
(C*e^2-2*f*(5*A*f+B*e))+3*f*(5*a^2*C*f^2-b^2*(2*C*e^2-2*f*(5*A*f+2*B*e)))*x
)*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4/f+1/16*a^2*(2*A*(a^2*
b^2*f^2+4*b^4*e^2)+a^2*(a^2*C*f^2+2*b^2*e*(2*B*f+C*e)))*arctan(b*x*c^(1/2)/
(-b^2*c*x^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^5/(-b^
2*c*x^2+a^2*c)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1624, 1668, 847, 794, 201, 223, 209}

$$-\frac{\sqrt{a+bx}\,(a^2-b^2x^2)\sqrt{ac-bcx}\left(8\left(2a^2f^2(Bf+2Ce)-\frac{1}{8}b^2(8Ce^3-16ef(5Af+Be))\right)+3fx(5a^2Cf^2-b^2(2Ce^2-2f(5Af+2Be)))\right)}{120b^4f}+\frac{\sqrt{a+bx}\,(a^2-b^2x^2)(e+fx)^2\sqrt{ac-bcx}\,(Ce-2Bf)}{10b^2f}-\frac{C\sqrt{a+bx}\,(a^2-b^2x^2)(e+fx)^3\sqrt{ac-bcx}}{6b^2f}+\frac{a^2\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)(a^4Cf^2+2A(a^2b^2f^2+4b^4e^2)+2a^2b^2e(2Bf+Ce))}{16b^5\sqrt{a^2c-b^2cx^2}}+\frac{x\sqrt{a+bx}\,\sqrt{ac-bcx}\,(a^4Cf^2+2A(a^2b^2f^2+4b^4e^2)+2a^2b^2e(2Bf+Ce))}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] ((a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*x*
Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((C*e - 2*B*f)*Sqrt[a + b*x]*Sq
rt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(10*b^2*f) - (C*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(6*b^2*f) - (Sqrt[a + b*x]*S
qrt[a*c - b*c*x]*(8*(2*a^2*f^2*(2*C*e + B*f) - (b^2*(8*C*e^3 - 16*e*f*(B*e
+ 5*A*f)))/8) + 3*f*(5*a^2*C*f^2 - b^2*(2*C*e^2 - 2*f*(2*B*e + 5*A*f)))*x)*
(a^2 - b^2*x^2))/(120*b^4*f) + (a^2*Sqrt[c]*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e +
 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*Ar
cTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2
])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
```

```
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\right)\int (e+fx)^2\sqrt{a^2c-b^2cx^2}\,(}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f} - \frac{(}{}$$

$$= \frac{(Ce-2Bf)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(a^2-b^2\right.}{10b^2f}$$

$$= \frac{(Ce-2Bf)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(a^2-b^2\right.}{10b^2f}$$

$$= \frac{(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2)}{16b^4}$$

$$= \frac{(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2)}{16b^4}$$

$$= \frac{(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2)}{16b^4}$$

Mathematica [A]
time = 0.81, size = 286, normalized size = 0.63

$$\frac{\sqrt{c(a-bx)}\left(b\sqrt{a-bx}\,\sqrt{a+bx}\,(-a^4f(64Ce+32Bf+15Cfx)-2a^2b^2(5Af(16e+3fx)+Cx(15e^2+16efx+5f^2x^2)+B(40e^2+30efx+8f^2x^2))+4b^4x(5A(6e^2+8efx+3f^2x^2)+x(2B(10e^2+15efx+6f^2x^2)+Cx(15e^2+24efx+10f^2x^2))))+30a^2(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2))\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{240b^5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(-(a^4*f*(64*C*e + 32*B*f
 + 15*C*f*x)) - 2*a^2*b^2*(5*A*f*(16*e + 3*f*x) + C*x*(15*e^2 + 16*e*f*x +
5*f^2*x^2) + B*(40*e^2 + 30*e*f*x + 8*f^2*x^2)) + 4*b^4*x*(5*A*(6*e^2 + 8*e
*f*x + 3*f^2*x^2) + x*(2*B*(10*e^2 + 15*e*f*x + 6*f^2*x^2) + C*x*(15*e^2 +
24*e*f*x + 10*f^2*x^2)))) + 30*a^2*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) +
```

```
 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(240*
b^5*Sqrt[a - b*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(410) = 820$.
time = 0.11, size = 933, normalized size = 2.07

method	result
risch	$-\frac{\left(-40C f^2x^5b^4-48B b^4f^2x^4-96C b^4ef x^4-60A b^4f^2x^3-120B b^4ef x^3+10C a^2b^2f^2x^3-60C b^4e^2x^3-160A b^4ef x^2+16B a^2b^2f^2 \right.}{}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c\left(-bx+a\right)}\left(-32B a^4f^2\sqrt{c\left(-b^2x^2+a^2\right)}\,\sqrt{b^2c}+15C \arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)a^6cf^2-3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/240*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)*(-32*B*a^4*f^2*(c*(-b^2*x^2+a^2))^(1
/2)*(b^2*c)^(1/2)+15*C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^6
*c*f^2-30*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*b^2*e^2*x+96*C*b^4*e
*f*x^4*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+120*B*b^4*e*f*x^3*(c*(-b^2*x^
2+a^2))^(1/2)*(b^2*c)^(1/2)-10*C*a^2*b^2*f^2*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(
b^2*c)^(1/2)+160*A*b^4*e*f*x^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-16*B*
a^2*b^2*f^2*x^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-160*A*a^2*b^2*e*f*(c
*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-30*A*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^
(1/2)*a^2*b^2*f^2*x+60*B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a
^4*b^2*c*e*f-15*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^4*f^2*x+30*A*arc
tan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*b^2*c*f^2+120*A*arctan((b
^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*b^4*c*e^2+30*C*arctan((b^2*c)^(
1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*b^2*c*e^2+40*C*b^4*f^2*x^5*(c*(-b^2*x^
2+a^2))^(1/2)*(b^2*c)^(1/2)+48*B*b^4*f^2*x^4*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*
c)^(1/2)+60*A*b^4*f^2*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+60*C*b^4*e
^2*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+80*B*b^4*e^2*x^2*(c*(-b^2*x^2
+a^2))^(1/2)*(b^2*c)^(1/2)-80*B*a^2*b^2*e^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c
)^(1/2)+120*A*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*b^4*e^2*x-64*C*a^4*e*f
*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-32*C*a^2*b^2*e*f*x^2*(c*(-b^2*x^2+a
^2))^(1/2)*(b^2*c)^(1/2)-60*B*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*b^
2*e*f*x)/(c*(-b^2*x^2+a^2))^(1/2)/b^4/(b^2*c)^(1/2)
```

Maxima [A]

time = 0.52, size = 417, normalized size = 0.92

$$\frac{Ca^6\sqrt{c}\,f^2\arcsin\left(\frac{bx}{a}\right)}{16\,b^5}+\frac{\sqrt{-b^2cx^2+a^2c}\,Ca^4f^2x}{16\,b^4}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cf^2x^3}{6\,b^2c}+\frac{Aa^2\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)e^2}{2\,b}+\frac{(Af^2+2\,Bfe+Ce^2)a^4\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}\,Axe^2+\frac{\sqrt{-b^2cx^2+a^2c}\,(Af^2+2\,Bfe+Ce^2)a^2x}{8\,b^2}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f^2x}{8\,b^4c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Bf^2+2\,Cfe)x^2}{5\,b^2c}-\frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Afe}{3\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Af^2+2\,Bfe+Ce^2)x}{4\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be^2}{3\,b^2c}-\frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}(Bf^2+2\,Cfe)a^2}{15\,b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*C*a^6*sqrt(c)*f^2*arcsin(b*x/a)/b^5 + 1/16*sqrt(-b^2*c*x^2 + a^2*c)*C*
a^4*f^2*x/b^4 - 1/6*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^2*x^3/(b^2*c) + 1/2*A*a^
2*sqrt(c)*arcsin(b*x/a)*e^2/b + 1/8*(A*f^2 + 2*B*f*e + C*e^2)*a^4*sqrt(c)*a
rcsin(b*x/a)/b^3 + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*x*e^2 + 1/8*sqrt(-b^2*c*x
^2 + a^2*c)*(A*f^2 + 2*B*f*e + C*e^2)*a^2*x/b^2 - 1/8*(-b^2*c*x^2 + a^2*c)^
(3/2)*C*a^2*f^2*x/(b^4*c) - 1/5*(-b^2*c*x^2 + a^2*c)^(3/2)*(B*f^2 + 2*C*f*e
)*x^2/(b^2*c) - 2/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*f*e/(b^2*c) - 1/4*(-b^2*c*
x^2 + a^2*c)^(3/2)*(A*f^2 + 2*B*f*e + C*e^2)*x/(b^2*c) - 1/3*(-b^2*c*x^2 +
a^2*c)^(3/2)*B*e^2/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^(3/2)*(B*f^2 + 2*C*f
*e)*a^2/(b^4*c)
```

<u>Fricas</u> [A]
time = 1.45, size = 703, normalized size = 1.56

$$\left[\frac{15\,(4\,Ba^4b^2fe+(Ca^6+2\,Aa^4b^2)f^2+2\,(Ca^4b^2+4\,Aa^2b^4)e^2)\sqrt{-c}\,\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,(40\,Cb^5f^2x^5+48\,Bb^5f^2x^4-16\,Ba^2b^3f^2x^2-32\,Ba^4bf^2-10\,(Ca^2b^3-6\,Ab^5)f^2x^3-15\,(Ca^4b+2\,Aa^2b^3)f^2x+10\,(6\,Cb^5x^3+8\,Bb^5x^2-8\,Ba^2b^3-3\,(Ca^2b^3-4\,Ab^5)x)e^2+4\,(24\,Cb^5fx^4+30\,Bb^5fx^3-15\,Ba^2b^3fx-8\,(Ca^2b^3-5\,Ab^5)fx^2-8\,(2\,Ca^4b+5\,Aa^2b^3)f)e)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{480\,b^5},\ -\frac{15\,(4\,Ba^4b^2fe+(Ca^6+2\,Aa^4b^2)f^2+2\,(Ca^4b^2+4\,Aa^2b^4)e^2)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-(40\,Cb^5f^2x^5+48\,Bb^5f^2x^4-16\,Ba^2b^3f^2x^2-32\,Ba^4bf^2-10\,(Ca^2b^3-6\,Ab^5)f^2x^3-15\,(Ca^4b+2\,Aa^2b^3)f^2x+10\,(6\,Cb^5x^3+8\,Bb^5x^2-8\,Ba^2b^3-3\,(Ca^2b^3-4\,Ab^5)x)e^2+4\,(24\,Cb^5fx^4+30\,Bb^5fx^3-15\,Ba^2b^3fx-8\,(Ca^2b^3-5\,Ab^5)fx^2-8\,(2\,Ca^4b+5\,Aa^2b^3)f)e)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{240\,b^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/480*(15*(4*B*a^4*b^2*f*e + (C*a^6 + 2*A*a^4*b^2)*f^2 + 2*(C*a^4*b^2 + 4*
A*a^2*b^4)*e^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(-c)*x - a^2*c) + 2*(40*C*b^5*f^2*x^5 + 48*B*b^5*f^2*x^4 - 16*B*a^
2*b^3*f^2*x^2 - 32*B*a^4*b*f^2 - 10*(C*a^2*b^3 - 6*A*b^5)*f^2*x^3 - 15*(C*a
^4*b + 2*A*a^2*b^3)*f^2*x + 10*(6*C*b^5*x^3 + 8*B*b^5*x^2 - 8*B*a^2*b^3 - 3
*(C*a^2*b^3 - 4*A*b^5)*x)*e^2 + 4*(24*C*b^5*f*x^4 + 30*B*b^5*f*x^3 - 15*B*a
^2*b^3*f*x - 8*(C*a^2*b^3 - 5*A*b^5)*f*x^2 - 8*(2*C*a^4*b + 5*A*a^2*b^3)*f)
*e)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5, -1/240*(15*(4*B*a^4*b^2*f*e + (C
*a^6 + 2*A*a^4*b^2)*f^2 + 2*(C*a^4*b^2 + 4*A*a^2*b^4)*e^2)*sqrt(c)*arctan(s
qrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (40*C*b^
5*f^2*x^5 + 48*B*b^5*f^2*x^4 - 16*B*a^2*b^3*f^2*x^2 - 32*B*a^4*b*f^2 - 10*(
C*a^2*b^3 - 6*A*b^5)*f^2*x^3 - 15*(C*a^4*b + 2*A*a^2*b^3)*f^2*x + 10*(6*C*b
^5*x^3 + 8*B*b^5*x^2 - 8*B*a^2*b^3 - 3*(C*a^2*b^3 - 4*A*b^5)*x)*e^2 + 4*(24
*C*b^5*f*x^4 + 30*B*b^5*f*x^3 - 15*B*a^2*b^3*f*x - 8*(C*a^2*b^3 - 5*A*b^5)*
f*x^2 - 8*(2*C*a^4*b + 5*A*a^2*b^3)*f)*e)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))
/b^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-a+bx\right)}\,\sqrt{a+bx}\,\left(e+fx\right)^{2}\left(A+Bx+Cx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2*(A + B*x + C*x**2),
 x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1868 vs. $2(421) = 842$.
time = 1.54, size = 1868, normalized size = 4.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/240*(240*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*
a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^4*e^2 - 2
40*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/
sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*a*b^3*f*
e + 40*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c
)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)
*sqrt(b*x + a))*A*a*b^2*f^2 - 120*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c)
+ sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b
*x + a)*(b*x - 2*a))*B*a*b^3*e^2 - 120*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*s
qrt(b*x + a)*(b*x - 2*a))*A*b^4*e^2 + 80*(6*a^3*c*log(abs(-sqrt(b*x + a)*sq
rt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) +
 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*a*b^2*f*e + 80*(6*a^3*c
*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) -
((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))
*A*b^3*f*e - 10*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b
*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*a*b*f^2 - 10*(18*a^4*c
*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) -
(39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*
c + 2*a*c)*sqrt(b*x + a))*A*b^2*f^2 + 40*(6*a^3*c*log(abs(-sqrt(b*x + a)*sq
rt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) +
 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b^2*e^2 + 40*(6*a^3*c
```

```
*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) -
((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))
*B*b^3*e^2 - 20*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b
*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b*f*e - 20*(18*a^4*c
*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) -
(39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*
c + 2*a*c)*sqrt(b*x + a))*B*b^2*f*e + 2*(90*a^5*c*log(abs(-sqrt(b*x + a)*sq
rt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3
*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*
c + 2*a*c)*sqrt(b*x + a))*C*a*f^2 + 2*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(
4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c
+ 2*a*c)*sqrt(b*x + a))*B*b*f^2 - 10*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(
-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(
b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*b
^2*e^2 + 4*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 1
33*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*b
*f*e - (150*a^6*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a
*c)))/sqrt(-c) - (405*a^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x
+ a) + 321*a^2)*(b*x + a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)
*sqrt(b*x + a))*C*f^2)/b^5
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] \text{Hanged}
```

3.22 $\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=300

$$\frac{(4Ab^2e+a^2(Ce+Bf))\,x\sqrt{a+bx}\,\sqrt{ac-bcx}}{8b^2}-\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\,(a^2-b^2x^2)}{5b^2f}-\frac{\sqrt{a+bx}\,\sqrt{}}{}$$

```
[Out] 1/8*(4*A*b^2*e+a^2*(B*f+C*e))*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2-1/5*C*
(f*x+e)^2*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/60*(8*a^2
*C*f^2-4*b^2*(3*C*e^2-5*f*(A*f+B*e))-3*b^2*f*(-5*B*f+3*C*e)*x)*(-b^2*x^2+a^
2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4/f+1/8*a^2*(4*A*b^2*e+a^2*(B*f+C*e))
*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x
+a*c)^(1/2)/b^3/(-b^2*c*x^2+a^2*c)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 297, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1624, 1668, 794, 201, 223, 209}

$$\frac{a^2\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)(a^2(Bf+Ce)+4Ab^2e)}{8b^3\sqrt{a^2c-b^2cx^2}}+\frac{1}{8}x\sqrt{a+bx}\,\sqrt{ac-bcx}\left(\frac{a^2(Bf+Ce)}{b^2}+4Ae\right)-\frac{\sqrt{a+bx}\,(a^2-b^2x^2)\sqrt{ac-bcx}\,(4(2a^2Cf^2-b^2(3Ce^2-5f(Af+Be)))-3b^2fx(3Ce-5Bf))}{60b^4f}-\frac{C\sqrt{a+bx}\,(a^2-b^2x^2)\,(e+fx)^2\sqrt{ac-bcx}}{5b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] ((4*A*e + (a^2*(C*e + B*f))/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (C*
Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(5*b^2*f) - (S
qrt[a + b*x]*Sqrt[a*c - b*c*x]*(4*(2*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(B*e +
A*f))) - 3*b^2*f*(3*C*e - 5*B*f)*x)*(a^2 - b^2*x^2))/(60*b^4*f) + (a^2*Sqrt
[c]*(4*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b
*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)\left(A+Bx+Cx^2\right)\,dx = \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\right)\int(e+fx)\sqrt{a^2c-b^2cx^2}\,(}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - $$

$$= -\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - $$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - $$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - $$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - $$

Mathematica [A]
time = 0.50, size = 183, normalized size = 0.61

$$\frac{\sqrt{c(a-bx)}\left(\sqrt{a-bx}\,\sqrt{a+bx}\,(-16a^4Cf-a^2b^2(40Af+5B(8e+3fx)+Cx(15e+8fx))+2b^4x(10A(3e+2fx)+x(5B(4e+3fx)+3Cx(5e+4fx))))+30a^2b(4Ab^2e+a^2(Ce+Bf))\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{120b^4\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(Sqrt[a - b*x]*Sqrt[a + b*x]*(-16*a^4*C*f - a^2*b^2*(40*
A*f + 5*B*(8*e + 3*f*x) + C*x*(15*e + 8*f*x)) + 2*b^4*x*(10*A*(3*e + 2*f*x)
 + x*(5*B*(4*e + 3*f*x) + 3*C*x*(5*e + 4*f*x)))) + 30*a^2*b*(4*A*b^2*e + a^
2*(C*e + B*f))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(120*b^4*Sqrt[a - b*x]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(265) = 530$.
time = 0.10, size = 554, normalized size = 1.85

method	result

risch	$-\frac{(-24fC\,x^4b^4-30B\,b^4f\,x^3-30C\,b^4e\,x^3-40A\,b^4f\,x^2-40B\,b^4e\,x^2+8C\,a^2b^2f\,x^2-60A\,b^4ex+15B\,a^2b^2fx+15C\,a^2b^2ex+40A\,a^2f\,b^2+}{120b^4\sqrt{-c\,(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c\,(-bx+a)}\,\Big(24C\,b^4f\,x^4\sqrt{c\,(-b^2x^2+a^2)}\,\sqrt{b^2c}+30B\,b^4f\,x^3\sqrt{c\,(-b^2x^2+a^2)}\,\sqrt{b^2c}+30C\,b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/120*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)*(24*C*b^4*f*x^4*(c*(-b^2*x^2+a^2))^(
1/2)*(b^2*c)^(1/2)+30*B*b^4*f*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+30
*C*b^4*e*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+60*A*arctan((b^2*c)^(1/
2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*b^4*c*e+40*A*b^4*f*x^2*(c*(-b^2*x^2+a^2)
)^(1/2)*(b^2*c)^(1/2)+15*B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))
*a^4*b^2*c*f+40*B*b^4*e*x^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+15*C*arc
tan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*b^2*c*e-8*C*a^2*b^2*f*x^2
*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+60*A*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2
))^(1/2)*b^4*e*x-15*B*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*b^2*f*x-15
*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*b^2*e*x-40*A*a^2*b^2*f*(c*(-b
^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-40*B*a^2*b^2*e*(c*(-b^2*x^2+a^2))^(1/2)*(b
^2*c)^(1/2)-16*C*a^4*f*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2))/(c*(-b^2*x^2
+a^2))^(1/2)/b^4/(b^2*c)^(1/2)
```

Maxima [A]
time = 0.56, size = 254, normalized size = 0.85

$$\frac{Aa^2\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)e}{2\,b}+\frac{(Bf+Ce)a^4\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}\,Axe+\frac{\sqrt{-b^2cx^2+a^2c}\,(Bf+Ce)a^2x}{8\,b^2}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cfx^2}{5\,b^2c}-\frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f}{15\,b^4c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Af}{3\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Bf+Ce)x}{4\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be}{3\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*A*a^2*sqrt(c)*arcsin(b*x/a)*e/b + 1/8*(B*f + C*e)*a^4*sqrt(c)*arcsin(b*
x/a)/b^3 + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*x*e + 1/8*sqrt(-b^2*c*x^2 + a^2*c
)*(B*f + C*e)*a^2*x/b^2 - 1/5*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f*x^2/(b^2*c) -
2/15*(-b^2*c*x^2 + a^2*c)^(3/2)*C*a^2*f/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^
(3/2)*A*f/(b^2*c) - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*(B*f + C*e)*x/(b^2*c) -
1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e/(b^2*c)
```

Fricas [A]
time = 1.46, size = 449, normalized size = 1.50

$$\left[\frac{15\,(Ba^4bf+(Ca^4b+4\,Aa^2b^3)e)\sqrt{-c}\,\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,(24\,Cb^4fx^4+30\,Bb^4fx^3-15\,Ba^2b^2fx-8\,(Ca^2b^2-5\,Ab^4)fx^2-8\,(2\,Ca^4+5\,Aa^2b^2)f+5\,(6\,Cb^4x^3+8\,Bb^4x^2-8\,Ba^2b^2-3\,(Ca^2b^2-4\,Ab^4)x)e)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{240\,b^4}, -\frac{15\,(Ba^4bf+(Ca^4b+4\,Aa^2b^3)e)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-(24\,Cb^4fx^4+30\,Bb^4fx^3-15\,Ba^2b^2fx-8\,(Ca^2b^2-5\,Ab^4)fx^2-8\,(2\,Ca^4+5\,Aa^2b^2)f+5\,(6\,Cb^4x^3+8\,Bb^4x^2-8\,Ba^2b^2-3\,(Ca^2b^2-4\,Ab^4)x)e)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{120\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/240*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2
 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f
*x^4 + 30*B*b^4*f*x^3 - 15*B*a^2*b^2*f*x - 8*(C*a^2*b^2 - 5*A*b^4)*f*x^2 -
8*(2*C*a^4 + 5*A*a^2*b^2)*f + 5*(6*C*b^4*x^3 + 8*B*b^4*x^2 - 8*B*a^2*b^2 -
3*(C*a^2*b^2 - 4*A*b^4)*x)*e)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4, -1/120
*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(c)*arctan(sqrt(-b*c*x + a
*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (24*C*b^4*f*x^4 + 30*B
*b^4*f*x^3 - 15*B*a^2*b^2*f*x - 8*(C*a^2*b^2 - 5*A*b^4)*f*x^2 - 8*(2*C*a^4
+ 5*A*a^2*b^2)*f + 5*(6*C*b^4*x^3 + 8*B*b^4*x^2 - 8*B*a^2*b^2 - 3*(C*a^2*b^
2 - 4*A*b^4)*x)*e)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\,(-a+bx)}\,\sqrt{a+bx}\,(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(273) = 546$.
time = 1.06, size = 1148, normalized size = 3.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/120*(120*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*
a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^3*e - 60*
(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqr
t(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*a*b^2*f - 6
0*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/s
```

```
qrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b^2*e -
 60*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^3*e +
 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sq
rt(b*x + a))*B*a*b*f + 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-
(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(
b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*b^2*f + 20*(6*a^3*c*log(abs(-sqrt(b*x
+ a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x
 + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b*e + 20*(6*a^
3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
 - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x +
a))*B*b^2*e - 5*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b
*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*f - 5*(18*a^4*c*log(
abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a
^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2
*a*c)*sqrt(b*x + a))*B*b*f - 5*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) +
sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x +
a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*b*e + (
90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqr
t(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x
 + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*f)/b^4
```

Mupad [B]
time = 30.58, size = 1765, normalized size = 5.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((B*a^4*c^8*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a^
(1/2))) - (B*a^4*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x)^
(1/2) - a^(1/2))^15) - (35*B*a^4*c^7*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*B*a^4*c^6*f*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*B*a^4*c^5*f*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7) + (7
15*B*a^4*c^4*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^(1/2) -
 a^(1/2))^9) - (273*B*a^4*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(2*
((a + b*x)^(1/2) - a^(1/2))^11) + (35*B*a^4*c^2*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^13)/(2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^3*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 + (8*b^3*c^7
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28
*b^3*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))
```

```
^4 + (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) -
a^(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^
(1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((
a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14) - (a*c - b*c*x)^(1/2)*((2*C*
a^4*f*(a + b*x)^(1/2))/(15*b^4) - (C*f*x^4*(a + b*x)^(1/2))/5 + (C*a^2*f*x^
2*(a + b*x)^(1/2))/(15*b^2)) + ((C*a^4*c^8*e*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2)))/(2*((a + b*x)^(1/2) - a^(1/2))) - (C*a^4*c*e*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^15)/(2*((a + b*x)^(1/2) - a^(1/2))^15) - (35*C*a^4*c^7*e*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*
C*a^4*c^6*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^
(1/2))^5) - (715*C*a^4*c^5*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a
+ b*x)^(1/2) - a^(1/2))^7) + (715*C*a^4*c^4*e*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^9)/(2*((a + b*x)^(1/2) - a^(1/2))^9) - (273*C*a^4*c^3*e*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^11)/(2*((a + b*x)^(1/2) - a^(1/2))^11) + (35*C*a^4*
c^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(2*((a + b*x)^(1/2) - a^(1/2)
)^13))/(b^3*c^8 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(
1/2) - a^(1/2))^16 + (8*b^3*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a
+ b*x)^(1/2) - a^(1/2))^2 + (28*b^3*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*
b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^1
4) + (A*e*x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/2 - (A*f*(a^2 - b^2*x^2)*(
a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (B*e*(a^2 - b^2*x^2)*(a*c - b
*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (B*a^4*c^(1/2)*f*atan(((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(2*b^3) - (C*
a^4*c^(1/2)*e*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(2*b^3) - (A*a^2*b^(1/2)*c^2*e*log((-b*c)^(1/2)*(c*(a -
 b*x))^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

3.23 $\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(A+Bx+Cx^2)\,dx$

Optimal. Leaf size=221

$$\frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx}-\frac{B\sqrt{a+bx}\,\sqrt{ac-bcx}\,(a^2-b^2x^2)}{3b^2}-\frac{Cx\sqrt{a+bx}\,\sqrt{ac-bcx}\,(a^2-b^2}{4b^2}$$

```
[Out] 1/8*(4*A+a^2*C/b^2)*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)-1/3*B*(-b^2*x^2+a^2)
*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2-1/4*C*x*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*
(-b*c*x+a*c)^(1/2)/b^2+1/8*a^2*(4*A*b^2+C*a^2)*arctan(b*x*c^(1/2)/(-b^2*c*x
^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^3/(-b^2*c*x^2+a
^2*c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$,
Rules used = {915, 1829, 655, 201, 223, 209}

$$\frac{a^2\sqrt{c}\,\sqrt{a+bx}\,(a^2C+4Ab^2)\sqrt{ac-bcx}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}+\frac{1}{8}x\sqrt{a+bx}\left(\frac{a^2C}{b^2}+4A\right)\sqrt{ac-bcx}-\frac{B\sqrt{a+bx}\,(a^2-b^2x^2)\sqrt{ac-bcx}}{3b^2}-\frac{Cx\sqrt{a+bx}\,(a^2-b^2x^2)\sqrt{ac-bcx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((4*A + (a^2*C)/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (B*Sqrt[a + b*x
]*Sqrt[a*c - b*c*x]*(a^2 - b^2*x^2))/(3*b^2) - (C*x*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(a^2 - b^2*x^2))/(4*b^2) + (a^2*Sqrt[c]*(4*A*b^2 + a^2*C)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b
^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(A+Bx+Cx^2\right)\,dx = \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\,\right)\int \sqrt{a^2c-b^2cx^2}\,\left(A+Bx+Cx^2\right)\,dx}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{Cx\sqrt{a+bx}\,\sqrt{ac-bcx}\,(a^2-b^2x^2)}{4b^2} - \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bc\right.}{}$$

$$= -\frac{B\sqrt{a+bx}\,\sqrt{ac-bcx}\,(a^2-b^2x^2)}{3b^2} - \frac{Cx\sqrt{a+bx}\,\sqrt{ac-bc}}{4b^2}$$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\,\sqrt{ac-b}}{3b^2}$$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\,\sqrt{ac-b}}{3b^2}$$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\,\sqrt{ac-b}}{3b^2}$$

Mathematica [A]
time = 0.24, size = 123, normalized size = 0.56

$$\frac{\sqrt{c(a-bx)}\left(b\sqrt{a-bx}\,\sqrt{a+bx}\,(-a^2(8B+3Cx)+2b^2x(6A+x(4B+3Cx)))+6a^2(4Ab^2+a^2C)\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{24b^3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(-(a^2*(8*B + 3*C*x)) + 2
*b^2*x*(6*A + x*(4*B + 3*C*x))) + 6*a^2*(4*A*b^2 + a^2*C)*ArcTan[Sqrt[a + b
*x]/Sqrt[a - b*x]]))/(24*b^3*Sqrt[a - b*x])
```

Maple [A]
time = 0.11, size = 269, normalized size = 1.22

method	result
risch	$\frac{(6C\,b^2x^3+8B\,b^2x^2+12A\,b^2x-3C\,a^2x-8a^2B)\sqrt{bx+a}\,(-bx+a)c}{24b^2\sqrt{-c\,(bx-a)}}+\frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+c\,a^2}}\right)A}{2\sqrt{b^2c}}+\frac{a^4\arctan\left(\right.}{8b^2\sqrt{}}\right.}{\sqrt{bx+a}\,\sqrt{-}}$

default	$\frac{\sqrt{bx+a}\,\sqrt{c\left(-bx+a\right)}\,\left(6C\,b^{2}x^{3}\sqrt{b^{2}c}\,\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}+12A\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}}\right)a^{2}b^{2}c+8B\,b^{2}\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)*(6*C*b^2*x^3*(b^2*c)^(1/2)*(c*(-b^2*x
^2+a^2))^(1/2)+12*A*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*b^
2*c+8*B*b^2*x^2*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+3*C*arctan((b^2*c)^(
1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*c+12*A*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2)
)^(1/2)*b^2*x-3*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*x-8*B*a^2*(b^2
*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2))/(c*(-b^2*x^2+a^2))^(1/2)/b^2/(b^2*c)^(1
/2)
```

Maxima [A]
time = 0.55, size = 140, normalized size = 0.63

$$\frac{Ca^{4}\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{8\,b^{3}}+\frac{Aa^{2}\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\,\sqrt{-b^{2}cx^{2}+a^{2}c}\,Ax+\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\,Ca^{2}x}{8\,b^{2}}-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}Cx}{4\,b^{2}c}-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}B}{3\,b^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/8*C*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/2*A*a^2*sqrt(c)*arcsin(b*x/a)/b + 1
/2*sqrt(-b^2*c*x^2 + a^2*c)*A*x + 1/8*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*x/b^2
 - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*C*x/(b^2*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/
2)*B/(b^2*c)
```

Fricas [A]
time = 0.77, size = 265, normalized size = 1.20

$$\left[\frac{3\left(Ca^{4}+4\,Aa^{2}b^{2}\right)\sqrt{-c}\,\log\left(2\,b^{2}cx^{2}+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^{2}c\right)+2\left(6\,Cb^{3}x^{3}+8\,Bb^{3}x^{2}-8\,Ba^{2}b-3\left(Ca^{2}b-4\,Ab^{3}\right)x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{48\,b^{3}},\ -\frac{3\left(Ca^{4}+4\,Aa^{2}b^{2}\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^{2}cx^{2}-a^{2}c}\right)-\left(6\,Cb^{3}x^{3}+8\,Bb^{3}x^{2}-8\,Ba^{2}b-3\left(Ca^{2}b-4\,Ab^{3}\right)x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{24\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/48*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a
*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*
B*a^2*b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3, -
1/24*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*B*a^2*
b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c(-a+bx)}\,\sqrt{a+bx}\,\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(191) = 382.
time = 1.06, size = 527, normalized size = 2.38

$$-\frac{24\left(\frac{2ac\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-\sqrt{-(bx+a)c+2ac}\,\sqrt{bx+a}\right)Aab^2-12\left(\frac{2a^2c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}+\sqrt{-(bx+a)c+2ac}\,\sqrt{bx+a}\,(bx-2a)\right)Bab-12\left(\frac{2a^2c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}+\sqrt{-(bx+a)c+2ac}\,\sqrt{bx+a}\,(bx-2a)\right)Ab^2+4\left(\frac{6a^3c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-\left((2bx-5a)(bx+a)+9a^2\right)\sqrt{-(bx+a)c+2ac}\,\sqrt{bx+a}\right)Ca+4\left(\frac{6a^3c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-\left((2bx-5a)(bx+a)+9a^2\right)\sqrt{-(bx+a)c+2ac}\,\sqrt{bx+a}\right)Bb-\left(\frac{18a^4c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-\left(39a^3-\left(2(3bx-10a)(bx+a)+43a^2\right)(bx+a)\right)\sqrt{-(bx+a)c+2ac}\,\sqrt{bx+a}\right)C}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/24*(24*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^2 - 12*(2*a
^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c
) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b - 12*(2*a^2
*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
+ sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^2 + 4*(6*a^3*c*
log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (
(2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*
C*a + 4*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c
)*sqrt(b*x + a))*B*b - (18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b
*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*
a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C)/b^3
```

Mupad [B]
time = 16.52, size = 876, normalized size = 3.96

$$\frac{\frac{Ca^4c^8\left(\sqrt{ac-bcx}-\sqrt{ac}\right)}{2\left(\sqrt{a+bx}-\sqrt{a}\right)}-\frac{Ca^4c\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^{15}}{2\left(\sqrt{a+bx}-\sqrt{a}\right)^{15}}-\frac{35Ca^4c^7\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^3}{2\left(\sqrt{a+bx}-\sqrt{a}\right)^3}+\frac{273Ca^4c^6\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^5}{2\left(\sqrt{a+bx}-\sqrt{a}\right)^5}-\frac{715Ca^4c^5\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^7}{2\left(\sqrt{a+bx}-\sqrt{a}\right)^7}+\frac{715Ca^4c^4\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^9}{2\left(\sqrt{a+bx}-\sqrt{a}\right)^9}-\frac{273Ca^4c^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^{11}}{2\left(\sqrt{a+bx}-\sqrt{a}\right)^{11}}+\frac{35Ca^4c^2\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^{13}}{2\left(\sqrt{a+bx}-\sqrt{a}\right)^{13}}}{b^3c^8+\frac{b^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^{16}}{\left(\sqrt{a+bx}-\sqrt{a}\right)^{16}}+\frac{8b^3c^7\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^2}{\left(\sqrt{a+bx}-\sqrt{a}\right)^2}+\frac{28b^3c^6\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^4}{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}+\frac{56b^3c^5\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^6}{\left(\sqrt{a+bx}-\sqrt{a}\right)^6}+\frac{70b^3c^4\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^8}{\left(\sqrt{a+bx}-\sqrt{a}\right)^8}+\frac{56b^3c^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^{10}}{\left(\sqrt{a+bx}-\sqrt{a}\right)^{10}}+\frac{28b^3c^2\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^{12}}{\left(\sqrt{a+bx}-\sqrt{a}\right)^{12}}+\frac{8b^3c\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^{14}}{\left(\sqrt{a+bx}-\sqrt{a}\right)^{14}}}+\frac{Ax\sqrt{ac-bcx}\,\sqrt{a+bx}}{2}-\frac{B\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\,\sqrt{a+bx}}{3b^2}-\frac{Ca^4\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{ac-bcx}-\sqrt{ac}}{\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{2b^3}-\frac{Aa^2\sqrt{b}\,c^2\ln\left(\sqrt{-bc}\,\sqrt{c\,(a-bx)}\,\sqrt{a+bx}-b^{3/2}cx\right)}{2(-bc)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((C*a^4*c^8*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a^(1
/2))) - (C*a^4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x)^(1/2
```

```
) - a^(1/2))^15) - (35*C*a^4*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(2*
((a + b*x)^(1/2) - a^(1/2))^3) + (273*C*a^4*c^6*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*C*a^4*c^5*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7) + (715*C*a^4*c
^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^(1/2) - a^(1/2))^9)
 - (273*C*a^4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(2*((a + b*x)^(1/
2) - a^(1/2))^11) + (35*C*a^4*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(
2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^3*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 + (8*b^3*c^7*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28*b^3*c^6*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*b^3*c^5
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70
*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))
^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) -
 a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*
x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/(
(a + b*x)^(1/2) - a^(1/2))^14) + (A*x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/
2 - (B*(a^2 - b^2*x^2)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (C*a^
4*c^(1/2)*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2
) - a^(1/2)))))/(2*b^3) - (A*a^2*b^(1/2)*c^2*log((-b*c)^(1/2)*(c*(a - b*x))
^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

3.24 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)}\,dx$

Optimal. Leaf size=278

$$-\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{(Ce^2-Bef+Af^2)\sqrt{a^2c-\cdots}}{\sqrt{c}\,f^2\sqrt{b^2\cdots}}$$

```
[Out] -C*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-(-B*f+C*e)*arctan(
b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2
)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(A*f^2-B*e*f+C*e^2)*arctan((b^2*e*x+a^2*
f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a
^2*c)^(1/2)/f^2/c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)
^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1624, 1668, 858, 223, 209, 739, 210}

$$\frac{\sqrt{a^2c-b^2cx^2}\,(Af^2-Bef+Ce^2)\,\text{ArcTan}\left(\frac{\sqrt{c}\,(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}\,(Ce-Bf)\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{-Ab^2cf^2+b^2cf(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b}}}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{}{1}\right.}{f^2\sqrt{a+bx}\sqrt{ac-}}$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b}{\sqrt{a^2c}}\right.}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A]
time = 0.51, size = 178, normalized size = 0.64

$$\frac{\frac{Cf(-a+bx)\sqrt{a+bx}}{b^2} - \frac{2(Ce-Bf)\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b} + \frac{2(Ce^2+f(-Be+Af))\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{\sqrt{be-af}\sqrt{be+af}}}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] ((C*f*(-a + b*x)*Sqrt[a + b*x])/b^2 - (2*(C*e - B*f)*Sqrt[a - b*x]*ArcTan[S
qrt[a + b*x]/Sqrt[a - b*x]])/b + (2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x
]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/
(Sqrt[b*e - a*f]*Sqrt[b*e + a*f]))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [A]
time = 0.13, size = 487, normalized size = 1.75

method	result
default	$\frac{\left(-A\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c(a^2f^2-b^2e^2)}{f^2}}\sqrt{c(-b^2x^2+a^2)}f}{fx+e}\right)b^2cf^2\sqrt{b^2c}+B\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c(a^2f^2-b^2e^2)}{f^2}}}{fx+e}\right.\right.}{}$

risch	$-\frac{C\sqrt{bx+a}\,(-bx+a)}{f\,b^2\sqrt{-c\,(bx-a)}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+c\,a^2}}\right)B}{f\sqrt{b^2c}}-\frac{\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+c\,a^2}}\right)Ce}{f^2\sqrt{b^2c}}-\frac{\ln\left(\frac{\frac{2c\left(a^2f^2-b^2e^2\right)}{f^2}+\frac{2b}{}}{}\right)}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] (-A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^
2))^(1/2)*f)/(f*x+e))*b^2*c*f^2*(b^2*c)^(1/2)+B*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e*f
*(b^2*c)^(1/2)+B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*f^2
*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*(b^2*c)^(1/2
)-C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e*f*(c*(a^2*f^2-
b^2*e^2)/f^2)^(1/2)-C*f^2*(b^2*c)^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2))*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(a^2*f^2-b^2*e^2
)/f^2)^(1/2)/f^3/(b^2*c)^(1/2)/b^2/c/(c*(-b^2*x^2+a^2))^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e detai
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-c\,(-a+bx)}\,\sqrt{a+bx}\,\,(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)),
x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B]
time = 44.56, size = 2500, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] (B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*
a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c
^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (
B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
 (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4
*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B
*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((
a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*
```

```
b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^
4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*
x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a
^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(
a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(
1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*
e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^
6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) +
(16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f
*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^
(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*
b^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^
8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)
) + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
 + b*x)^(1/2) - a^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a
*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*
e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5
/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*
B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(2
4*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c
)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2
*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) -
a^(1/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))
/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2
))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e
^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a +
 b*x)^(1/2) - a^(1/2))))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2)
 - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(
17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))
 + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144
*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^
(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a
^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a
^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)))/((131072*B^4*a^4*c^5)/
(b^8*e^4) - (B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a
^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^
2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (B
*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
(B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*
```

```
c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*
a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a
 + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b
^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4
*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x
)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^
(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a
^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1
/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e
*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6
*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (
16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*
(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)...
```

3.25 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} dx$

Optimal. Leaf size=322

$$\frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}+\frac{C\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{(a^2f^2(2Ce-Bf)}{}$$

```
[Out] f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)/(b*x+a)^(1
/2)/(-b*c*x+a*c)^(1/2)+C*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2
*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(a^2*f^2
*(-B*f+2*C*e)-b^2*(-A*e*f^2+C*e^3))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^
2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/f^2/(-a
^2*f^2+b^2*e^2)^(3/2)/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1624, 1665, 858, 223, 209, 739, 210}

$$\frac{\sqrt{a^2c-b^2cx^2}\,(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\,\text{ArcTan}\left(\frac{\sqrt{c}\,(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}\,f^2\sqrt{a+bx}\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^{3/2}}+\frac{f(a^2-b^2x^2)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)}+\frac{C\sqrt{a^2c-b^2cx^2}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\,\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\sqrt{a^2c-b^2cx^2}\,\int \frac{c(Ab}{}}{c\,(b^2e^2-a^2f^2}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a}\right.}{f\,(b^2e^2-a^2}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a}\right.}{f\,(}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\,\tan}{b\sqrt{c}\,f^2\sqrt{a+}}$$

Mathematica [**A**]
time = 0.97, size = 229, normalized size = 0.71

$$\frac{2\left(\frac{f(Ce^2+f(-Be+Af))(-a+bx)\sqrt{a+bx}}{2(-be+af)(be+af)(e+fx)}+\frac{C\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b}-\frac{(a^2f^2(-2Ce+Bf)+b^2(Ce^3-Aef^2))\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{be+af}\,\sqrt{a+bx}}{\sqrt{be-af}\,\sqrt{a-bx}}\right)}{(be-af)^{3/2}(be+af)^{3/2}}\right)}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x
]
```

```
[Out] (2*((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/(2*(-(b*e) + a*
f)*(b*e + a*f)*(e + f*x)) + (C*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a -
b*x]])/b - ((a^2*f^2*(-2*C*e + B*f) + b^2*(C*e^3 - A*e*f^2))*Sqrt[a - b*x]*
ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((
b*e - a*f)^(3/2)*(b*e + a*f)^(3/2))))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. 2(290) = 580.
time = 0.11, size = 1166, normalized size = 3.62

method	result

default	$\left(A\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c(a^2f^2-b^2e^2)}{f^2}}\,\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)b^2cef^3x\sqrt{b^2c}-B\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c(a^2f^2-b^2e^2)}{f^2}}\,\sqrt{}}{fx+e}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2
))^(1/2)*f)/(f*x+e))*b^2*c*e*f^3*x*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*f+
(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*
f^4*x*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*e*f^3*x*(b^2*c)^(1/2)-C*ln(
2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)*f)/(f*x+e))*b^2*c*e^3*f*x*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^
2*x^2+a^2))^(1/2))*a^2*c*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^
2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-
b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e
*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x
+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^
(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^
2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^4*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/
(c*(-b^2*x^2+a^2))^(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arc
tan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e
^2)/f^2)^(1/2)-A*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)*(b^2*c)^(1/2)+B*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))
^(1/2)*(b^2*c)^(1/2)-C*e^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2
+a^2))^(1/2)*(b^2*c)^(1/2))/c*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(-b^2*x^2
+a^2))^(1/2)/(a*f-b*e)/(b^2*c)^(1/2)/(a*f+b*e)/(f*x+e)/(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)/f^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e detai
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-c\left(-a+bx\right)}\,\sqrt{a+bx}\,\left(e+fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2
), x)
```

Giac [**A**]
time = 1.34, size = 523, normalized size = 1.62

$$-\frac{\frac{2\left(Ba^2b\sqrt{-c}f^3-2\,Ca^2b\sqrt{-c}f^2e-Ab^3\sqrt{-c}f^2e+Cb^3\sqrt{-c}e^3\right)\arctan\left(\frac{\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2f-2\,bce}{2\sqrt{a^2f^2-b^2e^2}\,c}\right)}{\left(a^2f^4-b^2f^2e^2\right)\sqrt{a^2f^2-b^2e^2}\,c}+\frac{C\log\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2\right)}{\sqrt{-c}f^2}+\frac{4\left(2\,Aa^2b^2\sqrt{-c}cf^3-Ab^3\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2\sqrt{-c}f^2e-2\,Ba^2b^2\sqrt{-c}cf^2e+Bb^3\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2\sqrt{-c}fe^2+2\,Ca^2b^2\sqrt{-c}cfe^2-Cb^3\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2\sqrt{-c}e^3\right)}{\left(a^2f^4-b^2f^2e^2\right)\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^4f+4\,a^2c^2f-4\,b\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2ce\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -(2*(B*a^2*b*sqrt(-c)*f^3 - 2*C*a^2*b*sqrt(-c)*f^2*e - A*b^3*sqrt(-c)*f^2*e
 + C*b^3*sqrt(-c)*e^3)*arctan(1/2*((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a
)*c + 2*a*c))^2*f - 2*b*c*e)/(sqrt(a^2*f^2 - b^2*e^2)*c))/((a^2*f^4 - b^2*f
^2*e^2)*sqrt(a^2*f^2 - b^2*e^2)*c) + C*log((sqrt(b*x + a)*sqrt(-c) - sqrt(-
(b*x + a)*c + 2*a*c))^2)/(sqrt(-c)*f^2) + 4*(2*A*a^2*b^2*sqrt(-c)*c*f^3 - A
*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*f^2*e
 - 2*B*a^2*b^2*sqrt(-c)*c*f^2*e + B*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^2*sqrt(-c)*f*e^2 + 2*C*a^2*b^2*sqrt(-c)*c*f*e^2 - C*b^3*
```

```
(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^3)/((a^2
*f^4 - b^2*f^2*e^2)*((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^
4*f + 4*a^2*c^2*f - 4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c
))^2*c*e)))/b
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.26 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3}\,dx$

Optimal. Leaf size=363

$$\frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2}+\frac{(2a^2f^2(2Ce-Bf)-b^2e(Ce^2+f(Be-3Af)))\,(a^2-b^2x^2}{2f\,(b^2e^2-a^2f^2)^2\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)}$$

```
[Out] 1/2*f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)^2/(b*x
+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*a^2*f^2*(-B*f+2*C*e)-b^2*e*(C*e^2+f*(-3
*A*f+B*e)))*(-b^2*x^2+a^2)/f/(-a^2*f^2+b^2*e^2)^2/(f*x+e)/(b*x+a)^(1/2)/(-b
*c*x+a*c)^(1/2)+1/2*(A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(2*a^2*C*f^2+b^2*e*(-3*B
*f+C*e)))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x
^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/(-a^2*f^2+b^2*e^2)^(5/2)/c^(1/2)/
(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 361, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1624, 1665, 821, 739, 210}

$$\frac{f(a^2-b^2x^2)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}\,(e+fx)^2\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)}+\frac{(a^2-b^2x^2)\,(2a^2f^2(2Ce-Bf)-b^2(ef(Be-3Af)+Ce^3))}{2f\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^2}+\frac{\sqrt{a^2c-b^2cx^2}\,(2a^4Cf^2+A(a^2b^2f^2+2b^4e^2)+a^2b^2e(Ce-3Bf))\,\text{ArcTan}\left(\frac{\sqrt{c}\,(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{2\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*(C
*e^3 + e*f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3
*B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c
]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(2
*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{2}{-}}{2c\,(b^2}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf}{2f\,(b^2e^2-a}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf}{2f\,(b^2e^2-a}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf}{2f\,(b^2e^2-a}$$

Mathematica [A]
time = 1.44, size = 252, normalized size = 0.69

$$\frac{\frac{(-a+bx)\sqrt{a+bx}\,(b^2e(Ce^2x+Be(2e+fx)-Af(4e+3fx))+a^2f(-Ce(3e+4fx)+f(Af+B(e+2fx))))}{2(be-af)^2(be+af)^2(e+fx)^2}+\frac{(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2))\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{be+af}\,\sqrt{a+bx}}{\sqrt{be-af}\,\sqrt{a-bx}}\right)}{(be-af)^{5/2}(be+af)^{5/2}}}{\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x
]
```

```
[Out] (((-a + b*x)*Sqrt[a + b*x]*(b^2*e*(C*e^2*x + B*e*(2*e + f*x) - A*f*(4*e + 3
*f*x)) + a^2*f*(-(C*e*(3*e + 4*f*x)) + f*(A*f + B*(e + 2*f*x)))))/(2*(b*e -
 a*f)^2*(b*e + a*f)^2*(e + f*x)^2) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*B*f
) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt
[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((b*e - a*f)^(5/2)*(b*e + a*f)
^(5/2)))/Sqrt[c*(a - b*x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. $2(333) = 666$.
time = 0.11, size = 1794, normalized size = 4.94

method	result	size
default	Expression too large to display	1794

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2*(-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2
*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e*f^3*x^2+C*ln(2*(b^2*c*e*x+a^2*c*f+
(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^
2*c*e^2*f^2*x^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)
*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e*f^3*x-6*B*ln(2*(b^2*c*e*x
+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e
))*a^2*b^2*c*e^2*f^2*x+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2
)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^3*f*x+A*a^2*f^4*(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+2*A*ln(2*(b^2*c*e*x+
a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e)
)*b^4*c*e^4+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a^2*c*f+(
c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2
*c*e^4+2*B*a^2*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)-4*A*b^2*e^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)
+B*a^2*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+2*B*b
^2*e^3*f*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)-3*C*a^2*e
^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+2*C*ln(2*(b
^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f
)/(f*x+e))*a^4*c*f^4*x^2+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2
)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*f^4*x^2+2*A*ln(2*(b^
2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)
/(f*x+e))*b^4*c*e^2*f^2*x^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2
)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^3*f*x+4*C*ln(2*(b
^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f
)/(f*x+e))*a^4*c*e*f^3*x+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2
)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2-3*B*ln(2*(b^
2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)
/(f*x+e))*a^2*b^2*c*e^3*f-3*A*b^2*e*f^3*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(
c*(-b^2*x^2+a^2))^(1/2)+B*b^2*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)-4*C*a^2*e*f^3*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-
b^2*x^2+a^2))^(1/2)+C*b^2*e^3*f*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*
x^2+a^2))^(1/2))/c*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(-b^2*x^2+a^2))^(1/2
)/(a*f-b*e)/(a*f+b*e)/(a^2*f^2-b^2*e^2)/(f*x+e)^2/(c*(a^2*f^2-b^2*e^2)/f^2)
^(1/2)/f
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*f-%e*b>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 61.83, size = 1347, normalized size = 3.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(((2*C*a^4 + A*a^2*b^2)*f^4*x^2 + (C*a^2*b^2 + 2*A*b^4)*e^4 - (3*B*a^2
*b^2*f - 2*(C*a^2*b^2 + 2*A*b^4)*f*x)*e^3 - (6*B*a^2*b^2*f^2*x - (C*a^2*b^2
 + 2*A*b^4)*f^2*x^2 - (2*C*a^4 + A*a^2*b^2)*f^2)*e^2 - (3*B*a^2*b^2*f^3*x^2
 - 2*(2*C*a^4 + A*a^2*b^2)*f^3*x)*e)*sqrt(a^2*c*f^2 - b^2*c*e^2)*log(-(a^2*
b^2*c*f^2*x^2 - 2*a^2*b^2*c*f*x*e - 2*a^4*c*f^2 + 2*sqrt(a^2*c*f^2 - b^2*c*
e^2)*(b^2*x*e + a^2*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a) - (2*b^4*c*x^2 - a^
2*b^2*c)*e^2)/(f^2*x^2 + 2*f*x*e + e^2)) - 2*(2*B*a^4*f^5*x + A*a^4*f^5 - (
C*b^4*x + 2*B*b^4)*e^5 - (B*b^4*f*x - (3*C*a^2*b^2 + 4*A*b^4)*f)*e^4 + (B*a
^2*b^2*f^2 + (5*C*a^2*b^2 + 3*A*b^4)*f^2*x)*e^3 - (B*a^2*b^2*f^3*x + (3*C*a
^4 + 5*A*a^2*b^2)*f^3)*e^2 + (B*a^4*f^4 - (4*C*a^4 + 3*A*a^2*b^2)*f^4*x)*e)
*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(a^6*c*f^8*x^2 + 2*a^6*c*f^7*x*e - 6*a^4
*b^2*c*f^5*x*e^3 + 6*a^2*b^4*c*f^3*x*e^5 - 2*b^6*c*f*x*e^7 - b^6*c*e^8 - (b
^6*c*f^2*x^2 - 3*a^2*b^4*c*f^2)*e^6 + 3*(a^2*b^4*c*f^4*x^2 - a^4*b^2*c*f^4)
*e^4 - (3*a^4*b^2*c*f^6*x^2 - a^6*c*f^6)*e^2), 1/2*(((2*C*a^4 + A*a^2*b^2)*
f^4*x^2 + (C*a^2*b^2 + 2*A*b^4)*e^4 - (3*B*a^2*b^2*f - 2*(C*a^2*b^2 + 2*A*b
^4)*f*x)*e^3 - (6*B*a^2*b^2*f^2*x - (C*a^2*b^2 + 2*A*b^4)*f^2*x^2 - (2*C*a^
4 + A*a^2*b^2)*f^2)*e^2 - (3*B*a^2*b^2*f^3*x^2 - 2*(2*C*a^4 + A*a^2*b^2)*f^
3*x)*e)*sqrt(-a^2*c*f^2 + b^2*c*e^2)*arctan(-sqrt(-a^2*c*f^2 + b^2*c*e^2)*(
b^2*x*e + a^2*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^2*b^2*c*f^2*x^2 - a^4*
c*f^2 - (b^4*c*x^2 - a^2*b^2*c)*e^2)) - (2*B*a^4*f^5*x + A*a^4*f^5 - (C*b^4
*x + 2*B*b^4)*e^5 - (B*b^4*f*x - (3*C*a^2*b^2 + 4*A*b^4)*f)*e^4 + (B*a^2*b^
2*f^2 + (5*C*a^2*b^2 + 3*A*b^4)*f^2*x)*e^3 - (B*a^2*b^2*f^3*x + (3*C*a^4 +
5*A*a^2*b^2)*f^3)*e^2 + (B*a^4*f^4 - (4*C*a^4 + 3*A*a^2*b^2)*f^4*x)*e)*sqrt
(-b*c*x + a*c)*sqrt(b*x + a))/(a^6*c*f^8*x^2 + 2*a^6*c*f^7*x*e - 6*a^4*b^2*
c*f^5*x*e^3 + 6*a^2*b^4*c*f^3*x*e^5 - 2*b^6*c*f*x*e^7 - b^6*c*e^8 - (b^6*c*
f^2*x^2 - 3*a^2*b^4*c*f^2)*e^6 + 3*(a^2*b^4*c*f^4*x^2 - a^4*b^2*c*f^4)*e^4
- (3*a^4*b^2*c*f^6*x^2 - a^6*c*f^6)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. 2(343) = 686.
time = 2.54, size = 1410, normalized size = 3.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -((2*C*a^4*b*sqrt(-c)*f^2 + A*a^2*b^3*sqrt(-c)*f^2 - 3*B*a^2*b^3*sqrt(-c)*f
*e + C*a^2*b^3*sqrt(-c)*e^2 + 2*A*b^5*sqrt(-c)*e^2)*arctan(1/2*((sqrt(b*x +
 a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*f - 2*b*c*e)/(sqrt(a^2*f^2 - b
^2*e^2)*c))/((a^4*f^4 - 2*a^2*b^2*f^2*e^2 + b^4*e^4)*sqrt(a^2*f^2 - b^2*e^2
)*c) + 2*(A*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6
*sqrt(-c)*f^5 + 4*B*a^4*b^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2
*a*c))^4*sqrt(-c)*c*f^5 - 4*A*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x
+ a)*c + 2*a*c))^2*sqrt(-c)*c^2*f^5 + 16*B*a^6*b^2*sqrt(-c)*c^3*f^5 - 3*B*a
^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f^4
*e - 8*C*a^4*b^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sq
rt(-c)*c*f^4*e - 6*A*a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
2*a*c))^4*sqrt(-c)*c*f^4*e - 20*B*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(
b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*f^4*e - 32*C*a^6*b^2*sqrt(-c)*c^3*f^4*e
 - 24*A*a^4*b^4*sqrt(-c)*c^3*f^4*e + 5*C*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) -
sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f^3*e^2 + 2*A*b^5*(sqrt(b*x + a)*sqr
t(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f^3*e^2 + 10*B*a^2*b^4*(sqrt
(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*f^3*e^2 + 44*
C*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*
c^2*f^3*e^2 + 40*A*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*
a*c))^2*sqrt(-c)*c^2*f^3*e^2 + 8*B*a^4*b^4*sqrt(-c)*c^3*f^3*e^2 - 14*C*a^2*
b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*f^2*
e^3 - 12*A*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt
(-c)*c*f^2*e^3 - 16*B*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
 2*a*c))^2*sqrt(-c)*c^2*f^2*e^3 + 8*C*a^4*b^4*sqrt(-c)*c^3*f^2*e^3 - 2*C*b^
5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f*e^4 +
```

```
4*B*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*
f*e^4 - 8*C*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2
*sqrt(-c)*c^2*f*e^4 + 4*C*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
 2*a*c))^4*sqrt(-c)*c*e^5)/((a^4*f^6 - 2*a^2*b^2*f^4*e^2 + b^4*f^2*e^4)*((s
qrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*f + 4*a^2*c^2*f - 4*b
*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*c*e)^2))/b
```

Mupad [B]
time = 86.67, size = 2500, normalized size = 6.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e^
2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f^4
)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2*b^
2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a
^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a^2*b^2*c*
e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) - a^(1/2))^
4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b^2*e^2*f))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^
4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c^2*e^2*f)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^6
- 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6
) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2
)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/
2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b
*e*((a + b*x)^(1/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c
*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2)
 - a^(1/2))^5) + (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) + (((4*A*a^4*f^4 - 10*A*a^2*
b^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(
1/2))^7*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a^4*c^3*f^4
- 10*A*a^2*b^2*c^3*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x
```

)^(1/2) - a^(1/2))*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a
^4*c^2*f^4 - 58*A*a^2*b^2*c^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*
f^2)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5*(4*A*a^4*c*f^4 - 58*A*a^2*b^
2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a
^2*b^3*e^5*f^2)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^6*(16*A*b^4*e^4*f - 8*A*a^4*f^5 + 28*A*a^2*b^2*e^2*f^3))/(((a + b*x)^(1/2)
 - a^(1/2))^6*(b^6*e^8 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(
a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*A*a^4*c*f^5 + 32*A*b^4
*c*e^4*f - 72*A*a^2*b^2*c*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^4*(b^6*e^8
 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2*(16*A*b^4*c^2*e^4*f - 8*A*a^4*c^2*f^5 + 28*A*a^2
*b^2*c^2*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^8 - 2*a^2*b^4*e^6*
f^2 + a^4*b^2*e^4*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^
(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c
*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3
*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a +
 b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1
/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1
/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) +
 (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*(
(a + b*x)^(1/2) - a^(1/2))^3)) - (((32*B*a^4*c^2*f^3 + 22*B*a^2*b^2*c^2*e^2
f)((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(1/2))^3*(
b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) - ((32*B*a^4*c*f^3 + 22*B*a^2
*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^
(1/2))^5*(b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) + (a^(1/2)*(a*c)^(1
/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(8*B*a^4*c^2*f^4 + 8*B*b^4*c^2*e^
4 + 20*B*a^2*b^2*c^2*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^7 - 2*
a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^6*(8*B*a^4*f^4 + 8*B*b^4*e^4 + 20*B*a^2*b^2*e^2*f^2))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^
4)) - (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*B*a^4*
c*f^4 - 16*B*b^4*c*e^4 + 24*B*a^2*b^2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/
2))^4*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^...

3.27 $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=501

$$-\frac{(16a^2Cf^2-b^2(3Ce^2-5f(3Be+4Af)))\,(e+fx)^2\,(a^2-b^2x^2)}{60b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{(Ce-5Bf)(e+fx)^3\,(a^2-b^2x^2)}{20b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C(}{5b^2f}$$

```
[Out] -1/60*(16*a^2*C*f^2-b^2*(3*C*e^2-5*f*(4*A*f+3*B*e)))*(f*x+e)^2*(-b^2*x^2+a^
2)/b^4/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/20*(-5*B*f+C*e)*(f*x+e)^3*(-b^2
*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/5*C*(f*x+e)^4*(-b^2*x^2+
a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/120*(64*a^4*C*f^4+16*a^2*b^2*
f^2*(13*C*e^2+5*f*(A*f+3*B*e))-4*b^4*e^2*(3*C*e^2-5*f*(16*A*f+3*B*e))+b^2*f
*(a^2*f^2*(45*B*f+71*C*e)-2*b^2*e*(3*C*e^2-5*f*(10*A*f+3*B*e)))*x)*(-b^2*x^
2+a^2)/b^6/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/8*(4*A*(3*a^2*b^2*e*f^2+2*b
^4*e^3)+a^2*(3*a^2*f^2*(B*f+3*C*e)+4*b^2*e^2*(3*B*f+C*e)))*arctan(b*x*c^(1/
2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^5/c^(1/2)/(b*x+a)^(
1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 496, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1624, 1668, 847, 794, 223, 209}

$$\frac{(a^2-b^2x^2)\,(e+fx)^2\left(-\frac{16a^2Cf^2}{b^2}-5f(4Af+3Be)+3Ce^2\right)}{60b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{(a^2-b^2x^2)\,(e+fx)^3(Ce-5Bf)}{20b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C(a^2-b^2x^2)\,(e+fx)^4}{5b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)(3a^4f^2(Bf+3Ce)+4A(3a^2b^2ef^2+2b^4e^3)+4a^2b^2e^2(3Bf+Ce))}{8b^5\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{(a^2-b^2x^2)\,(b^2fx(a^2f^2(45Bf+71Ce)-b^2(6Ce^3-10ef(10Af+3Be)))+4(16a^4Cf^4+4a^2b^2f^2(5f(Af+3Be)+13Ce^2)+b^4(-e^2)\,(3Ce^2-5f(16Af+3Be))))}{120b^6f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] ((3*C*e^2 - (16*a^2*C*f^2)/b^2 - 5*f*(3*B*e + 4*A*f))*(e + f*x)^2*(a^2 - b^
2*x^2))/(60*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e - 5*B*f)*(e + f*
x)^3*(a^2 - b^2*x^2))/(20*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - (C*(e +
f*x)^4*(a^2 - b^2*x^2))/(5*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - ((4*(16
*a^4*C*f^4 + 4*a^2*b^2*f^2*(13*C*e^2 + 5*f*(3*B*e + A*f)) - b^4*e^2*(3*C*e^
2 - 5*f*(3*B*e + 16*A*f))) + b^2*f*(a^2*f^2*(71*C*e + 45*B*f) - b^2*(6*C*e^
3 - 10*e*f*(3*B*e + 10*A*f)))*x)*(a^2 - b^2*x^2))/(120*b^6*f*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]) + ((3*a^4*f^2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f
) + 4*A*(2*b^4*e^3 + 3*a^2*b^2*e*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sq
rt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3\left(-c\left(5Ab^2+4a^2C\right)f^2+ \cdots}{\sqrt{a^2c-b^2cx \cdots}}}{5b^2cf^2\sqrt{a+bx}\,\sqrt{ac-bc \cdots}}$$

$$= \frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{\sqrt{a^2c \cdots}}{\cdots}$$

$$= -\frac{\left(16a^2Cf^2-b^2(3Ce^2-5f(3Be+4Af))\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{(C \cdots}{\cdots}$$

$$= -\frac{\left(16a^2Cf^2-b^2(3Ce^2-5f(3Be+4Af))\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{(C \cdots}{\cdots}$$

$$= -\frac{\left(16a^2Cf^2-b^2(3Ce^2-5f(3Be+4Af))\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{(C \cdots}{\cdots}$$

$$= -\frac{\left(16a^2Cf^2-b^2(3Ce^2-5f(3Be+4Af))\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{(C \cdots}{\cdots}$$

Mathematica [A]
time = 0.80, size = 283, normalized size = 0.56

$$\frac{-\left((a-bx)\sqrt{a+bx}\left(64a^4Cf^3+a^2b^2f\left(5f(48Be+16Af+9Bfx)+C\left(240e^2+135efx+32f^2x^2\right)\right)+2b^4\left(10Af\left(18e^2+9efx+2f^2x^2\right)+15B\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)+3Cx\left(10e^3+20e^2fx+15ef^2x^2+4f^3x^3\right)\right)\right)+30b\left(3a^4f^2(3Ce+Bf)+4a^2b^2e^2(Ce+3Bf)+4A\left(2b^4e^3+3a^2b^2ef^2\right)\right)\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{120b^6\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
,x]
```

```
[Out] (-((a - b*x)*Sqrt[a + b*x]*(64*a^4*C*f^3 + a^2*b^2*f*(5*f*(48*B*e + 16*A*f
+ 9*B*f*x) + C*(240*e^2 + 135*e*f*x + 32*f^2*x^2)) + 2*b^4*(10*A*f*(18*e^2
+ 9*e*f*x + 2*f^2*x^2) + 15*B*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) +
 3*C*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 30*b*(3*a^4*f^
2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f) + 4*A*(2*b^4*e^3 + 3*a^2*b^2*
e*f^2))*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(120*b^6*Sqrt[c*
(a - b*x)])
```

Maple [A]
time = 0.11, size = 913, normalized size = 1.82

method	result
risch	$-\frac{(24Cf^3x^4b^4+30Bb^4f^3x^3+90Cb^4ef^2x^3+40Ab^4f^3x^2+120Bb^4ef^2x^2+32Ca^2b^2f^3x^2+120Cb^4e^2fx^2+180Ab^4ef^2x+45Ba^2b^2f^3}{120}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(-24Cb^4f^3x^4\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}-30Bb^4f^3x^3\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}-90\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/120*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/c*(-24*C*b^4*f^3*x^4*(c*(-b^2*x^2+a^
2))^(1/2)*(b^2*c)^(1/2)-30*B*b^4*f^3*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(
1/2)-90*C*b^4*e*f^2*x^3*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+180*A*arctan
((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*b^4*c*e*f^2+120*A*arctan((b^
2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^6*c*e^3-40*A*b^4*f^3*x^2*(c*(-b^2*
x^2+a^2))^(1/2)*(b^2*c)^(1/2)+45*B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2)
)^(1/2))*a^4*b^2*c*f^3+180*B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2
))*a^2*b^4*c*e^2*f-120*B*b^4*e*f^2*x^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/
2)+135*C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*b^2*c*e*f^2+6
0*C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*b^4*c*e^3-32*C*a^2
*b^2*f^3*x^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-120*C*b^4*e^2*f*x^2*(c*
(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)-180*A*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^
(1/2)*b^4*e*f^2*x-45*B*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)*a^2*b^2*f^3*x
-180*B*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)*b^4*e^2*f*x-135*C*(c*(-b^2*x^
2+a^2))^(1/2)*(b^2*c)^(1/2)*a^2*b^2*e*f^2*x-60*C*(c*(-b^2*x^2+a^2))^(1/2)*(
b^2*c)^(1/2)*b^4*e^3*x-80*A*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*b^2*
f^3-360*A*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*b^4*e^2*f-240*B*(b^2*c)^(1
/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*b^2*e*f^2-120*B*(b^2*c)^(1/2)*(c*(-b^2*x^2
+a^2))^(1/2)*b^4*e^3-64*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^4*f^3-24
0*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*b^2*e^2*f)/b^6/(c*(-b^2*x^2+
a^2))^(1/2)/(b^2*c)^(1/2)
```

Maxima [A]

time = 0.53, size = 469, normalized size = 0.94

$$-\frac{\sqrt{-b^2cx^2+a^2c}\,Cf^3x^4}{5\,b^2c}-\frac{4\sqrt{-b^2cx^2+a^2c}\,Ca^2f^3x^2}{15\,b^4c}-\frac{8\sqrt{-b^2cx^2+a^2c}\,Ca^4f^3}{15\,b^6c}-\frac{\sqrt{-b^2cx^2+a^2c}\,(Bf^3+3\,Cf^2e)x^3}{4\,b^2c}+\frac{A\arcsin\left(\frac{bx}{a}\right)e^3}{b\sqrt{c}}-\frac{\sqrt{-b^2cx^2+a^2c}\,(Af^3+3\,Bf^2e+3\,Cfe^2)x^2}{3\,b^2c}+\frac{3\,(Bf^3+3\,Cf^2e)a^4\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}}+\frac{(3\,Af^2e+3\,Bfe^2+Ce^3)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}}-\frac{3\sqrt{-b^2cx^2+a^2c}\,Afe^2}{b^2c}-\frac{3\sqrt{-b^2cx^2+a^2c}\,(Bf^3+3\,Cf^2e)a^2x}{8\,b^4c}-\frac{\sqrt{-b^2cx^2+a^2c}\,(3\,Af^2e+3\,Bfe^2+Ce^3)x}{2\,b^2c}-\frac{\sqrt{-b^2cx^2+a^2c}\,Be^3}{b^2c}-\frac{2\sqrt{-b^2cx^2+a^2c}\,(Af^3+3\,Bf^2e+3\,Cfe^2)a^2}{3\,b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

[Out] -1/5*sqrt(-b^2*c*x^2 + a^2*c)*C*f^3*x^4/(b^2*c) - 4/15*sqrt(-b^2*c*x^2 + a^
2*c)*C*a^2*f^3*x^2/(b^4*c) - 8/15*sqrt(-b^2*c*x^2 + a^2*c)*C*a^4*f^3/(b^6*c
) - 1/4*sqrt(-b^2*c*x^2 + a^2*c)*(B*f^3 + 3*C*f^2*e)*x^3/(b^2*c) + A*arcsin
(b*x/a)*e^3/(b*sqrt(c)) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(A*f^3 + 3*B*f^2*e +
 3*C*f*e^2)*x^2/(b^2*c) + 3/8*(B*f^3 + 3*C*f^2*e)*a^4*arcsin(b*x/a)/(b^5*sq
rt(c)) + 1/2*(3*A*f^2*e + 3*B*f*e^2 + C*e^3)*a^2*arcsin(b*x/a)/(b^3*sqrt(c)
) - 3*sqrt(-b^2*c*x^2 + a^2*c)*A*f*e^2/(b^2*c) - 3/8*sqrt(-b^2*c*x^2 + a^2*
c)*(B*f^3 + 3*C*f^2*e)*a^2*x/(b^4*c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(3*A*f^
2*e + 3*B*f*e^2 + C*e^3)*x/(b^2*c) - sqrt(-b^2*c*x^2 + a^2*c)*B*e^3/(b^2*c)
 - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(A*f^3 + 3*B*f^2*e + 3*C*f*e^2)*a^2/(b^4*c)

Fricas [**A**]
time = 1.58, size = 698, normalized size = 1.39

$$\left[-\frac{15\left(3Ba^4bf^3+12Ba^2b^3fe^2+3\left(3Ca^4b+4Aa^2b^3\right)f^2e+4\left(Ca^2b^3+2Ab^5\right)e^3\right)\sqrt{-c}\log\left(2b^2cx^2-2\sqrt{-bcx+ac}\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\left(24Cb^4f^3x^4+30Bb^4f^3x^3+45Ba^2b^2f^3x+8\left(4Ca^2b^2+5Ab^4\right)f^3x^2+16\left(4Ca^4+5Aa^2b^2\right)f^3+60\left(Cb^4x+2Bb^4\right)e^3+60\left(2Cb^4fx^2+3Bb^4fx+2\left(2Ca^2b^2+3Ab^4\right)f\right)e^2+15\left(6Cb^4f^2x^3+8Bb^4f^2x^2+16Ba^2b^2f^2+3\left(3Ca^2b^2+4Ab^4\right)f^2x\right)e\right)\sqrt{-bcx+ac}\sqrt{bx+a}}{240\,b^6c},\ -\frac{15\left(3Ba^4bf^3+12Ba^2b^3fe^2+3\left(3Ca^4b+4Aa^2b^3\right)f^2e+4\left(Ca^2b^3+2Ab^5\right)e^3\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)+\left(24Cb^4f^3x^4+30Bb^4f^3x^3+45Ba^2b^2f^3x+8\left(4Ca^2b^2+5Ab^4\right)f^3x^2+16\left(4Ca^4+5Aa^2b^2\right)f^3+60\left(Cb^4x+2Bb^4\right)e^3+60\left(2Cb^4fx^2+3Bb^4fx+2\left(2Ca^2b^2+3Ab^4\right)f\right)e^2+15\left(6Cb^4f^2x^3+8Bb^4f^2x^2+16Ba^2b^2f^2+3\left(3Ca^2b^2+4Ab^4\right)f^2x\right)e\right)\sqrt{-bcx+ac}\sqrt{bx+a}}{120\,b^6c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")

[Out] [-1/240*(15*(3*B*a^4*b*f^3 + 12*B*a^2*b^3*f*e^2 + 3*(3*C*a^4*b + 4*A*a^2*b^
3)*f^2*e + 4*(C*a^2*b^3 + 2*A*b^5)*e^3)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f^3*x^4 + 30
*B*b^4*f^3*x^3 + 45*B*a^2*b^2*f^3*x + 8*(4*C*a^2*b^2 + 5*A*b^4)*f^3*x^2 + 1
6*(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 60*(C*b^4*x + 2*B*b^4)*e^3 + 60*(2*C*b^4*f*
x^2 + 3*B*b^4*f*x + 2*(2*C*a^2*b^2 + 3*A*b^4)*f)*e^2 + 15*(6*C*b^4*f^2*x^3
+ 8*B*b^4*f^2*x^2 + 16*B*a^2*b^2*f^2 + 3*(3*C*a^2*b^2 + 4*A*b^4)*f^2*x)*e)*
sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c), -1/120*(15*(3*B*a^4*b*f^3 + 12*B
*a^2*b^3*f*e^2 + 3*(3*C*a^4*b + 4*A*a^2*b^3)*f^2*e + 4*(C*a^2*b^3 + 2*A*b^5
)*e^3)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x
^2 - a^2*c)) + (24*C*b^4*f^3*x^4 + 30*B*b^4*f^3*x^3 + 45*B*a^2*b^2*f^3*x +
8*(4*C*a^2*b^2 + 5*A*b^4)*f^3*x^2 + 16*(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 60*(C*
b^4*x + 2*B*b^4)*e^3 + 60*(2*C*b^4*f*x^2 + 3*B*b^4*f*x + 2*(2*C*a^2*b^2 + 3
*A*b^4)*f)*e^2 + 15*(6*C*b^4*f^2*x^3 + 8*B*b^4*f^2*x^2 + 16*B*a^2*b^2*f^2 +
 3*(3*C*a^2*b^2 + 4*A*b^4)*f^2*x)*e)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6
*c)]

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)**3*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)

[Out] Timed out

Giac [A]
time = 1.39, size = 571, normalized size = 1.14

$$-\frac{\left(\left(2\left(3\left(\frac{4(bx+a)Cf^3}{c} - \frac{16Cac^4f^3-5Bbc^4f^3-15Cbc^4f^2e}{c^5}\right)(bx+a) + \frac{88Ca^2c^4f^3-45Babc^4f^3+20Ab^2c^4f^3-135Cabc^4f^2e+60Bb^2c^4f^2e+60Cb^2c^4fe^2}{c^5}\right)(bx+a) - \frac{5(32Ca^3c^4f^3-27Ba^2bc^4f^3+16Aab^2c^4f^3-81Ca^2bc^4f^2e+48Bab^2c^4f^2e-36Ab^3c^4f^2e+48Cab^2c^4fe^2-36Bb^3c^4fe^2-12Cb^3c^4e^3)}{c^5}\right)(bx+a) + \frac{15(8Ca^4c^4f^3-5Ba^3bc^4f^3+8Aa^2b^2c^4f^3-15Ca^3bc^4f^2e+24Ba^2b^2c^4f^2e-12Aab^3c^4f^2e+24Ca^2b^2c^4fe^2-12Bab^3c^4fe^2+24Ab^4c^4fe^2-4Cab^3c^4e^3+8Bb^4c^4e^3)}{c^5}\right)\sqrt{-(bx+a)c+2ac}\sqrt{bx+a} + \frac{30(3Ba^4bf^3+9Ca^4bf^2e+12Aa^2b^3f^2e+12Ba^2b^3fe^2+4Ca^2b^3e^3+8Ab^5e^3)\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}}{120b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/120*(((2*(3*(4*(b*x + a)*C*f^3/c - (16*C*a*c^4*f^3 - 5*B*b*c^4*f^3 - 15*
C*b*c^4*f^2*e)/c^5)*(b*x + a) + (88*C*a^2*c^4*f^3 - 45*B*a*b*c^4*f^3 + 20*A
*b^2*c^4*f^3 - 135*C*a*b*c^4*f^2*e + 60*B*b^2*c^4*f^2*e + 60*C*b^2*c^4*f*e^
2)/c^5)*(b*x + a) - 5*(32*C*a^3*c^4*f^3 - 27*B*a^2*b*c^4*f^3 + 16*A*a*b^2*c
^4*f^3 - 81*C*a^2*b*c^4*f^2*e + 48*B*a*b^2*c^4*f^2*e - 36*A*b^3*c^4*f^2*e +
 48*C*a*b^2*c^4*f*e^2 - 36*B*b^3*c^4*f*e^2 - 12*C*b^3*c^4*e^3)/c^5)*(b*x +
a) + 15*(8*C*a^4*c^4*f^3 - 5*B*a^3*b*c^4*f^3 + 8*A*a^2*b^2*c^4*f^3 - 15*C*a
^3*b*c^4*f^2*e + 24*B*a^2*b^2*c^4*f^2*e - 12*A*a*b^3*c^4*f^2*e + 24*C*a^2*b
^2*c^4*f*e^2 - 12*B*a*b^3*c^4*f*e^2 + 24*A*b^4*c^4*f*e^2 - 4*C*a*b^3*c^4*e^
3 + 8*B*b^4*c^4*e^3)/c^5)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 30*(3*
B*a^4*b*f^3 + 9*C*a^4*b*f^2*e + 12*A*a^2*b^3*f^2*e + 12*B*a^2*b^3*f*e^2 + 4
*C*a^2*b^3*e^3 + 8*A*b^5*e^3)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
+ a)*c + 2*a*c)))/sqrt(-c))/b^6
```

Mupad [B]
time = 161.43, size = 2500, normalized size = 4.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^3*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x
)
```

```
[Out] - ((((23*B*a^4*c*f^3)/2 - 18*B*a^2*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^13)/(b^5*((a + b*x)^(1/2) - a^(1/2))^13) + (((a*c - b*c*x)^(1/2) -
 (a*c)^(1/2))^15*((3*B*a^4*f^3)/2 + 6*B*a^2*b^2*e^2*f))/(b^5*((a + b*x)^(1/
2) - a^(1/2))^15) - (((3*B*a^4*c^7*f^3)/2 + 6*B*a^2*b^2*c^7*e^2*f)*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2)))/(b^5*((a + b*x)^(1/2) - a^(1/2))) - (((23*B*a^
4*c^6*f^3)/2 - 18*B*a^2*b^2*c^6*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) + (((333*B*a^4*c^5*f^3)/2 + 90*B*a^2
*b^2*c^5*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^5*((a + b*x)^(1/2
) - a^(1/2))^5) - (((333*B*a^4*c^2*f^3)/2 + 90*B*a^2*b^2*c^2*e^2*f)*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^11)/(b^5*((a + b*x)^(1/2) - a^(1/2))^11) - (((
671*B*a^4*c^4*f^3)/2 - 66*B*a^2*b^2*c^4*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^7)/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + (((671*B*a^4*c^3*f^3)/2 -
66*B*a^2*b^2*c^3*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^5*((a + b
```

x)^(1/2) - a^(1/2))^9) + (a^(1/2)(a*c)^(1/2)*(48*B*b^2*c^5*e^3 + 192*B*a^2*c^5*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) + (a^(1/2)*(a*c)^(1/2)*(160*B*b^2*c^3*e^3 + 128*B*a^2*c^3*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (a^(1/2)*(a*c)^(1/2)*(120*B*b^2*c^4*e^3 + 256*B*a^2*c^4*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (a^(1/2)*(a*c)^(1/2)*(120*B*b^2*c^2*e^3 + 256*B*a^2*c^2*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(b^4*((a + b*x)^(1/2) - a^(1/2))^10) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12*(48*B*b^2*c*e^3 + 192*B*a^2*c*e*f^2))/(b^4*((a + b*x)^(1/2) - a^(1/2))^12) + (8*B*a^(1/2)*e^3*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/(b^2*((a + b*x)^(1/2) - a^(1/2))^14) + (8*B*a^(1/2)*c^6*e^3*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16/((a + b*x)^(1/2) - a^(1/2))^16 + c^8 + (8*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14 + (8*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12) - ((a^(1/2)*(a*c)^(1/2)*(64*A*a^2*c^3*f^3 + 96*A*b^2*c^3*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) - (a^(1/2)*(a*c)^(1/2)*((128*A*a^2*c^2*f^3)/3 - 144*A*b^2*c^2*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8*(64*A*a^2*c*f^3 + 96*A*b^2*c*e^2*f))/(b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (6*A*a^2*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(b^3*((a + b*x)^(1/2) - a^(1/2))^11) - (6*A*a^2*c^5*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (30*A*a^2*c*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^3*((a + b*x)^(1/2) - a^(1/2))^9) + (24*A*a^(1/2)*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(b^2*((a + b*x)^(1/2) - a^(1/2))^10) + (30*A*a^2*c^4*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^3*((a + b*x)^(1/2) - a^(1/2))^3) + (36*A*a^2*c^3*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^3*((a + b*x)^(1/2) - a^(1/2))^5) - (36*A*a^2*c^2*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a + b*x)^(1/2) - a^(1/2))^7) + (24*A*a^(1/2)*c^4*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12/((a + b*x)^(1/2) - a^(1/2))^12 + c^6 + (6*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (6*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (15*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (20*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (15*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8) - ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^19*((9*C*a^4*e*f^2)/2 + 2*C*a^2*b^2*e^3))/(b^5*((a + b*x)^(1/2) - a^(1/2))^1

```
9) - ((2*C*a^2*b^2*c*e^3 - (87*C*a^4*c*e*f^2)/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^17)/(b^5*((a + b*x)^(1/2) - a^(1/2))^17) - (((9*C*a^4*c^9*e*f^2)/
2 + 2*C*a^2*b^2*c^9*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^5*((a + b*
x)^(1/2) - a^(1/2))) - (((87*C*a^4*c^8*e*f^2)/2 - 2*C*a^2*b^2*c^8*e^3)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) - ((
42*C*a^4*c^6*e*f^2 - 88*C*a^2*b^2*c^6*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^7)/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + ((42*C*a^4*c^3*e*f^2 - 88*C*a^
2*b^2*c^3*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(b^5*((a + b*x)^(1/2
) - a^(1/2))^13) + ((426*C*a^4*c^7*e*f^2 + 40*C...
```

3.28 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=368

$$\frac{(Ce-4Bf)(e+fx)^2\,(a^2-b^2x^2)}{12b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C(e+fx)^3\,(a^2-b^2x^2)}{4b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{(4(4a^2f^2(2Ce+Bf)-b^2e(Ce^2-4f(Be+}{24}$$

```
[Out] 1/12*(-4*B*f+C*e)*(f*x+e)^2*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)
^(1/2)-1/4*C*(f*x+e)^3*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2
)-1/24*(16*a^2*f^2*(B*f+2*C*e)-4*b^2*e*(C*e^2-4*f*(3*A*f+B*e))+f*(9*a^2*C*f
^2-b^2*(2*C*e^2-4*f*(3*A*f+2*B*e)))*x)*(-b^2*x^2+a^2)/b^4/f/(b*x+a)^(1/2)/(
-b*c*x+a*c)^(1/2)+1/8*(4*A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(3*a^2*C*f^2+4*b^2*e
*(2*B*f+C*e)))*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2
*c)^(1/2)/b^5/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1624, 1668, 847, 794, 223, 209}

$$-\frac{(a^2-b^2x^2)\left(4\left(4a^2f^2(Bf+2Ce)-\frac{1}{4}b^2(4Ce^3-16ef(3Af+Be))\right)+fx(9a^2Cf^2-b^2(2Ce^2-4f(3Af+2Be)))\right)}{24b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{(a^2-b^2x^2)\,(e+fx)^2(Ce-4Bf)}{12b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C(a^2-b^2x^2)\,(e+fx)^3}{4b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\,\mathrm{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)(3a^4Cf^2+4A(a^2b^2f^2+2b^4e^2)+4a^2b^2e(2Bf+Ce))}{8b^5\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*(a^2 - b^2*x^2))/(12*b^2*f*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]) - (C*(e + f*x)^3*(a^2 - b^2*x^2))/(4*b^2*f*Sqrt[a + b*x]*Sqrt[a
*c - b*c*x]) - ((4*(4*a^2*f^2*(2*C*e + B*f) - (b^2*(4*C*e^3 - 16*e*f*(B*e +
 3*A*f)))/4) + f*(9*a^2*C*f^2 - b^2*(2*C*e^2 - 4*f*(2*B*e + 3*A*f)))*x)*(a^
2 - b^2*x^2))/(24*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((3*a^4*C*f^2 +
4*a^2*b^2*e*(C*e + 2*B*f) + 4*A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2
*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[
a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2(-c(4Ab^2+3a^2C)f^2+\cdots}{\sqrt{a^2c-b^2c\cdots}}}{4b^2cf^2\sqrt{a+bx}\,\sqrt{ac-bc\cdots}}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{\sqrt{a^2c\cdots}}{\cdots}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{(4(4\cdots}{\cdots}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{(4(4\cdots}{\cdots}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{(4(4\cdots}{\cdots}$$

Mathematica [A]
time = 0.49, size = 200, normalized size = 0.54

$$\frac{-b(a-bx)\sqrt{a+bx}\left(a^2f(32Ce+16Bf+9Cfx)+2b^2(6Af(4e+fx)+4B(3e^2+3efx+f^2x^2)+Cx(6e^2+8efx+3f^2x^2))\right)+6(3a^4Cf^2+4a^2b^2e(Ce+2Bf)+4A(2b^4e^2+a^2b^2f^2))\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{24b^5\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
,x]
```

```
[Out] (-(b*(a - b*x)*Sqrt[a + b*x]*(a^2*f*(32*C*e + 16*B*f + 9*C*f*x) + 2*b^2*(6*
A*f*(4*e + f*x) + 4*B*(3*e^2 + 3*e*f*x + f^2*x^2) + C*x*(6*e^2 + 8*e*f*x +
3*f^2*x^2)))) + 6*(3*a^4*C*f^2 + 4*a^2*b^2*e*(C*e + 2*B*f) + 4*A*(2*b^4*e^2
 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(24*b^5
*Sqrt[c*(a - b*x)])
```

Maple [A]
time = 0.10, size = 599, normalized size = 1.63

method	result

risch	$-\frac{(6C\,f^2x^3b^2+8B\,b^2f^2x^2+16C\,b^2ef\,x^2+12A\,b^2f^2x+24B\,b^2efx+9C\,a^2f^2x+12C\,b^2e^2x+48A\,b^2ef+16B\,a^2f^2+24B\,b^2e^2+32C\,a^2ef}{24b^4\sqrt{-c\,(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c\,(-bx+a)}\,\left(-6C\,b^2f^2x^3\sqrt{c\,(-b^2x^2+a^2)}\,\sqrt{b^2c}+12A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\,(-b^2x^2+a^2)}}\right)a^2b^2c\,f^2+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/24*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/c*(-6*C*b^2*f^2*x^3*(c*(-b^2*x^2+a^2)
)^(1/2)*(b^2*c)^(1/2)+12*A*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))
*a^2*b^2*c*f^2+24*A*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^4*c*
e^2+24*B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*b^2*c*e*f-8*B
*b^2*f^2*x^2*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+9*C*arctan((b^2*c)^(1/2
)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^4*c*f^2+12*C*arctan((b^2*c)^(1/2)*x/(c*(-b^
2*x^2+a^2))^(1/2))*a^2*b^2*c*e^2-16*C*b^2*e*f*x^2*(c*(-b^2*x^2+a^2))^(1/2)*
(b^2*c)^(1/2)-12*A*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)*b^2*f^2*x-24*B*(c
*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)*b^2*e*f*x-9*C*(c*(-b^2*x^2+a^2))^(1/2)
*(b^2*c)^(1/2)*a^2*f^2*x-12*C*(c*(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)*b^2*e^
2*x-48*A*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*b^2*e*f-16*B*(b^2*c)^(1/2)*
(c*(-b^2*x^2+a^2))^(1/2)*a^2*f^2-24*B*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2
)*b^2*e^2-32*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*a^2*e*f)/b^4/(c*(-b^2
*x^2+a^2))^(1/2)/(b^2*c)^(1/2)
```

<u>Maxima</u> [A]

time = 0.51, size = 318, normalized size = 0.86

$$-\frac{\sqrt{-b^2cx^2+a^2c}\,Cf^2x^3}{4\,b^2c}+\frac{3\,Ca^4f^2\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}}-\frac{3\,\sqrt{-b^2cx^2+a^2c}\,Ca^2f^2x}{8\,b^4c}+\frac{A\arcsin\left(\frac{bx}{a}\right)e^2}{b\sqrt{c}}-\frac{\sqrt{-b^2cx^2+a^2c}\,(Bf^2+2\,Cfe)x^2}{3\,b^2c}+\frac{(Af^2+2\,Bfe+Ce^2)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}}-\frac{2\,\sqrt{-b^2cx^2+a^2c}\,Afe}{b^2c}-\frac{\sqrt{-b^2cx^2+a^2c}\,(Af^2+2\,Bfe+Ce^2)x}{2\,b^2c}-\frac{\sqrt{-b^2cx^2+a^2c}\,Be^2}{b^2c}-\frac{2\,\sqrt{-b^2cx^2+a^2c}\,(Bf^2+2\,Cfe)a^2}{3\,b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*sqrt(-b^2*c*x^2 + a^2*c)*C*f^2*x^3/(b^2*c) + 3/8*C*a^4*f^2*arcsin(b*x/
a)/(b^5*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*f^2*x/(b^4*c) + A*arc
sin(b*x/a)*e^2/(b*sqrt(c)) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(B*f^2 + 2*C*f*e)
*x^2/(b^2*c) + 1/2*(A*f^2 + 2*B*f*e + C*e^2)*a^2*arcsin(b*x/a)/(b^3*sqrt(c)
) - 2*sqrt(-b^2*c*x^2 + a^2*c)*A*f*e/(b^2*c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)
*(A*f^2 + 2*B*f*e + C*e^2)*x/(b^2*c) - sqrt(-b^2*c*x^2 + a^2*c)*B*e^2/(b^2*
c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(B*f^2 + 2*C*f*e)*a^2/(b^4*c)
```

Fricas [**A**]
time = 1.32, size = 484, normalized size = 1.32

$$\left[-\frac{3\,(8\,Ba^2b^2fe+(3\,Ca^4+4\,Aa^2b^2)f^2+4\,(Ca^2b^2+2\,Ab^4)e^2)\sqrt{-c}\,\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,(6\,Cb^3f^2x^3+8\,Bb^3f^2x^2+16\,Ba^2bf^2+3\,(3\,Ca^2b+4\,Ab^3)f^2x+12\,(Cb^3x+2\,Bb^3)e^2+8\,(2\,Cb^3fx^2+3\,Bb^3fx+2\,(2\,Ca^2b+3\,Ab^3)f)e)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{48\,b^5c},\ -\frac{3\,(8\,Ba^2b^2fe+(3\,Ca^4+4\,Aa^2b^2)f^2+4\,(Ca^2b^2+2\,Ab^4)e^2)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)+(6\,Cb^3f^2x^3+8\,Bb^3f^2x^2+16\,Ba^2bf^2+3\,(3\,Ca^2b+4\,Ab^3)f^2x+12\,(Cb^3x+2\,Bb^3)e^2+8\,(2\,Cb^3fx^2+3\,Bb^3fx+2\,(2\,Ca^2b+3\,Ab^3)f)e)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{24\,b^5c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/48*(3*(8*B*a^2*b^2*f*e + (3*C*a^4 + 4*A*a^2*b^2)*f^2 + 4*(C*a^2*b^2 + 2
*A*b^4)*e^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*
b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*f^2*x^3 + 8*B*b^3*f^2*x^2 + 16*B*a^2*b*f
^2 + 3*(3*C*a^2*b + 4*A*b^3)*f^2*x + 12*(C*b^3*x + 2*B*b^3)*e^2 + 8*(2*C*b^
3*f*x^2 + 3*B*b^3*f*x + 2*(2*C*a^2*b + 3*A*b^3)*f)*e)*sqrt(-b*c*x + a*c)*sq
rt(b*x + a))/(b^5*c), -1/24*(3*(8*B*a^2*b^2*f*e + (3*C*a^4 + 4*A*a^2*b^2)*f
^2 + 4*(C*a^2*b^2 + 2*A*b^4)*e^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*
x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (6*C*b^3*f^2*x^3 + 8*B*b^3*f^2*x^
2 + 16*B*a^2*b*f^2 + 3*(3*C*a^2*b + 4*A*b^3)*f^2*x + 12*(C*b^3*x + 2*B*b^3)
*e^2 + 8*(2*C*b^3*f*x^2 + 3*B*b^3*f*x + 2*(2*C*a^2*b + 3*A*b^3)*f)*e)*sqrt(
-b*c*x + a*c)*sqrt(b*x + a))/(b^5*c)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.01, size = 366, normalized size = 0.99

$$-\frac{\left(\left(2\left(\frac{3\,(bx+a)Cf^2}{c}-\frac{9\,Cac^3f^2-4\,Bbc^3f^2-8\,Cbc^3fe}{c^4}\right)(bx+a)+\frac{27\,Ca^2c^3f^2-16\,Babc^3f^2+12\,Ab^2c^3f^2-32\,Cabc^3fe+24\,Bb^2c^3fe+12\,Cb^2c^3e^2}{c^4}\right)(bx+a)-\frac{3\,(5\,Ca^3c^3f^2-8\,Ba^2bc^3f^2+4\,Aab^2c^3f^2-16\,Ca^2bc^3fe+8\,Bab^2c^3fe-16\,Ab^3c^3fe+4\,Cab^2c^3e^2-8\,Bb^3c^3e^2)}{c^4}\right)\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}+\frac{6\,(3\,Ca^4f^2+4\,Aa^2b^2f^2+8\,Ba^2b^2fe+4\,Ca^2b^2e^2+8\,Ab^4e^2)\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/24*(((2*(3*(b*x + a)*C*f^2/c - (9*C*a*c^3*f^2 - 4*B*b*c^3*f^2 - 8*C*b*c^
3*f*e)/c^4)*(b*x + a) + (27*C*a^2*c^3*f^2 - 16*B*a*b*c^3*f^2 + 12*A*b^2*c^3
*f^2 - 32*C*a*b*c^3*f*e + 24*B*b^2*c^3*f*e + 12*C*b^2*c^3*e^2)/c^4)*(b*x +
a) - 3*(5*C*a^3*c^3*f^2 - 8*B*a^2*b*c^3*f^2 + 4*A*a*b^2*c^3*f^2 - 16*C*a^2*
b*c^3*f*e + 8*B*a*b^2*c^3*f*e - 16*A*b^3*c^3*f*e + 4*C*a*b^2*c^3*e^2 - 8*B*
```

```
b^3*c^3*e^2)/c^4)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 6*(3*C*a^4*f^2
 + 4*A*a^2*b^2*f^2 + 8*B*a^2*b^2*f*e + 4*C*a^2*b^2*e^2 + 8*A*b^4*e^2)*log(a
bs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c))/b^5
```

Mupad [B]
time = 81.65, size = 2500, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^2*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x
)
```

```
[Out] - ((a^(1/2)*(a*c)^(1/2)*(64*B*a^2*c*f^2 + 32*B*b^2*c*e^2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (a^(1/2)*(a*c)^
(1/2)*(64*B*a^2*c^3*f^2 + 32*B*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) - (a^(1/2)*(a*c)^(1/2)*((128*B*
a^2*c^2*f^2)/3 - 48*B*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(
b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (4*B*a^2*e*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^11)/(b^3*((a + b*x)^(1/2) - a^(1/2))^11) + (8*B*a^(1/2)*e^2*(a*c
)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(b^2*((a + b*x)^(1/2) - a^(
1/2))^10) + (20*B*a^2*c^4*e*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^3*(
(a + b*x)^(1/2) - a^(1/2))^3) + (24*B*a^2*c^3*e*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^5)/(b^3*((a + b*x)^(1/2) - a^(1/2))^5) - (24*B*a^2*c^2*e*f*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a + b*x)^(1/2) - a^(1/2))^7) + (8*
B*a^(1/2)*c^4*e^2*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*(
(a + b*x)^(1/2) - a^(1/2))^2) - (4*B*a^2*c^5*e*f*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (20*B*a^2*c*e*f*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^9)/(b^3*((a + b*x)^(1/2) - a^(1/2))^9))/(((a*c - b*
c*x)^(1/2) - (a*c)^(1/2))^12/((a + b*x)^(1/2) - a^(1/2))^12 + c^6 + (6*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (6*c
^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (
15*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4
 + (20*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2
))^6 + (15*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^
(1/2))^8) - ((2*A*a^2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a +
 b*x)^(1/2) - a^(1/2))^7) + (14*A*a^2*c^2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^3)/(b^3*((a + b*x)^(1/2) - a^(1/2))^3) - (2*A*a^2*c^3*f^2*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (14*A*a^2*c*
f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^3*((a + b*x)^(1/2) - a^(1/2))
^5) + (16*A*a^(1/2)*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/
(b^2*((a + b*x)^(1/2) - a^(1/2))^6) + (32*A*a^(1/2)*c*e*f*(a*c)^(1/2)*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*((a + b*x)^(1/2) - a^(1/2))^4) + (16
*A*a^(1/2)*c^2*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*
((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a
```

```
+ b*x)^(1/2) - a^(1/2))^8 + c^4 + (4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
6)/((a + b*x)^(1/2) - a^(1/2))^6 + (4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4) - ((((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^5*((333*C*a^4*c^5*f^2)/2 + 30*C*a^2*b^2*c^5*e^2))/(b^5*((a + b*x)^(1/2
) - a^(1/2))^5) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*((23*C*a^4*c^6*f^2
)/2 - 6*C*a^2*b^2*c^6*e^2))/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) - (((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))*((3*C*a^4*c^7*f^2)/2 + 2*C*a^2*b^2*c^7*e^2))/(b
^5*((a + b*x)^(1/2) - a^(1/2))) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11*(
(333*C*a^4*c^2*f^2)/2 + 30*C*a^2*b^2*c^2*e^2))/(b^5*((a + b*x)^(1/2) - a^(1
/2))^11) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7*((671*C*a^4*c^4*f^2)/2 -
22*C*a^2*b^2*c^4*e^2))/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + (((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^9*((671*C*a^4*c^3*f^2)/2 - 22*C*a^2*b^2*c^3*e^2))/(b
^5*((a + b*x)^(1/2) - a^(1/2))^9) + (((23*C*a^4*c*f^2)/2 - 6*C*a^2*b^2*c*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(b^5*((a + b*x)^(1/2) - a^(1/2))
^13) + (((3*C*a^4*f^2)/2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^15)/(b^5*((a + b*x)^(1/2) - a^(1/2))^15) + (128*C*a^(5/2)*c*e*f*(a*c)^
(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/(b^4*((a + b*x)^(1/2) - a^(1/
2))^12) + (128*C*a^(5/2)*c^5*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) + (512*C*a^(5/2)*c^4*e*f*(a*c)
^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(3*b^4*((a + b*x)^(1/2) - a^(
1/2))^6) + (256*C*a^(5/2)*c^3*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^8)/(3*b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (512*C*a^(5/2)*c^2*e*f*(a
*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(3*b^4*((a + b*x)^(1/2) -
 a^(1/2))^10))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16/((a + b*x)^(1/2) - a
^(1/2))^16 + c^8 + (8*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^
(1/2) - a^(1/2))^14 + (8*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b
*x)^(1/2) - a^(1/2))^2 + (28*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a
 + b*x)^(1/2) - a^(1/2))^4 + (56*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)
/((a + b*x)^(1/2) - a^(1/2))^6 + (70*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
)^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*c^2*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12) - (2*A*atan((A*(a^2*f^2 + 2
*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(c^(1/2)*(A*a^2*f^2 + 2*A*b^
2*e^2)*((a + b*x)^(1/2) - a^(1/2))))*(a^2*f^2 +...
```

3.29 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=246

$$-\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{\left(2\left(2a^2Cf^2-b^2\left(Ce^2-3f(Be+Af)\right)\right)-b^2f(Ce-3Bf)x\right)\left(a^2-b^2x^2\right)}{6b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{(2Ab^2}{}$$

```
[Out] -1/3*C*(f*x+e)^2*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/6*
(4*a^2*C*f^2-2*b^2*(C*e^2-3*f*(A*f+B*e))-b^2*f*(-3*B*f+C*e)*x)*(-b^2*x^2+a^
2)/b^4/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*A*b^2*e+a^2*(B*f+C*e))*arc
tan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^3/c^(1
/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1624, 1668, 794, 223, 209}

$$\frac{\sqrt{a^2c-b^2cx^2}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2(Bf+Ce)+2Ab^2e\right)}{2b^3\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{\left(a^2-b^2x^2\right)\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Af+Be)\right)\right)-b^2fx(Ce-3Bf)\right)}{6b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C\left(a^2-b^2x^2\right)(e+fx)^2}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -1/3*(C*(e + f*x)^2*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]
) - ((2*(2*a^2*C*f^2 - (b^2*(2*C*e^2 - 6*f*(B*e + A*f)))/2) - b^2*f*(C*e -
3*B*f)*x)*(a^2 - b^2*x^2))/(6*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*
A*b^2*e + a^2*(C*e + B*f))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqr
t[a^2*c - b^2*c*x^2]])/(2*b^3*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] && !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\,\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\,\int \frac{(e+fx)\left(-c\left(3Ab^2+2a^2C\right)f^2+b^2c \cdots}{\sqrt{a^2c-b^2cx^2}\cdots}}{3b^2cf^2\sqrt{a+bx}\,\sqrt{ac-bcx}\cdots}$$

$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2(2Ce^2-6f(Be+Af))\right)\cdots\right.}{6b^4f\sqrt{a+bx}\,\sqrt{\cdots}}$$

$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2(2Ce^2-6f(Be+Af))\right)\cdots\right.}{6b^4f\sqrt{a+bx}\,\sqrt{\cdots}}$$

$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2(2Ce^2-6f(Be+Af))\right)\cdots\right.}{6b^4f\sqrt{a+bx}\,\sqrt{\cdots}}$$

Mathematica [A]
time = 0.27, size = 128, normalized size = 0.52

$$\frac{-\left((a-bx)\sqrt{a+bx}\,(4a^2Cf+b^2(6Be+6Af+3Cex+3Bfx+2Cfx^2))\right)+6b(2Ab^2e+a^2(Ce+Bf))\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{6b^4\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x
]
```

```
[Out] (-((a - b*x)*Sqrt[a + b*x]*(4*a^2*C*f + b^2*(6*B*e + 6*A*f + 3*C*e*x + 3*B*
f*x + 2*C*f*x^2))) + 6*b*(2*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a - b*x]*ArcTan
[Sqrt[a + b*x]/Sqrt[a - b*x]])/(6*b^4*Sqrt[c*(a - b*x)])
```

Maple [A]
time = 0.11, size = 343, normalized size = 1.39

method	result
risch	$-\frac{(2Cfx^2b^2+3Bb^2fx+3Cb^2ex+6Ab^2f+6Bb^2e+4a^2Cf)\sqrt{bx+a}\,(-bx+a)}{6b^4\sqrt{-c(bx-a)}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2cx^2+ca^2}}\right)Ae}{\sqrt{b^2c}}+\frac{\arctan(\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(6A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)b^4ce+3B\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)a^2b^2cf+3C\,a\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/6*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/c*(6*A*arctan((b^2*c)^(1/2)*x/(c*(-b^2
*x^2+a^2))^(1/2))*b^4*c*e+3*B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/
2))*a^2*b^2*c*f+3*C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*b^
2*c*e-2*C*b^2*f*x^2*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)-3*B*(b^2*c)^(1/2
)*(c*(-b^2*x^2+a^2))^(1/2)*b^2*f*x-3*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)*b^2*e*x-6*A*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*b^2*f-6*B*(b^2*c)^(1/
2)*(c*(-b^2*x^2+a^2))^(1/2)*b^2*e-4*C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2
)*a^2*f)/(c*(-b^2*x^2+a^2))^(1/2)/b^4/(b^2*c)^(1/2)
```

Maxima [A]
time = 0.51, size = 193, normalized size = 0.78

$$-\frac{\sqrt{-b^2cx^2+a^2c}\,Cfx^2}{3\,b^2c}+\frac{A\arcsin\left(\frac{bx}{a}\right)e}{b\sqrt{c}}+\frac{(Bf+Ce)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}}-\frac{2\sqrt{-b^2cx^2+a^2c}\,Ca^2f}{3\,b^4c}-\frac{\sqrt{-b^2cx^2+a^2c}\,Af}{b^2c}-\frac{\sqrt{-b^2cx^2+a^2c}\,(Bf+Ce)x}{2\,b^2c}-\frac{\sqrt{-b^2cx^2+a^2c}\,Be}{b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*sqrt(-b^2*c*x^2 + a^2*c)*C*f*x^2/(b^2*c) + A*arcsin(b*x/a)*e/(b*sqrt(c
)) + 1/2*(B*f + C*e)*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) - 2/3*sqrt(-b^2*c*x^2
+ a^2*c)*C*a^2*f/(b^4*c) - sqrt(-b^2*c*x^2 + a^2*c)*A*f/(b^2*c) - 1/2*sqrt(
-b^2*c*x^2 + a^2*c)*(B*f + C*e)*x/(b^2*c) - sqrt(-b^2*c*x^2 + a^2*c)*B*e/(b
^2*c)
```

Fricas [A]
time = 0.98, size = 308, normalized size = 1.25

$$\left[-\frac{3\,(Ba^2bf + (Ca^2b + 2\,Ab^3)e)\sqrt{-c}\,\log\left(2\,b^2cx^2 - 2\,\sqrt{-bcx + ac}\,\sqrt{bx + a}\,b\sqrt{-c}\,x - a^2c\right) + 2\,(2\,Cb^2fx^2 + 3\,Bb^2fx + 2\,(2\,Ca^2 + 3\,Ab^2)f + 3\,(Cb^2x + 2\,Bb^2)e)\sqrt{-bcx + ac}\,\sqrt{bx + a}}{12\,b^4c}, -\frac{3\,(Ba^2bf + (Ca^2b + 2\,Ab^3)e)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx + ac}\,\sqrt{bx + a}\,b\sqrt{c}\,x}{b^2cx^2 - a^2c}\right) + (2\,Cb^2fx^2 + 3\,Bb^2fx + 2\,(2\,Ca^2 + 3\,Ab^2)f + 3\,(Cb^2x + 2\,Bb^2)e)\sqrt{-bcx + ac}\,\sqrt{bx + a}}{6\,b^4c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2 - 2*
sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(2*C*b^2*f*x^2 +
 3*B*b^2*f*x + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*x + 2*B*b^2)*e)*sqrt(-b*c
*x + a*c)*sqrt(b*x + a))/(b^4*c), -1/6*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*
e)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 -
 a^2*c)) + (2*C*b^2*f*x^2 + 3*B*b^2*f*x + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^
2*x + 2*B*b^2)*e)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^4*c)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.70, size = 199, normalized size = 0.81

$$-\frac{\left(\left(\frac{2\,(bx+a)Cf}{c} - \frac{4\,Cac^2f-3\,Bbc^2f-3\,Cbc^2e}{c^3}\right)(bx + a) + \frac{3\,(2\,Ca^2c^2f-Babc^2f+2\,Ab^2c^2f-Cabc^2e+2\,Bb^2c^2e)}{c^3}\right)\sqrt{-(bx + a)c + 2\,ac}\,\sqrt{bx + a} + \frac{6\,(Ba^2bf+Ca^2be+2\,Ab^3e)\log\left(\left|-\sqrt{bx + a}\,\sqrt{-c} + \sqrt{-(bx + a)c + 2\,ac}\right|\right)}{\sqrt{-c}}}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

[Out] -1/6*(((2*(b*x + a)*C*f/c - (4*C*a*c^2*f - 3*B*b*c^2*f - 3*C*b*c^2*e)/c^3)*
(b*x + a) + 3*(2*C*a^2*c^2*f - B*a*b*c^2*f + 2*A*b^2*c^2*f - C*a*b*c^2*e +
2*B*b^2*c^2*e)/c^3)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 6*(B*a^2*b*f
 + C*a^2*b*e + 2*A*b^3*e)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c))/b^4

Mupad [B]
time = 30.74, size = 1011, normalized size = 4.11

$$-\frac{\frac{2Ba^2f\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^7}{\left(\sqrt{a+bx}-\sqrt{a}\right)^7}-\frac{2Ba^2c^3f\left(\sqrt{ac-bcx}-\sqrt{ac}\right)}{\sqrt{a+bx}-\sqrt{a}}-\frac{14Ba^2cf\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^5}{\left(\sqrt{a+bx}-\sqrt{a}\right)^5}+\frac{14Ba^2c^2f\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^3}{\left(\sqrt{a+bx}-\sqrt{a}\right)^3}}{b^3c^4+\frac{b^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^8}{\left(\sqrt{a+bx}-\sqrt{a}\right)^8}+\frac{4b^3c^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^2}{\left(\sqrt{a+bx}-\sqrt{a}\right)^2}+\frac{6b^3c^2\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^4}{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}+\frac{4b^3c\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^6}{\left(\sqrt{a+bx}-\sqrt{a}\right)^6}}-\frac{\frac{2Ca^2e\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^7}{\left(\sqrt{a+bx}-\sqrt{a}\right)^7}-\frac{2Ca^2c^3e\left(\sqrt{ac-bcx}-\sqrt{ac}\right)}{\sqrt{a+bx}-\sqrt{a}}-\frac{14Ca^2ce\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^5}{\left(\sqrt{a+bx}-\sqrt{a}\right)^5}+\frac{14Ca^2c^2e\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^3}{\left(\sqrt{a+bx}-\sqrt{a}\right)^3}}{b^3c^4+\frac{b^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^8}{\left(\sqrt{a+bx}-\sqrt{a}\right)^8}+\frac{4b^3c^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^2}{\left(\sqrt{a+bx}-\sqrt{a}\right)^2}+\frac{6b^3c^2\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^4}{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}+\frac{4b^3c\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^6}{\left(\sqrt{a+bx}-\sqrt{a}\right)^6}}-\frac{\sqrt{ac-bcx}\left(\frac{2Ca^3f}{3b^4c}+\frac{Cfx^3}{3bc}+\frac{Cafx^2}{3b^2c}+\frac{2Ca^2fx}{3b^3c}\right)}{\sqrt{a+bx}}-\frac{4Ae\,\mathrm{atan}\left(\frac{b\left(\sqrt{ac-bcx}-\sqrt{ac}\right)}{\sqrt{b^2c}\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{\sqrt{b^2c}}-\frac{Af\sqrt{ac-bcx}\sqrt{a+bx}}{b^2c}-\frac{Be\sqrt{ac-bcx}\sqrt{a+bx}}{b^2c}-\frac{2Ba^2f\,\mathrm{atan}\left(\frac{\sqrt{ac-bcx}-\sqrt{ac}}{\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{b^3\sqrt{c}}-\frac{2Ca^2e\,\mathrm{atan}\left(\frac{\sqrt{ac-bcx}-\sqrt{ac}}{\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((e + f*x)*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)

[Out] - ((2*B*a^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(
1/2))^7 - (2*B*a^2*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1
/2) - a^(1/2)) - (14*B*a^2*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a +
 b*x)^(1/2) - a^(1/2))^5 + (14*B*a^2*c^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - ((2*C
*a^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(1/2))^7
 - (2*C*a^2*c^3*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a
^(1/2)) - (14*C*a^2*c*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a + b*x)^(
1/2) - a^(1/2))^5 + (14*C*a^2*c^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/
((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((a*c - b
*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - ((a*c - b*c*
x)^(1/2)*((2*C*a^3*f)/(3*b^4*c) + (C*f*x^3)/(3*b*c) + (C*a*f*x^2)/(3*b^2*c)
 + (2*C*a^2*f*x)/(3*b^3*c)))/(a + b*x)^(1/2) - (4*A*e*atan((b*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^2*
c)^(1/2) - (A*f*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(b^2*c) - (B*e*(a*c -
b*c*x)^(1/2)*(a + b*x)^(1/2))/(b^2*c) - (2*B*a^2*f*atan(((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2)) - (2
*C*a^2*e*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2)
 - a^(1/2)))))/(b^3*c^(1/2))

3.30 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=177

$$-\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{(2Ab^2+a^2C)\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

```
[Out] -B*(-b^2*x^2+a^2)/b^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/2*C*x*(-b^2*x^2+a^
2)/b^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*A*b^2+C*a^2)*arctan(b*x*c^(1
/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^3/c^(1/2)/(b*x+a)^
(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {915, 1829, 655, 223, 209}

$$\frac{(a^2C+2Ab^2)\sqrt{a^2c-b^2cx^2}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -((B*(a^2 - b^2*x^2))/(b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - (C*x*(a^2 -
b^2*x^2))/(2*b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*A*b^2 + a^2*C)*Sqrt
[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b^3*S
qrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx &= \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\\
&= -\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{-c(2Ab^2+a^2C)-2b^2Bcx}{\sqrt{a^2c-b^2cx^2}}\,dx}{2b^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}\\
&= -\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{\left((2Ab^2+a^2C)\sqrt{a^2c-\cdots}\right.}{2b^2\sqrt{a+\cdots}}\\
&= -\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{\left((2Ab^2+a^2C)\sqrt{a^2c-\cdots}\right.}{\cdots}\\
&= -\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{(2Ab^2+a^2C)\sqrt{a^2c-\cdots}}{2b^3\sqrt{c}\,\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 90, normalized size = 0.51

$$
\frac{b(-a+bx)\sqrt{a+bx}\,(2B+Cx)+2(2Ab^2+a^2C)\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{2b^3\sqrt{c(a-bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (b*(-a + b*x)*Sqrt[a + b*x]*(2*B + C*x) + 2*(2*A*b^2 + a^2*C)*Sqrt[a - b*x]
*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(2*b^3*Sqrt[c*(a - b*x)])
```

Maple [A]
time = 0.10, size = 168, normalized size = 0.95

method	result
risch	$-\frac{(Cx+2B)\sqrt{bx+a}\,(-bx+a)}{2b^2\sqrt{-c\,(bx-a)}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+c\,a^2}}\right)A}{\sqrt{b^2c}}+\frac{\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+c\,a^2}}\right)C\,a^2}{2b^2\sqrt{b^2c}}\right)\sqrt{-\,(bx+}}{\sqrt{bx+a}\,\sqrt{-c\,(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c\,(-bx+a)}\,\left(2A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\,(-b^2x^2+a^2)}}\right)b^2c+C\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\,(-b^2x^2+a^2)}}\right)a^2c-C\sqrt{b^2c}\right.}{2b^2\sqrt{c\,(-b^2x^2+a^2)}\,c\sqrt{b^2c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/b^2*(2*A*arctan((b^2*c)^(1/2)*x/(c*(-b
^2*x^2+a^2))^(1/2))*b^2*c+C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2)
)*a^2*c-C*(b^2*c)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*x-2*B*(b^2*c)^(1/2)*(c*(-b
^2*x^2+a^2))^(1/2))/(c*(-b^2*x^2+a^2))^(1/2)/c/(b^2*c)^(1/2)
```

Maxima [A]
time = 0.49, size = 88, normalized size = 0.50

$$\frac{Ca^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}}+\frac{A\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}-\frac{\sqrt{-b^2cx^2+a^2c}\,Cx}{2\,b^2c}-\frac{\sqrt{-b^2cx^2+a^2c}\,B}{b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*C*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) + A*arcsin(b*x/a)/(b*sqrt(c)) - 1/2*s
qrt(-b^2*c*x^2 + a^2*c)*C*x/(b^2*c) - sqrt(-b^2*c*x^2 + a^2*c)*B/(b^2*c)
```

Fricas [A]
time = 1.58, size = 196, normalized size = 1.11

$$\left[-\frac{(Ca^2+2\,Ab^2)\sqrt{-c}\,\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,(Cbx+2\,Bb)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{4\,b^3c},\,-\frac{(Ca^2+2\,Ab^2)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)+(Cbx+2\,Bb)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{2\,b^3c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/4*((C*a^2 + 2*A*b^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sq
rt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(C*b*x + 2*B*b)*sqrt(-b*c*x + a*c)*sq
rt(b*x + a))/(b^3*c), -1/2*((C*a^2 + 2*A*b^2)*sqrt(c)*arctan(sqrt(-b*c*x +
a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (C*b*x + 2*B*b)*sqrt(
-b*c*x + a*c)*sqrt(b*x + a))/(b^3*c)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 106, normalized size = 0.60

$$-\frac{\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}\,\left(\frac{(bx+a)C}{c}-\frac{Cac-2\,Bbc}{c^2}\right)+\frac{2\,(Ca^2+2\,Ab^2)\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/2*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*((b*x + a)*C/c - (C*a*c - 2*
B*b*c)/c^2) + 2*(C*a^2 + 2*A*b^2)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(
b*x + a)*c + 2*a*c)))/sqrt(-c))/b^3
```

Mupad [**B**]
time = 14.95, size = 489, normalized size = 2.76

$$-\frac{\frac{2\,C\,a^2\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^7}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^7}-\frac{2\,C\,a^2\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,C\,a^2\,c\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^5}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^5}+\frac{14\,C\,a^2\,c^2\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^3}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3}}{b^3\,c^4+\frac{b^3\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^8}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^8}+\frac{4\,b^3\,c^3\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^2}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}+\frac{6\,b^3\,c^2\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^4}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}+\frac{4\,b^3\,c\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^6}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^6}}-\frac{4\,A\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}}-\frac{2\,C\,a^2\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^3\,\sqrt{c}}-\frac{B\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] - ((2*C*a^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(1/
2))^7 - (2*C*a^2*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2)
```

```
- a^(1/2)) - (14*C*a^2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a + b*x)^
(1/2) - a^(1/2))^5 + (14*C*a^2*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(
(a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)^(1/2) -
 (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - (4*A*atan((b*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^(1/2) - a^(1
/2)))))/(b^2*c)^(1/2) - (2*C*a^2*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(
c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2)) - (B*(a*c - b*c*x)^(1/
2)*(a + b*x)^(1/2))/(b^2*c)
```

3.31 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)}\,dx$

Optimal. Leaf size=278

$$-\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{(Ce^2-Bef+Af^2)\sqrt{a^2c-}}{\sqrt{c}\,f^2\sqrt{b^2}}$$

```
[Out] -C*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-(-B*f+C*e)*arctan(
b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2
)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(A*f^2-B*e*f+C*e^2)*arctan((b^2*e*x+a^2*
f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a
^2*c)^(1/2)/f^2/c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)
^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1624, 1668, 858, 223, 209, 739, 210}

$$\frac{\sqrt{a^2c-b^2cx^2}\,(Af^2-Bef+Ce^2)\,\text{ArcTan}\left(\frac{\sqrt{c}\,(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}\,(Ce-Bf)\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{-Ab^2cf^2+b^2cf(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{b^2cf^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b}}}{f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{}{1-}\right.}{f^2\sqrt{a+bx}\,\sqrt{ac-}}$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b}{\sqrt{a^2c}}\right.}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Mathematica [**A**]
time = 0.22, size = 178, normalized size = 0.64

$$\frac{\frac{Cf(-a+bx)\sqrt{a+bx}}{b^2} - \frac{2(Ce-Bf)\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b} + \frac{2(Ce^2+f(-Be+Af))\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{be+af}\,\sqrt{a+bx}}{\sqrt{be-af}\,\sqrt{a-bx}}\right)}{\sqrt{be-af}\,\sqrt{be+af}}}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] ((C*f*(-a + b*x)*Sqrt[a + b*x])/b^2 - (2*(C*e - B*f)*Sqrt[a - b*x]*ArcTan[S
qrt[a + b*x]/Sqrt[a - b*x]])/b + (2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x
]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/
(Sqrt[b*e - a*f]*Sqrt[b*e + a*f]))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [**A**]
time = 0.00, size = 487, normalized size = 1.75

method	result
default	$\frac{\left(-A\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c(a^2f^2-b^2e^2)}{f^2}}\,\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)b^2cf^2\sqrt{b^2c}+B\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c(a^2f^2-b^2e^2)}{f^2}}\,\sqrt{}}{fx+e}\right.\right.}{}$

risch	$-\frac{C\sqrt{bx+a}\,(-bx+a)}{f\,b^{2}\sqrt{-c\,(bx-a)}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+c\,a^{2}}}\right)B}{f\sqrt{b^{2}c}}-\frac{\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+c\,a^{2}}}\right)Ce}{f^{2}\sqrt{b^{2}c}}-\frac{\ln\left(\frac{\frac{2c\left(a^{2}f^{2}-b^{2}e^{2}\right)}{f^{2}}+\frac{2b \cdots}{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] (-A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^
2))^(1/2)*f)/(f*x+e))*b^2*c*f^2*(b^2*c)^(1/2)+B*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e*f
*(b^2*c)^(1/2)+B*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*f^2
*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*(b^2*c)^(1/2
)-C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e*f*(c*(a^2*f^2-
b^2*e^2)/f^2)^(1/2)-C*f^2*(b^2*c)^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2))*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(a^2*f^2-b^2*e^2
)/f^2)^(1/2)/f^3/(b^2*c)^(1/2)/b^2/c/(c*(-b^2*x^2+a^2))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e detai
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-c\,(-a+bx)}\,\sqrt{a+bx}\,\,(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)),
x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B]
time = 0.01, size = 2500, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] (B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*
a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c
^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (
B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
 (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4
*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B
*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((
a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*
```

```
b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^
4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*
x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a
^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(
a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(
1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*
e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^
6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) +
(16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f
*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^
(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*
b^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^
8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)
) + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
 + b*x)^(1/2) - a^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a
*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*
e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5
/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*
B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(2
4*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c
)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2
*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) -
a^(1/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))
/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2
))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e
^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a +
 b*x)^(1/2) - a^(1/2))))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2)
 - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(
17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))
 + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144
*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^
(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a
^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a
^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)))/((131072*B^4*a^4*c^5)/
(b^8*e^4) - (B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a
^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^
2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (B
*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
(B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*
```

```
c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*
a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a
 + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b
^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4
*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x
)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^
(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a
^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1
/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e
*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6
*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (
16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*
(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)...
```

3.32 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2}\,dx$

Optimal. Leaf size=322

$$\frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)}+\frac{C\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{(a^2f^2(2Ce-Bf)}{}$$

```
[Out] f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)/(b*x+a)^(1
/2)/(-b*c*x+a*c)^(1/2)+C*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2
*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(a^2*f^2
*(-B*f+2*C*e)-b^2*(-A*e*f^2+C*e^3))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^
2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/f^2/(-a
^2*f^2+b^2*e^2)^(3/2)/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1624, 1665, 858, 223, 209, 739, 210}

$$\frac{\sqrt{a^2c-b^2cx^2}\,(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\,\text{ArcTan}\left(\frac{\sqrt{c}\,(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^{3/2}}+\frac{f(a^2-b^2x^2)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)}+\frac{C\sqrt{a^2c-b^2cx^2}\,\text{ArcTan}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{c(Ab\cdots}{\cdots}}{c\,(b^2e^2-a^2f^2\cdots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a\cdots}\right.}{f\,(b^2e^2-a^2\cdots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a\cdots}\right.}{f\,(\cdots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\,\tan\cdots}{b\sqrt{c}\,f^2\sqrt{a+\cdots}}$$

Mathematica [A]
time = 0.34, size = 229, normalized size = 0.71

$$\frac{2\left(\frac{f(Ce^2+f(-Be+Af))(-a+bx)\sqrt{a+bx}}{2(-be+af)(be+af)(e+fx)}+\frac{C\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b}-\frac{(a^2f^2(-2Ce+Bf)+b^2(Ce^3-Aef^2))\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{be+af}\,\sqrt{a+bx}}{\sqrt{be-af}\,\sqrt{a-bx}}\right)}{(be-af)^{3/2}(be+af)^{3/2}}\right)}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x
]
```

```
[Out] (2*((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/(2*(-(b*e) + a*
f)*(b*e + a*f)*(e + f*x)) + (C*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a -
b*x]])/b - ((a^2*f^2*(-2*C*e + B*f) + b^2*(C*e^3 - A*e*f^2))*Sqrt[a - b*x]*
ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((
b*e - a*f)^(3/2)*(b*e + a*f)^(3/2))))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. 2(290) = 580.
time = 0.00, size = 1166, normalized size = 3.62

method	result

default	$\left(A\ln\left(\frac{2b^{2}cex+2a^{2}cf+2\sqrt{\frac{c(a^{2}f^{2}-b^{2}e^{2})}{f^{2}}}\,\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}\,f}{fx+e}\right)b^{2}cef^{3}x\sqrt{b^{2}c}-B\ln\left(\frac{2b^{2}cex+2a^{2}cf+2\sqrt{\frac{c(a^{2}f^{2}-b^{2}e^{2})}{f^{2}}}}{fx+e}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2
))^(1/2)*f)/(f*x+e))*b^2*c*e*f^3*x*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*f+
(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*
f^4*x*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*e*f^3*x*(b^2*c)^(1/2)-C*ln(
2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)*f)/(f*x+e))*b^2*c*e^3*f*x*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^
2*x^2+a^2))^(1/2))*a^2*c*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^
2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-
b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e
*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x
+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^
(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^
2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^4*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/
(c*(-b^2*x^2+a^2))^(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arc
tan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e
^2)/f^2)^(1/2)-A*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)*(b^2*c)^(1/2)+B*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))
^(1/2)*(b^2*c)^(1/2)-C*e^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2
+a^2))^(1/2)*(b^2*c)^(1/2))/c*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(-b^2*x^2
+a^2))^(1/2)/(a*f-b*e)/(b^2*c)^(1/2)/(a*f+b*e)/(f*x+e)/(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)/f^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e detai
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{\sqrt{-c\left(-a + bx\right)}\,\sqrt{a + bx}\,\left(e + fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2
), x)
```

Giac [A]
time = 0.78, size = 523, normalized size = 1.62

$$-\frac{\frac{2\left(Ba^2b\sqrt{-c}\,f^3-2\,Ca^2b\sqrt{-c}\,f^2e-Ab^3\sqrt{-c}\,f^2e+Cb^3\sqrt{-c}\,e^3\right)\arctan\left(\frac{\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2f-2\,bce}{2\,\sqrt{a^2f^2-b^2e^2}\,c}\right)}{\left(a^2f^4-b^2f^2e^2\right)\sqrt{a^2f^2-b^2e^2}\,c}+\frac{C\log\left(\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2\right)}{\sqrt{-c}\,f^2}+\frac{4\left(2\,Aa^2b^2\sqrt{-c}\,cf^3-Ab^3\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2\sqrt{-c}\,f^2e-2\,Ba^2b^2\sqrt{-c}\,cf^2e+Bb^3\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2\sqrt{-c}\,fe^2+2\,Ca^2b^2\sqrt{-c}\,cfe^2-Cb^3\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2\sqrt{-c}\,e^3\right)}{\left(a^2f^4-b^2f^2e^2\right)\left(\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^4f+4\,a^2c^2f-4\,b\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2ce\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -(2*(B*a^2*b*sqrt(-c)*f^3 - 2*C*a^2*b*sqrt(-c)*f^2*e - A*b^3*sqrt(-c)*f^2*e
 + C*b^3*sqrt(-c)*e^3)*arctan(1/2*((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a
)*c + 2*a*c))^2*f - 2*b*c*e)/(sqrt(a^2*f^2 - b^2*e^2)*c))/((a^2*f^4 - b^2*f
^2*e^2)*sqrt(a^2*f^2 - b^2*e^2)*c) + C*log((sqrt(b*x + a)*sqrt(-c) - sqrt(-
(b*x + a)*c + 2*a*c))^2)/(sqrt(-c)*f^2) + 4*(2*A*a^2*b^2*sqrt(-c)*c*f^3 - A
*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*f^2*e
 - 2*B*a^2*b^2*sqrt(-c)*c*f^2*e + B*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^2*sqrt(-c)*f*e^2 + 2*C*a^2*b^2*sqrt(-c)*c*f*e^2 - C*b^3*
```

```
(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^3)/((a^2
*f^4 - b^2*f^2*e^2)*((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^
4*f + 4*a^2*c^2*f - 4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c
))^2*c*e)))/b
```

Mupad [B]
time = 19.40, size = 2500, normalized size = 7.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
[Out] ((4*B*a^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(1
/2))^3*(b^3*e^3 - a^2*b*e*f^2)) + (8*B*a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^2)/((a^2*f^2 - b^2*e^2)*((a + b*x)^(1/2) - a^(1/2))^2)
 - (4*B*a^2*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(
1/2))*(b^3*e^3 - a^2*b*e*f^2)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4/((a
 + b*x)^(1/2) - a^(1/2))^4 + c^2 + (2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
2)/((a + b*x)^(1/2) - a^(1/2))^2 - (4*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3) + (4*a^(1/2)*c*
f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) -
a^(1/2)))) - ((4*C*a^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((b^3*e^2 - a
^2*b*f^2)*((a + b*x)^(1/2) - a^(1/2))^3) - (4*C*a^2*c*((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2)))/((b^3*e^2 - a^2*b*f^2)*((a + b*x)^(1/2) - a^(1/2))) + (8*C*
a^(1/2)*e*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a^2*f^3 - b^
2*e^2*f)*((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2 + (2*c*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (4*a^(1/2)*f*(a*c)^(1/2)*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3) + (4*a
^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)
^(1/2) - a^(1/2)))) + ((4*A*a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/
((b^3*e^4 - a^2*b*e^2*f^2)*((a + b*x)^(1/2) - a^(1/2))) - (4*A*a^2*f^2*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((b^3*e^4 - a^2*b*e^2*f^2)*((a + b*x)^(1
/2) - a^(1/2))^3) + (8*A*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^2)/((b^2*e^3 - a^2*e*f^2)*((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2 + (2*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (4*a^(
1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(
1/2) - a^(1/2))^3) + (4*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2)))) - (4*C*atan(((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b*c^(1/2)*f^2) +
 (2*A*b^2*e*(atan((2*b^3*c^3*e^3 + 2*b*c^2*e*(a^2*c*f^2 - b^2*c*e^2) + 2*a^
2*b*c^3*e*f^2 + (3*a^(3/2)*f^3*(a*c)^(3/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (2*b^3*c^2*e^3*((a*c - b*c*x)^(1/2)
```

```
- (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (3*a^(1/2)*f*(a*c)^(1/2)*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(
1/2) - a^(1/2))^3 - (a^(3/2)*c*f^3*(a*c)^(3/2)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + (2*b*c*e*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^2*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2))^2 + (a^(1/2
)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^
2))/((a + b*x)^(1/2) - a^(1/2)) - (10*a^2*b*c^2*e*f^2*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (7*a^(1/2)*b^2*c^2*e^2*f*
(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)
) - (a^(1/2)*b^2*c*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)
/((a + b*x)^(1/2) - a^(1/2))^3)/(4*a^(1/2)*b*c^2*e*f*(a*c)^(1/2)*(b^2*c*e^2
 - a^2*c*f^2)^(1/2))) - atan(((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f
^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(
1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))))/((a*f + b*e)*(a*f - b*e)*(
b^2*c*e^2 - a^2*c*f^2)^(1/2)) - (2*B*a^2*f*(atan((2*b^3*c^3*e^3 + 2*b*c^2*e
*(a^2*c*f^2 - b^2*c*e^2) + 2*a^2*b*c^3*e*f^2 + (3*a^(3/2)*f^3*(a*c)^(3/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (2*b^
3*c^2*e^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2)
)^2 - (3*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(a^2*c
*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2))^3 - (a^(3/2)*c*f^3*(a*c)^(3/
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + (2*b*
c*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(a^2*c*f^2 - b^2*c*e^2))/((a + b*
x)^(1/2) - a^(1/2))^2 + (a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (10*a^2*b*
c^2*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2)
)^2 + (7*a^(1/2)*b^2*c^2*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2)))/((a + b*x)^(1/2) - a^(1/2)) - (a^(1/2)*b^2*c*e^2*f*(a*c)^(1/2)*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(4*a^(1/2)*b
*c^2*e*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) - atan(((((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2))
 - (a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/
2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)
))))/((a*f + b*e)*(a*f - b*e)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) - (C*e*(2*a^2*
f^2 - b^2*e^2)*(2*atan((((a*c - b*c*x)^(1/2) - ...
```

3.33 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

Optimal. Leaf size=363

$$\frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}+\frac{(2a^2f^2(2Ce-Bf)-b^2e(Ce^2+f(Be-3Af)))(a^2-b^2x^2)}{2f(b^2e^2-a^2f^2)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}$$

```
[Out] 1/2*f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)^2/(b*x
+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*a^2*f^2*(-B*f+2*C*e)-b^2*e*(C*e^2+f*(-3
*A*f+B*e)))*(-b^2*x^2+a^2)/f/(-a^2*f^2+b^2*e^2)^2/(f*x+e)/(b*x+a)^(1/2)/(-b
*c*x+a*c)^(1/2)+1/2*(A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(2*a^2*C*f^2+b^2*e*(-3*B
*f+C*e)))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x
^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/(-a^2*f^2+b^2*e^2)^(5/2)/c^(1/2)/
(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 361, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1624, 1665, 821, 739, 210}

$$\frac{f(a^2-b^2x^2)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}(b^2e^2-a^2f^2)}+\frac{(a^2-b^2x^2)(2a^2f^2(2Ce-Bf)-b^2(ef(Be-3Af)+Ce^3))}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}(b^2e^2-a^2f^2)^2}+\frac{\sqrt{a^2c-b^2cx^2}(2a^4Cf^2+A(a^2b^2f^2+2b^4e^2)+a^2b^2e(Ce-3Bf))\,\text{ArcTan}\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}(b^2e^2-a^2f^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*(C
*e^3 + e*f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3
*B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c
]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(2
*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\,\int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\sqrt{a^2c-b^2cx^2}\,\int \frac{2c(}{}}{2c\,(b^2e}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf)}{2f\,(b^2e^2-a^2}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf)}{2f\,(b^2e^2-a^2}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf)}{2f\,(b^2e^2-a^2}$$

Mathematica [A]
time = 0.47, size = 252, normalized size = 0.69

$$\frac{\frac{(-a+bx)\sqrt{a+bx}\,(b^2e(Ce^2x+Be(2e+fx)-Af(4e+3fx))+a^2f(-Ce(3e+4fx)+f(Af+B(e+2fx))))}{2(be-af)^2(be+af)^2(e+fx)^2}+\frac{(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2))\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{be+af}\,\sqrt{a+bx}}{\sqrt{be-af}\,\sqrt{a-bx}}\right)}{(be-af)^{5/2}(be+af)^{5/2}}}{\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x
]
```

```
[Out] (((-a + b*x)*Sqrt[a + b*x]*(b^2*e*(C*e^2*x + B*e*(2*e + f*x) - A*f*(4*e + 3
*f*x)) + a^2*f*(-(C*e*(3*e + 4*f*x)) + f*(A*f + B*(e + 2*f*x)))))/(2*(b*e -
 a*f)^2*(b*e + a*f)^2*(e + f*x)^2) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*B*f
) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt
[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((b*e - a*f)^(5/2)*(b*e + a*f)
^(5/2)))/Sqrt[c*(a - b*x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. 2(333) = 666.
time = 0.00, size = 1794, normalized size = 4.94

method	result	size
default	Expression too large to display	1794

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2*(-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2
*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e*f^3*x^2+C*ln(2*(b^2*c*e*x+a^2*c*f+
(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^
2*c*e^2*f^2*x^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)
*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e*f^3*x-6*B*ln(2*(b^2*c*e*x
+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e
))*a^2*b^2*c*e^2*f^2*x+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2
)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^3*f*x+A*a^2*f^4*(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+2*A*ln(2*(b^2*c*e*x+
a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e)
)*b^4*c*e^4+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a^2*c*f+(
c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2
*c*e^4+2*B*a^2*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)-4*A*b^2*e^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)
+B*a^2*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+2*B*b
^2*e^3*f*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)-3*C*a^2*e
^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+2*C*ln(2*(b
^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f
)/(f*x+e))*a^4*c*f^4*x^2+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2
)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*f^4*x^2+2*A*ln(2*(b^
2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)
/(f*x+e))*b^4*c*e^2*f^2*x^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2
)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^3*f*x+4*C*ln(2*(b
^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f
)/(f*x+e))*a^4*c*e*f^3*x+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2
)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2-3*B*ln(2*(b^
2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)
/(f*x+e))*a^2*b^2*c*e^3*f-3*A*b^2*e*f^3*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(
c*(-b^2*x^2+a^2))^(1/2)+B*b^2*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)-4*C*a^2*e*f^3*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-
b^2*x^2+a^2))^(1/2)+C*b^2*e^3*f*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*
x^2+a^2))^(1/2))/c*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(-b^2*x^2+a^2))^(1/2
)/(a*f-b*e)/(a*f+b*e)/(a^2*f^2-b^2*e^2)/(f*x+e)^2/(c*(a^2*f^2-b^2*e^2)/f^2)
^(1/2)/f
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*f-%e*b>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 54.95, size = 1347, normalized size = 3.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(((2*C*a^4 + A*a^2*b^2)*f^4*x^2 + (C*a^2*b^2 + 2*A*b^4)*e^4 - (3*B*a^2
*b^2*f - 2*(C*a^2*b^2 + 2*A*b^4)*f*x)*e^3 - (6*B*a^2*b^2*f^2*x - (C*a^2*b^2
 + 2*A*b^4)*f^2*x^2 - (2*C*a^4 + A*a^2*b^2)*f^2)*e^2 - (3*B*a^2*b^2*f^3*x^2
 - 2*(2*C*a^4 + A*a^2*b^2)*f^3*x)*e)*sqrt(a^2*c*f^2 - b^2*c*e^2)*log(-(a^2*
b^2*c*f^2*x^2 - 2*a^2*b^2*c*f*x*e - 2*a^4*c*f^2 + 2*sqrt(a^2*c*f^2 - b^2*c*
e^2)*(b^2*x*e + a^2*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a) - (2*b^4*c*x^2 - a^
2*b^2*c)*e^2)/(f^2*x^2 + 2*f*x*e + e^2)) - 2*(2*B*a^4*f^5*x + A*a^4*f^5 - (
C*b^4*x + 2*B*b^4)*e^5 - (B*b^4*f*x - (3*C*a^2*b^2 + 4*A*b^4)*f)*e^4 + (B*a
^2*b^2*f^2 + (5*C*a^2*b^2 + 3*A*b^4)*f^2*x)*e^3 - (B*a^2*b^2*f^3*x + (3*C*a
^4 + 5*A*a^2*b^2)*f^3)*e^2 + (B*a^4*f^4 - (4*C*a^4 + 3*A*a^2*b^2)*f^4*x)*e)
*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(a^6*c*f^8*x^2 + 2*a^6*c*f^7*x*e - 6*a^4
*b^2*c*f^5*x*e^3 + 6*a^2*b^4*c*f^3*x*e^5 - 2*b^6*c*f*x*e^7 - b^6*c*e^8 - (b
^6*c*f^2*x^2 - 3*a^2*b^4*c*f^2)*e^6 + 3*(a^2*b^4*c*f^4*x^2 - a^4*b^2*c*f^4)
*e^4 - (3*a^4*b^2*c*f^6*x^2 - a^6*c*f^6)*e^2), 1/2*(((2*C*a^4 + A*a^2*b^2)*
f^4*x^2 + (C*a^2*b^2 + 2*A*b^4)*e^4 - (3*B*a^2*b^2*f - 2*(C*a^2*b^2 + 2*A*b
^4)*f*x)*e^3 - (6*B*a^2*b^2*f^2*x - (C*a^2*b^2 + 2*A*b^4)*f^2*x^2 - (2*C*a^
4 + A*a^2*b^2)*f^2)*e^2 - (3*B*a^2*b^2*f^3*x^2 - 2*(2*C*a^4 + A*a^2*b^2)*f^
3*x)*e)*sqrt(-a^2*c*f^2 + b^2*c*e^2)*arctan(-sqrt(-a^2*c*f^2 + b^2*c*e^2)*(
b^2*x*e + a^2*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^2*b^2*c*f^2*x^2 - a^4*
c*f^2 - (b^4*c*x^2 - a^2*b^2*c)*e^2)) - (2*B*a^4*f^5*x + A*a^4*f^5 - (C*b^4
*x + 2*B*b^4)*e^5 - (B*b^4*f*x - (3*C*a^2*b^2 + 4*A*b^4)*f)*e^4 + (B*a^2*b^
2*f^2 + (5*C*a^2*b^2 + 3*A*b^4)*f^2*x)*e^3 - (B*a^2*b^2*f^3*x + (3*C*a^4 +
5*A*a^2*b^2)*f^3)*e^2 + (B*a^4*f^4 - (4*C*a^4 + 3*A*a^2*b^2)*f^4*x)*e)*sqrt
(-b*c*x + a*c)*sqrt(b*x + a))/(a^6*c*f^8*x^2 + 2*a^6*c*f^7*x*e - 6*a^4*b^2*
c*f^5*x*e^3 + 6*a^2*b^4*c*f^3*x*e^5 - 2*b^6*c*f*x*e^7 - b^6*c*e^8 - (b^6*c*
f^2*x^2 - 3*a^2*b^4*c*f^2)*e^6 + 3*(a^2*b^4*c*f^4*x^2 - a^4*b^2*c*f^4)*e^4
- (3*a^4*b^2*c*f^6*x^2 - a^6*c*f^6)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. 2(343) = 686.
time = 0.99, size = 1410, normalized size = 3.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -((2*C*a^4*b*sqrt(-c)*f^2 + A*a^2*b^3*sqrt(-c)*f^2 - 3*B*a^2*b^3*sqrt(-c)*f
*e + C*a^2*b^3*sqrt(-c)*e^2 + 2*A*b^5*sqrt(-c)*e^2)*arctan(1/2*((sqrt(b*x +
 a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*f - 2*b*c*e)/(sqrt(a^2*f^2 - b
^2*e^2)*c))/((a^4*f^4 - 2*a^2*b^2*f^2*e^2 + b^4*e^4)*sqrt(a^2*f^2 - b^2*e^2
)*c) + 2*(A*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6
*sqrt(-c)*f^5 + 4*B*a^4*b^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2
*a*c))^4*sqrt(-c)*c*f^5 - 4*A*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x
+ a)*c + 2*a*c))^2*sqrt(-c)*c^2*f^5 + 16*B*a^6*b^2*sqrt(-c)*c^3*f^5 - 3*B*a
^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f^4
*e - 8*C*a^4*b^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sq
rt(-c)*c*f^4*e - 6*A*a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
2*a*c))^4*sqrt(-c)*c*f^4*e - 20*B*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(
b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*f^4*e - 32*C*a^6*b^2*sqrt(-c)*c^3*f^4*e
 - 24*A*a^4*b^4*sqrt(-c)*c^3*f^4*e + 5*C*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) -
sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f^3*e^2 + 2*A*b^5*(sqrt(b*x + a)*sqr
t(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f^3*e^2 + 10*B*a^2*b^4*(sqrt
(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*f^3*e^2 + 44*
C*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*
c^2*f^3*e^2 + 40*A*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*
a*c))^2*sqrt(-c)*c^2*f^3*e^2 + 8*B*a^4*b^4*sqrt(-c)*c^3*f^3*e^2 - 14*C*a^2*
b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*f^2*
e^3 - 12*A*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt
(-c)*c*f^2*e^3 - 16*B*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
 2*a*c))^2*sqrt(-c)*c^2*f^2*e^3 + 8*C*a^4*b^4*sqrt(-c)*c^3*f^2*e^3 - 2*C*b^
5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f*e^4 +
```

```
4*B*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*
f*e^4 - 8*C*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2
*sqrt(-c)*c^2*f*e^4 + 4*C*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
 2*a*c))^4*sqrt(-c)*c*e^5)/((a^4*f^6 - 2*a^2*b^2*f^4*e^2 + b^4*f^2*e^4)*((s
qrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*f + 4*a^2*c^2*f - 4*b
*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*c*e)^2))/b
```

Mupad [B]
time = 0.01, size = 2500, normalized size = 6.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e^
2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f^4
)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2*b^
2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a
^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a^2*b^2*c*
e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) - a^(1/2))^
4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b^2*e^2*f))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^
4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c^2*e^2*f)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^6
- 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6
) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2
)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/
2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b
*e*((a + b*x)^(1/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c
*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2)
 - a^(1/2))^5) + (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) + (((4*A*a^4*f^4 - 10*A*a^2*
b^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(
1/2))^7*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a^4*c^3*f^4
- 10*A*a^2*b^2*c^3*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x
```

```
)^(1/2) - a^(1/2))*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a
^4*c^2*f^4 - 58*A*a^2*b^2*c^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*
f^2)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5*(4*A*a^4*c*f^4 - 58*A*a^2*b^
2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a
^2*b^3*e^5*f^2)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^6*(16*A*b^4*e^4*f - 8*A*a^4*f^5 + 28*A*a^2*b^2*e^2*f^3))/(((a + b*x)^(1/2)
 - a^(1/2))^6*(b^6*e^8 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(
a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*A*a^4*c*f^5 + 32*A*b^4
*c*e^4*f - 72*A*a^2*b^2*c*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^4*(b^6*e^8
 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2*(16*A*b^4*c^2*e^4*f - 8*A*a^4*c^2*f^5 + 28*A*a^2
*b^2*c^2*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^8 - 2*a^2*b^4*e^6*
f^2 + a^4*b^2*e^4*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^
(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c
*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3
*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a +
 b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1
/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1
/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) +
 (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*(
(a + b*x)^(1/2) - a^(1/2))^3)) - (((32*B*a^4*c^2*f^3 + 22*B*a^2*b^2*c^2*e^2
*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(1/2))^3*(
b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) - ((32*B*a^4*c*f^3 + 22*B*a^2
*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^
(1/2))^5*(b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) + (a^(1/2)*(a*c)^(1
/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(8*B*a^4*c^2*f^4 + 8*B*b^4*c^2*e^
4 + 20*B*a^2*b^2*c^2*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^7 - 2*
a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^6*(8*B*a^4*f^4 + 8*B*b^4*e^4 + 20*B*a^2*b^2*e^2*f^2))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^
4)) - (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*B*a^4*
c*f^4 - 16*B*b^4*c*e^4 + 24*B*a^2*b^2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/
2))^4*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^...
```

3.34 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=87

$$\frac{cx^2\sqrt{-1+dx}\,\sqrt{1+dx}}{3d^2}+\frac{\sqrt{-1+dx}\,\sqrt{1+dx}\,(2(2c+3ad^2)+3bd^2x)}{6d^4}+\frac{b\cosh^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*b*arccosh(d*x)/d^3+1/3*c*x^2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/6*(3*b*d
^2*x+6*a*d^2+4*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^4
```

Rubi [A]
time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$,
Rules used = {1624, 1823, 794, 223, 212}

$$-\frac{(1-d^2x^2)\,(2(3ad^2+2c)+3bd^2x)}{6d^4\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(c*x^2*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c +
3*a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) +
 (b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 +
d*x]*Sqrt[1 + d*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{x(a+bx+cx^2)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{x(2c+3ad^2+3bd^2x)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+\right.}{2d^2}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+\right.}{}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1+}}{2}$$

Mathematica [**A**]
time = 0.08, size = 74, normalized size = 0.85

$$\frac{\sqrt{-1+dx}\,\sqrt{1+dx}\,(3d^2(2a+bx)+2c(2+d^2x^2))+6bd\tanh^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{6d^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2)) + 6*b
*d*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 137, normalized size = 1.57

method	result
risch	$\frac{(2c\,d^2x^2+3b\,d^2x+6a\,d^2+4c)\sqrt{dx+1}\,\sqrt{dx-1}}{6d^4}+\frac{b\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(dx+1)(dx-1)}}{2d^2\sqrt{d^2}\,\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\,\mathrm{csgn}(d)c\,d^2x^2\sqrt{d^2x^2-1}+3\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)b\,d^2x+6\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)a\,d^2+4\sqrt{d^2x^2}\right.}{6d^4\sqrt{d^2x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*c*d^2*x^2*(d^2*x^2-1)^(1/2)+3*(d
^2*x^2-1)^(1/2)*csgn(d)*b*d^2*x+6*(d^2*x^2-1)^(1/2)*csgn(d)*a*d^2+4*(d^2*x^
2-1)^(1/2)*csgn(d)*c+3*ln(((d^2*x^2-1)^(1/2)*csgn(d)+d*x)*csgn(d))*b*d)*csg
n(d)/d^4/(d^2*x^2-1)^(1/2)
```

Maxima [A]
time = 0.31, size = 100, normalized size = 1.15

$$\frac{\sqrt{d^2x^2-1}\,cx^2}{3\,d^2}+\frac{\sqrt{d^2x^2-1}\,bx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,a}{d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}+\frac{2\,\sqrt{d^2x^2-1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqrt(
d^2*x^2 - 1)*c/d^4
```

Fricas [A]
time = 1.14, size = 73, normalized size = 0.84

$$-\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 105, normalized size = 1.21

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\,(2\,ad^{11}-bd^{10}+2\,cd^9)}{d^{12}}\right)-\frac{6\,b\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
- 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d*x
 + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [**B**]
time = 14.76, size = 318, normalized size = 3.66

$$\frac{\sqrt{d\,x-1}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}+\frac{d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{d\,x+1}-1\right)}^8}}+\frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3*d^2) +
(2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.35 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=52

$$\frac{(2b+cx)\sqrt{-1+dx}\,\sqrt{1+dx}}{2d^2}+\frac{(c+2ad^2)\cosh^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arccosh(d*x)/d^3+1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/
d^2
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(52) = 104.
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 2.60, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {915, 1829, 655, 223, 212}

$$\frac{\sqrt{d^2x^2-1}\,(2ad^2+c)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
```

```
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left((c+2ad^2)\sqrt{-1+\right.}{2d^2\sqrt{-1}}$$

$$= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left((c+2ad^2)\sqrt{-1+\right.}{}$$

$$= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{(c+2ad^2)\sqrt{-1+d}}{2d^3\sqrt{-}}$$

Mathematica [**A**]
time = 0.05, size = 63, normalized size = 1.21

$$\frac{d(2b+cx)\sqrt{-1+dx}\,\sqrt{1+dx}+2(c+2ad^2)\tanh^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{2d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
[Out] (d*(2*b + c*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x] + 2*(c + 2*a*d^2)*ArcTanh[Sqrt[
(-1 + d*x)/(1 + d*x)]])/(2*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 120, normalized size = 2.31

method	result
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)dcx+2\ln\left(\left(\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)a\,d^2+2\,\mathrm{csgn}(d)d\sqrt{d^2x^2-}\right.}{2d^3\sqrt{d^2x^2-1}}$
risch	$\frac{(cx+2b)\sqrt{dx-1}\,\sqrt{dx+1}}{2d^2}+\frac{\left(\frac{\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)a}{\sqrt{d^2}}+\frac{\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)c}{2d^2\sqrt{d^2}}\right)\sqrt{(dx+1)\,(dx-}}{\sqrt{dx-1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^3*((d^2*x^2-1)^(1/2)*csgn(d)*d*c*x+2*ln((
(d^2*x^2-1)^(1/2)*csgn(d)+d*x)*csgn(d))*a*d^2+2*csgn(d)*d*(d^2*x^2-1)^(1/2)
*b+ln(((d^2*x^2-1)^(1/2)*csgn(d)+d*x)*csgn(d))*c)/(d^2*x^2-1)^(1/2)*csgn(d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(44) = 88.
time = 0.27, size = 90, normalized size = 1.73

$$\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,cx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,b}{d^2}+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

Fricas [A]
time = 1.26, size = 61, normalized size = 1.17

$$\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{dx-1}\,-(2\,ad^2+c)\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 80, normalized size = 1.54

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\,(2\,ad^2+c)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
- 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [**B**]
time = 14.59, size = 312, normalized size = 6.00

$$\frac{b\sqrt{dx-1}\,\sqrt{dx+1}}{d^2}+\frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{dx-1}-\mathrm{i}}{\sqrt{dx+1}-1}\right)}{d^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{dx-1}-\mathrm{i}\right)}{\left(\sqrt{dx+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-\frac{\frac{14\,c\left(\sqrt{dx-1}-\mathrm{i}\right)^3}{\left(\sqrt{dx+1}-1\right)^3}+\frac{14\,c\left(\sqrt{dx-1}-\mathrm{i}\right)^5}{\left(\sqrt{dx+1}-1\right)^5}+\frac{2\,c\left(\sqrt{dx-1}-\mathrm{i}\right)^7}{\left(\sqrt{dx+1}-1\right)^7}+\frac{2\,c\left(\sqrt{dx-1}-\mathrm{i}\right)}{\sqrt{dx+1}-1}}{d^3-\frac{4\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+\frac{6\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^4}{\left(\sqrt{dx+1}-1\right)^4}-\frac{4\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^6}{\left(\sqrt{dx+1}-1\right)^6}+\frac{d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^8}{\left(\sqrt{dx+1}-1\right)^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(
1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.36 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$\frac{c\sqrt{-1+dx}\,\sqrt{1+dx}}{d^2}+\frac{b\cosh^{-1}(dx)}{d}+a\tan^{-1}\left(\sqrt{-1+dx}\,\sqrt{1+dx}\right)$$

```
[Out] b*arccosh(d*x)/d+a*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+c*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/d^2
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1624, 1823, 858, 223, 212, 272, 65, 211}

$$\frac{a\sqrt{d^2x^2-1}\,\text{ArcTan}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{c(1-d^2x^2)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-\right.}{}$$

$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x,\right.}{2\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\sqrt{\right.}{}$$

$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{a\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1}}{}$$

Mathematica [A]
time = 0.05, size = 69, normalized size = 1.25

$$\frac{c\sqrt{-1+dx}\,\sqrt{1+dx}}{d^2} + 2a\tan^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2b\tanh^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (c*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + 2*a*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)
]] + (2*b*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 95, normalized size = 1.73

method	result
default	$\frac{\left(-\mathrm{csgn}(d)\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2+\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)c+\ln\left(\left(\sqrt{(dx+1)\,(dx-1)}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)bd\right)\sqrt{}}{d^2\sqrt{d^2x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+(d^2*x^2-1)^(1/2)*csgn(d)*c+ln(
(((d*x+1)*(d*x-1))^(1/2)*csgn(d)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

Maxima [A]
time = 0.49, size = 56, normalized size = 1.02

$$-a\arcsin\left(\frac{1}{d|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*c/d^2
```

Fricas [A]
time = 0.92, size = 73, normalized size = 1.33

$$\frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

Sympy [C] Result contains complex when optimal does not.
time = 41.46, size = 240, normalized size = 4.36

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
```

```
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [A]
time = 0.68, size = 71, normalized size = 1.29

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

Mupad [B]
time = 5.39, size = 118, normalized size = 2.15

$$\frac{c\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-a\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) - 1
i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((
d*x + 1)^(1/2) - 1)))*1i
```

3.37 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$\frac{a\sqrt{-1+dx}\,\sqrt{1+dx}}{x}+\frac{c\cosh^{-1}(dx)}{d}+b\tan^{-1}\left(\sqrt{-1+dx}\,\sqrt{1+dx}\right)$$

```
[Out] c*arccosh(d*x)/d+b*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+a*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/x
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(55) = 110. time = 0.11, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {1624, 1821, 858, 223, 212, 272, 65, 211}

$$-\frac{a(1-d^2x^2)}{x\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\text{ArcTan}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{c\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(c\sqrt{-\right.}{}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x\right.}{2\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\,\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{\right.}{}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\,\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{c\sqrt{-1}}{}$$

Mathematica [A]
time = 0.08, size = 69, normalized size = 1.25

$$\frac{a\sqrt{-1+dx}\,\sqrt{1+dx}}{x} + 2b\tan^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2c\tanh^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/x + 2*b*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
 + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 96, normalized size = 1.75

method	result
risch	$\frac{a\sqrt{dx-1}\,\sqrt{dx+1}}{x} + \frac{\left(\frac{c\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)}{\sqrt{d^2}} - b\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\right)\sqrt{(dx+1)(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$

default	$\frac{\left(-\operatorname{csgn}(d) d \arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right) bx + \sqrt{d^2x^2-1}\, \operatorname{csgn}(d) da + \ln\left(\left(\sqrt{d^2x^2-1}\, \operatorname{csgn}(d) + dx\right)\operatorname{csgn}(d)\right) cx\right)\sqrt{dx-1}}{\sqrt{d^2x^2-1}\, xd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(d)*d*arctan(1/(d^2*x^2-1)^(1/2))*b*x+(d^2*x^2-1)^(1/2)*csgn(d)*d*a+l
n(((d^2*x^2-1)^(1/2)*csgn(d)+d*x)*csgn(d))*c*x)*(d*x-1)^(1/2)*(d*x+1)^(1/2)
*csgn(d)/(d^2*x^2-1)^(1/2)/x/d
```

Maxima [A]
time = 0.51, size = 56, normalized size = 1.02

$$-b \arcsin\left(\frac{1}{d|x|}\right) + \frac{c \log\left(2\, d^2x + 2\, \sqrt{d^2x^2-1}\, d\right)}{d} + \frac{\sqrt{d^2x^2-1}\, a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*a/x
```

Fricas [A]
time = 0.90, size = 82, normalized size = 1.49

$$\frac{ad^2x + 2\, bdx \arctan\left(-dx + \sqrt{dx+1}\, \sqrt{dx-1}\right) + \sqrt{dx+1}\, \sqrt{dx-1}\, ad - cx \log\left(-dx + \sqrt{dx+1}\, \sqrt{dx-1}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 40.65, size = 216, normalized size = 3.93

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

Giac [A]
time = 0.55, size = 83, normalized size = 1.51

$$-\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)-\frac{8\,ad^{2}}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{4}+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

Mupad [B]
time = 5.15, size = 118, normalized size = 2.15

$$\frac{a\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^{2}}}\right)}{\sqrt{-d^{2}}}-b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^{2}}{\left(\sqrt{d\,x+1}-1\right)^{2}}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i)
)/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)^(
1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((d*
x + 1)^(1/2) - 1)))*1i
```

3.38 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=83

$$\frac{a\sqrt{-1+dx}\,\sqrt{1+dx}}{2x^2}+\frac{b\sqrt{-1+dx}\,\sqrt{1+dx}}{x}+\frac{1}{2}(2c+ad^2)\tan^{-1}\left(\sqrt{-1+dx}\,\sqrt{1+dx}\right)$$

```
[Out] 1/2*(a*d^2+2*c)*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/2*a*(d*x-1)^(1/2)*(d*
x+1)^(1/2)/x^2+b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1624, 1821, 821, 272, 65, 211}

$$\frac{\sqrt{d^2x^2-1}\,(ad^2+2c)\,\mathrm{ArcTan}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{2x^2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{x\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*(a*(1 - d^2*x^2))/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2
))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*Arc
Tan[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx &= \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+(2c+ad^2)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\,\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1 \ldots\right.}{2\sqrt{-1 \ldots}} \\
&= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1 \ldots\right.}{\ldots} \\
&= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1 \ldots\right.}{\ldots} \\
&= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{(2c+ad^2)\sqrt{-1+ \ldots}}{2\sqrt{-1 \ldots}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 0.72

$$\frac{(a+2bx)\sqrt{-1+dx}\,\sqrt{1+dx}}{2x^2}+\left(2c+ad^2\right)\tan^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((a + 2*b*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*x^2) + (2*c + a*d^2)*ArcTan[S
qrt[(-1 + d*x)/(1 + d*x)]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 103, normalized size = 1.24

method	result
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(2bx+a)}{2x^2}+\frac{\left(-\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)c-\frac{\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2}{2}\right)\sqrt{(dx+1)\,(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)^2\left(\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2x^2+2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)c\,x^2-2\sqrt{d^2x^2-1}\,bx-\sqrt{}\right.}{2\sqrt{d^2x^2-1}\,x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctan(1/(d^2*x^2-1)^(1/2))*a*d
^2*x^2+2*arctan(1/(d^2*x^2-1)^(1/2))*c*x^2-2*(d^2*x^2-1)^(1/2)*b*x-(d^2*x^2
-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^2
```

Maxima [A]
time = 0.51, size = 61, normalized size = 0.73

$$-\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right)-c\arcsin\left(\frac{1}{d|x|}\right)+\frac{\sqrt{d^2x^2-1}\,b}{x}+\frac{\sqrt{d^2x^2-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 - 1
)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

Fricas [A]
time = 1.34, size = 69, normalized size = 0.83

$$\frac{2\,bdx^2+2\,(ad^2+2\,c)x^2\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+(2\,bx+a)\sqrt{dx+1}\,\sqrt{dx-1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(67) = 134.
time = 0.83, size = 145, normalized size = 1.75

$$-\frac{(ad^3+2\,cd)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-16\,bd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

Mupad [B]
time = 12.77, size = 316, normalized size = 3.81

$$\frac{\frac{a\,d^2\,1\mathrm{i}}{32}+\frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}-c\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i}-\frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)1\mathrm{i}}{2}+\frac{a\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)1\mathrm{i}}{2}+\frac{b\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2 1\mathrm{i}}{32\left(\sqrt{d\,x+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/((((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x - 1)^(
1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2
))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.39 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=116

$$\frac{a\sqrt{-1+dx}\,\sqrt{1+dx}}{3x^3}+\frac{b\sqrt{-1+dx}\,\sqrt{1+dx}}{2x^2}+\frac{(3c+2ad^2)\,\sqrt{-1+dx}\,\sqrt{1+dx}}{3x}+\frac{1}{2}bd^2\tan^{-1}\left(\sqrt{-1+dx}\right.$$

```
[Out] 1/2*b*d^2*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/3*a*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/x^3+1/2*b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^2+1/3*(2*a*d^2+3*c)*(d*x-1)^(1
/2)*(d*x+1)^(1/2)/x
```

Rubi [A]
time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1624, 1821, 849, 821, 272, 65, 211}

$$-\frac{(1-d^2x^2)\,(2ad^2+3c)}{3x\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{3x^3\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\,\text{ArcTan}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(a*(1 - d^2*x^2))/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2
))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(
3*x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-
1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{3b+(3c+2ad^2)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int}{6\sqrt{-1+d}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-}{3x\sqrt{-1+dx}\,\sqrt{}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-}{3x\sqrt{-1+dx}\,\sqrt{}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-}{3x\sqrt{-1+dx}\,\sqrt{}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-}{3x\sqrt{-1+dx}\,\sqrt{}}$$

Mathematica [A]
time = 0.07, size = 71, normalized size = 0.61

$$\frac{\sqrt{-1+dx}\,\sqrt{1+dx}\,(3x(b+2cx)+a(2+4d^2x^2))}{6x^3} + bd^2\tan^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)))/(6*x^3
) + b*d^2*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 123, normalized size = 1.06

method	result
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(4a\,d^2x^2+6c\,x^2+3bx+2a)}{6x^3} - \frac{b\,d^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\sqrt{(dx+1)(dx-1)}}{2\sqrt{dx-1}\,\sqrt{dx+1}}$

default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)^2\left(3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)b\,d^2x^3-4\sqrt{d^2x^2-1}\,a\,d^2x^2-6\sqrt{d^2x^2-1}\,c\,x^2-3\sqrt{d^2x}\right.}{6\sqrt{d^2x^2-1}\,x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(3*arctan(1/(d^2*x^2-1)^(1/2))*b
*d^2*x^3-4*(d^2*x^2-1)^(1/2)*a*d^2*x^2-6*(d^2*x^2-1)^(1/2)*c*x^2-3*(d^2*x^2
-1)^(1/2)*b*x-2*(d^2*x^2-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^3
```

Maxima [A]
time = 0.50, size = 86, normalized size = 0.74

$$-\frac{1}{2}\,bd^2\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\sqrt{d^2x^2-1}\,ad^2}{3\,x}+\frac{\sqrt{d^2x^2-1}\,c}{x}+\frac{\sqrt{d^2x^2-1}\,b}{2\,x^2}+\frac{\sqrt{d^2x^2-1}\,a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2*
x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

Fricas [A]
time = 1.10, size = 90, normalized size = 0.78

$$\frac{6\,bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+2\,(2\,ad^3+3\,cd)x^3+(2\,(2\,ad^2+3\,c)x^2+3\,bx+2\,a)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(92) = 184.
time = 0.74, size = 197, normalized size = 1.70

$$-\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^8-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-96\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-48\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-128\,ad^4-192\,cd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
- 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

Mupad [B]
time = 11.82, size = 304, normalized size = 2.62

$$\frac{\frac{b\,d^2\,1\mathrm{i}}{32}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}}-\frac{b\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}+\frac{b\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{\sqrt{d\,x-1}\,\left(\frac{2\,a\,d^3\,x^3}{3}+\frac{2\,a\,d^2\,x^2}{3}+\frac{a\,d\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{d\,x+1}}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/((((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*
1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (c
*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d^2*x^2)
/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*((d*x - 1
)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.40 $\int \frac{a+bx+cx^2}{\sqrt{-1+x}\,\sqrt{1+x}\,(d+ex)^3}\,dx$

Optimal. Leaf size=199

$$-\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x}\,\sqrt{1+x}}{2e\left(d^2-e^2\right)(d+ex)^2}+\frac{\left(cd^3+bd^2e-(3a+4c)de^2+2be^3\right)\sqrt{-1+x}\,\sqrt{1+x}}{2e\left(d^2-e^2\right)^2(d+ex)}+\frac{((2a+c)d^2}{}$$

```
[Out] ((2*a+c)*d^2-3*b*d*e+(a+2*c)*e^2)*arctanh((d+e)^(1/2)*(1+x)^(1/2)/(d-e)^(1/
2)/(-1+x)^(1/2))/(d-e)^(5/2)/(d+e)^(5/2)-1/2*(a*e^2-b*d*e+c*d^2)*(-1+x)^(1/
2)*(1+x)^(1/2)/e/(d^2-e^2)/(e*x+d)^2+1/2*(c*d^3+b*d^2*e-(3*a+4*c)*d*e^2+2*b
*e^3)*(-1+x)^(1/2)*(1+x)^(1/2)/e/(d^2-e^2)^2/(e*x+d)
```

Rubi [A]
time = 0.23, antiderivative size = 242, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1624, 1665, 821, 739, 212}

$$\frac{\left(1-x^2\right)\left(ae^2-bde+cd^2\right)}{2e\sqrt{x-1}\,\sqrt{x+1}\,\left(d^2-e^2\right)(d+ex)^2}-\frac{\sqrt{x^2-1}\,\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\,\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{2\sqrt{x-1}\,\sqrt{x+1}\,\left(d^2-e^2\right)^{5/2}}-\frac{\left(1-x^2\right)\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\sqrt{x-1}\,\sqrt{x+1}\,\left(d^2-e^2\right)^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(1 - x^2))/(2*e*(d^2 - e^2)*Sqrt[-1 + x]*Sqrt[1 +
x]*(d + e*x)^2) - ((c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*(1 -
 x^2))/(2*e*(d^2 - e^2)^2*Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)) - ((3*b*d*e -
 a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*Sqrt[-1 + x^2]*ArcTanh[(e + d*x)/(Sqrt[
d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2)*Sqrt[-1 + x]*Sqrt[1 + x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\,\sqrt{1+x}\,(d+ex)^3}\,dx = \frac{\sqrt{-1+x^2}\,\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x}\,\sqrt{1+x}}$$

$$= \frac{(cd^2-bde+ae^2)\,(1-x^2)}{2e\,(d^2-e^2)\,\sqrt{-1+x}\,\sqrt{1+x}\,(d+ex)^2} - \frac{\sqrt{-1+x^2}\,\int \frac{-2(ad+cd-be)-}{(d+ex)^2}}{2\,(d^2-e^2)\,\sqrt{-1+}}$$

$$= \frac{(cd^2-bde+ae^2)\,(1-x^2)}{2e\,(d^2-e^2)\,\sqrt{-1+x}\,\sqrt{1+x}\,(d+ex)^2} - \frac{(c(d^3-4de^2)-e(3ade-}{2e\,(d^2-e^2)^2\,\sqrt{-1+x}}$$

$$= \frac{(cd^2-bde+ae^2)\,(1-x^2)}{2e\,(d^2-e^2)\,\sqrt{-1+x}\,\sqrt{1+x}\,(d+ex)^2} - \frac{(c(d^3-4de^2)-e(3ade-}{2e\,(d^2-e^2)^2\,\sqrt{-1+x}}$$

$$= \frac{(cd^2-bde+ae^2)\,(1-x^2)}{2e\,(d^2-e^2)\,\sqrt{-1+x}\,\sqrt{1+x}\,(d+ex)^2} - \frac{(c(d^3-4de^2)-e(3ade-}{2e\,(d^2-e^2)^2\,\sqrt{-1+x}}$$

Mathematica [A]

time = 0.74, size = 185, normalized size = 0.93

$$\frac{\sqrt{-1+x}\,\sqrt{1+x}\,\left(ae(-4d^2+e^2-3dex)+cd(-3de+d^2x-4e^2x)+b(2d^3+de^2+d^2ex+2e^3x)\right)}{2(d-e)^2(d+e)^2(d+ex)^2}-\frac{(-3bde+a(2d^2+e^2)+c(d^2+2e^2))\tan^{-1}\left(\frac{\sqrt{d-e}\,\sqrt{\frac{-1+x}{1+x}}}{\sqrt{-d-e}}\right)}{(-d-e)^{5/2}(d-e)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

```
[Out] (Sqrt[-1 + x]*Sqrt[1 + x]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^2
*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/(2*(d - e)^2*(d + e
)^2*(d + e*x)^2) - ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*ArcTan[(
Sqrt[d - e]*Sqrt[(-1 + x)/(1 + x)])/Sqrt[-d - e]])/((-d - e)^(5/2)*(d - e)^
(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(175) = 350$.
time = 0.12, size = 1095, normalized size = 5.50

method	result
default	$-\frac{\left(4cd\,e^3x\sqrt{x^2-1}\,\sqrt{\frac{d^2-e^2}{e^2}}+3ad\,e^3x\sqrt{x^2-1}\,\sqrt{\frac{d^2-e^2}{e^2}}-b\,d^2e^2x\sqrt{x^2-1}\,\sqrt{\frac{d^2-e^2}{e^2}}-c\,d^3ex\sqrt{x^2-1}\,\sqrt{\frac{d^2-}{e}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*(4*c*d*e^3*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+3*a*d*e^3*x*(x^2-1)^(
1/2)*((d^2-e^2)/e^2)^(1/2)-b*d^2*e^2*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)-
c*d^3*e*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+ln(-2*(-(x^2-1)^(1/2)*((d^2-e
^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*e^4*x^2+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^
2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*e^4*x^2+ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/
e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^2*e^2-3*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e
^2)^(1/2)*e+d*x+e)/(e*x+d))*b*d^3*e+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)
^(1/2)*e+d*x+e)/(e*x+d))*c*d^2*e^2-a*e^4*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2
)+ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^4+2*ln(
-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^4-2*b*e^4*x*
(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+4*a*d^2*e^2*(x^2-1)^(1/2)*((d^2-e^2)/e^
2)^(1/2)-2*b*d^3*e*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)-b*d*e^3*(x^2-1)^(1/2
)*((d^2-e^2)/e^2)^(1/2)+3*c*d^2*e^2*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+2*l
n(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^2*e^2*x^2-
```

```
3*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*b*d*e^3*x^2
+ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^2*e^2*x^
2+4*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^3*e*x
+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d*e^3*x-
6*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*b*d^2*e^2*x
+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^3*e*x+
4*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d*e^3*x)*
(-1+x)^(1/2)*(1+x)^(1/2)/(x^2-1)^(1/2)/(d+e)/(d-e)/(d^2-e^2)/(e*x+d)^2/((d^
2-e^2)/e^2)^(1/2)/e
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-%e>0)', see `assume?` for more de
tails)I
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(176) = 352.
time = 0.96, size = 1137, normalized size = 5.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(c*d^7 - 2*b*x^2*e^7 + ((2*a + c)*d^4*e^2 + (a + 2*c)*x^2*e^6 - (3*b*d
*x^2 - 2*(a + 2*c)*d*x)*e^5 + ((2*a + c)*d^2*x^2 - 6*b*d^2*x + (a + 2*c)*d^
2)*e^4 + (2*(2*a + c)*d^3*x - 3*b*d^3)*e^3)*sqrt(d^2 - e^2)*log((d^2*x + (d
^2 + sqrt(d^2 - e^2)*d - e^2)*sqrt(x + 1)*sqrt(x - 1) + d*e + sqrt(d^2 - e^
2)*(d*x + e))/(x*e + d)) - ((2*b*x + a)*e^7 - ((3*a + 4*c)*d*x - b*d)*e^6 -
 (b*d^2*x + (5*a + 3*c)*d^2)*e^5 + ((3*a + 5*c)*d^3*x + b*d^3)*e^4 - (b*d^4
*x - (4*a + 3*c)*d^4)*e^3 - (c*d^5*x + 2*b*d^5)*e^2)*sqrt(x + 1)*sqrt(x - 1
) + ((3*a + 4*c)*d*x^2 - 4*b*d*x)*e^6 + (b*d^2*x^2 + 2*(3*a + 4*c)*d^2*x -
2*b*d^2)*e^5 - ((3*a + 5*c)*d^3*x^2 - 2*b*d^3*x - (3*a + 4*c)*d^3)*e^4 + (b
*d^4*x^2 - 2*(3*a + 5*c)*d^4*x + b*d^4)*e^3 + (c*d^5*x^2 + 2*b*d^5*x - (3*a
 + 5*c)*d^5)*e^2 + (2*c*d^6*x + b*d^6)*e)/(2*d^7*x*e^3 + d^8*e^2 - 6*d^5*x*
e^5 + 6*d^3*x*e^7 - x^2*e^10 - 2*d*x*e^9 + (3*d^2*x^2 - d^2)*e^8 - 3*(d^4*x
```

```
^2 - d^4)*e^6 + (d^6*x^2 - 3*d^6)*e^4), 1/2*(c*d^7 - 2*b*x^2*e^7 - 2*((2*a
+ c)*d^4*e^2 + (a + 2*c)*x^2*e^6 - (3*b*d*x^2 - 2*(a + 2*c)*d*x)*e^5 + ((2*
a + c)*d^2*x^2 - 6*b*d^2*x + (a + 2*c)*d^2)*e^4 + (2*(2*a + c)*d^3*x - 3*b*
d^3)*e^3)*sqrt(-d^2 + e^2)*arctan(-sqrt(-d^2 + e^2)*(sqrt(x + 1)*sqrt(x - 1
)*e - x*e - d)/(d^2 - e^2)) - ((2*b*x + a)*e^7 - ((3*a + 4*c)*d*x - b*d)*e^
6 - (b*d^2*x + (5*a + 3*c)*d^2)*e^5 + ((3*a + 5*c)*d^3*x + b*d^3)*e^4 - (b*
d^4*x - (4*a + 3*c)*d^4)*e^3 - (c*d^5*x + 2*b*d^5)*e^2)*sqrt(x + 1)*sqrt(x
- 1) + ((3*a + 4*c)*d*x^2 - 4*b*d*x)*e^6 + (b*d^2*x^2 + 2*(3*a + 4*c)*d^2*x
 - 2*b*d^2)*e^5 - ((3*a + 5*c)*d^3*x^2 - 2*b*d^3*x - (3*a + 4*c)*d^3)*e^4 +
 (b*d^4*x^2 - 2*(3*a + 5*c)*d^4*x + b*d^4)*e^3 + (c*d^5*x^2 + 2*b*d^5*x - (
3*a + 5*c)*d^5)*e^2 + (2*c*d^6*x + b*d^6)*e)/(2*d^7*x*e^3 + d^8*e^2 - 6*d^5
*x*e^5 + 6*d^3*x*e^7 - x^2*e^10 - 2*d*x*e^9 + (3*d^2*x^2 - d^2)*e^8 - 3*(d^
4*x^2 - d^4)*e^6 + (d^6*x^2 - 3*d^6)*e^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(176) = 352.
time = 0.87, size = 605, normalized size = 3.04

$$-\frac{(2ad^2 + cd^2 - 3bde + ae^2 + 2ce^2)\arctan\left(\frac{\left(\sqrt{x+1}-\sqrt{x-1}\right)^2 e + 2d}{2\sqrt{-d^2+e^2}}\right)}{(d^4 - 2d^2e^2 + e^4)\sqrt{-d^2+e^2}} + \frac{2\left(2cd^4\left(\sqrt{x+1}-\sqrt{x-1}\right)^6 e + 4cd^5\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 - 2ad^2\left(\sqrt{x+1}-\sqrt{x-1}\right)^6 e^3 - 5cd^2\left(\sqrt{x+1}-\sqrt{x-1}\right)^6 e^3 + 4bd^4\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 e + 3bd\left(\sqrt{x+1}-\sqrt{x-1}\right)^6 e^4 - 12ad^3\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 e^2 - 14cd^3\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 e^2 - a\left(\sqrt{x+1}-\sqrt{x-1}\right)^6 e^5 + 10bd^2\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 e^3 + 8cd^4\left(\sqrt{x+1}-\sqrt{x-1}\right)^2 e - 6ad\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 e^4 - 8cd\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 e^4 + 16bd^3\left(\sqrt{x+1}-\sqrt{x-1}\right)^2 e^2 + 4b\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 e^5 - 40ad^2\left(\sqrt{x+1}-\sqrt{x-1}\right)^2 e^3 - 44cd^2\left(\sqrt{x+1}-\sqrt{x-1}\right)^2 e^3 + 20bd\left(\sqrt{x+1}-\sqrt{x-1}\right)^2 e^4 + 8cd^3e^2 + 4a\left(\sqrt{x+1}-\sqrt{x-1}\right)^2 e^5 + 8bd^2e^3 - 24ade^4 - 32cde^4 + 16be^5\right)}{(d^4e^2 - 2d^2e^4 + e^6)\left(\left(\sqrt{x+1}-\sqrt{x-1}\right)^4 e + 4d\left(\sqrt{x+1}-\sqrt{x-1}\right)^2 + 4e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -(2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(1/2*((sqrt(x + 1) - s
qrt(x - 1))^2*e + 2*d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2
 + e^2)) + 2*(2*c*d^4*(sqrt(x + 1) - sqrt(x - 1))^6*e + 4*c*d^5*(sqrt(x + 1
) - sqrt(x - 1))^4 - 2*a*d^2*(sqrt(x + 1) - sqrt(x - 1))^6*e^3 - 5*c*d^2*(s
qrt(x + 1) - sqrt(x - 1))^6*e^3 + 4*b*d^4*(sqrt(x + 1) - sqrt(x - 1))^4*e +
 3*b*d*(sqrt(x + 1) - sqrt(x - 1))^6*e^4 - 12*a*d^3*(sqrt(x + 1) - sqrt(x -
 1))^4*e^2 - 14*c*d^3*(sqrt(x + 1) - sqrt(x - 1))^4*e^2 - a*(sqrt(x + 1) -
sqrt(x - 1))^6*e^5 + 10*b*d^2*(sqrt(x + 1) - sqrt(x - 1))^4*e^3 + 8*c*d^4*(
sqrt(x + 1) - sqrt(x - 1))^2*e - 6*a*d*(sqrt(x + 1) - sqrt(x - 1))^4*e^4 -
8*c*d*(sqrt(x + 1) - sqrt(x - 1))^4*e^4 + 16*b*d^3*(sqrt(x + 1) - sqrt(x -
1))^2*e^2 + 4*b*(sqrt(x + 1) - sqrt(x - 1))^4*e^5 - 40*a*d^2*(sqrt(x + 1) -
 sqrt(x - 1))^2*e^3 - 44*c*d^2*(sqrt(x + 1) - sqrt(x - 1))^2*e^3 + 20*b*d*(
```

```
sqrt(x + 1) - sqrt(x - 1))^2*e^4 + 8*c*d^3*e^2 + 4*a*(sqrt(x + 1) - sqrt(x
- 1))^2*e^5 + 8*b*d^2*e^3 - 24*a*d*e^4 - 32*c*d*e^4 + 16*b*e^5)/((d^4*e^2 -
 2*d^2*e^4 + e^6)*((sqrt(x + 1) - sqrt(x - 1))^4*e + 4*d*(sqrt(x + 1) - sqr
t(x - 1))^2 + 4*e)^2)
```

Mupad [B]
time = 66.85, size = 2500, normalized size = 12.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((x - 1)^(1/2)*(x + 1)^(1/2)*(d + e*x)^3),x)
```

```
[Out] ((((x - 1)^(1/2) - 1i)^2*(2*c*e^3 + c*d^2*e)*12i)/(d^2*((x + 1)^(1/2) - 1)^
2*(d^4 + e^4 - 2*d^2*e^2)) - (2*(7*c*d^4 + 14*c*d^2*e^2)*((x - 1)^(1/2) - 1
i))/(7*d^3*((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) + (((x - 1)^(1/2) -
 1i)^4*(2*c*e^3 - c*d^2*e)*24i)/(d^2*((x + 1)^(1/2) - 1)^4*(d^4 + e^4 - 2*d
^2*e^2)) - (2*(21*c*d^4 - 102*c*d^2*e^2)*((x - 1)^(1/2) - 1i)^5)/(3*d^3*((x
 + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)) - (2*(35*c*d^4 - 170*c*d^2*e^2)
*((x - 1)^(1/2) - 1i)^3)/(5*d^3*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2*d^2*e^
2)) + (c*((x - 1)^(1/2) - 1i)^7*(d^2*1i + e^2*2i)*2i)/(d*((x + 1)^(1/2) - 1
)^7*(d^4 + e^4 - 2*d^2*e^2)) + (12*c*e*((x - 1)^(1/2) - 1i)^6*(d^2*1i + e^2
*2i))/(d^2*((x + 1)^(1/2) - 1)^6*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/2)
- 1i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^(1/
2) - 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*((x
 - 1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) - 1i)
^7*8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e^2)
)/(d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2))/(
d^2*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2 - 32*e^2))/(d^2
*((x + 1)^(1/2) - 1)^4) + 1) - ((2*((x - 1)^(1/2) - 1i)^3*(16*b*e^3 + 11*b*
d^2*e))/(d^2*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2*d^2*e^2)) - (6*b*e*((x -
1)^(1/2) - 1i)^7)/(((x + 1)^(1/2) - 1)^7*(d^4 + e^4 - 2*d^2*e^2)) - (6*b*e*
((x - 1)^(1/2) - 1i))/(((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) + (((x
- 1)^(1/2) - 1i)^4*(2*b*e^4 - 2*b*d^4 + 3*b*d^2*e^2)*8i)/(d^3*((x + 1)^(1/2
) - 1)^4*(d^4 + e^4 - 2*d^2*e^2)) + (b*((x - 1)^(1/2) - 1i)^2*(2*d^4 + 2*e^
4 + 5*d^2*e^2)*4i)/(d^3*((x + 1)^(1/2) - 1)^2*(d^4 + e^4 - 2*d^2*e^2)) + (b
*((x - 1)^(1/2) - 1i)^6*(2*d^4 + 2*e^4 + 5*d^2*e^2)*4i)/(d^3*((x + 1)^(1/2)
 - 1)^6*(d^4 + e^4 - 2*d^2*e^2)) + (2*b*e*((x - 1)^(1/2) - 1i)^5*(11*d^2 +
16*e^2))/(d^2*((x + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/
2) - 1i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^
(1/2) - 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*
((x - 1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) -
1i)^7*8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e
^2))/(d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2)
)/(d^2*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2 - 32*e^2))/(
```

```
d^2*((x + 1)^(1/2) - 1)^4) + 1) + ((2*(2*a*e^4 - 5*a*d^2*e^2)*((x - 1)^(1/2
) - 1i))/(d^3*((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) - (((x - 1)^(1/2
) - 1i)^4*(2*a*e^5 - 9*a*d^2*e^3 + 4*a*d^4*e)*8i)/(d^4*((x + 1)^(1/2) - 1)^
4*(d^4 + e^4 - 2*d^2*e^2)) + (2*(2*a*e^4 - 5*a*d^2*e^2)*((x - 1)^(1/2) - 1i
)^7)/(d^3*((x + 1)^(1/2) - 1)^7*(d^4 + e^4 - 2*d^2*e^2)) - (2*(2*a*e^4 - 29
*a*d^2*e^2)*((x - 1)^(1/2) - 1i)^3)/(d^3*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 -
 2*d^2*e^2)) - (2*(2*a*e^4 - 29*a*d^2*e^2)*((x - 1)^(1/2) - 1i)^5)/(d^3*((x
 + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)) + (e*((x - 1)^(1/2) - 1i)^2*(4*
a*d^4 - 2*a*e^4 + 7*a*d^2*e^2)*4i)/(d^4*((x + 1)^(1/2) - 1)^2*(d^4 + e^4 -
2*d^2*e^2)) + (e*((x - 1)^(1/2) - 1i)^6*(4*a*d^4 - 2*a*e^4 + 7*a*d^2*e^2)*4
i)/(d^4*((x + 1)^(1/2) - 1)^6*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/2) - 1
i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^(1/2)
- 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*((x -
1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) - 1i)^7*
8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e^2))/(
d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2))/(d^2
*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2 - 32*e^2))/(d^2*((
x + 1)^(1/2) - 1)^4) + 1) - (c*atan(((c*(d^2 + 2*e^2)*((4*(c*e^7*8i - c*d^2
*e^5*12i + c*d^6*e*4i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2
) + (4*((x - 1)^(1/2) - 1i)^2*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(((x
 + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) -
(c*(d^2 + 2*e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4
*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10
 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2
*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/((
(x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)))
)/(2*(d + e)^(5/2)*(d - e)^(5/2)))*1i)/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (c
*(d^2 + 2*e^2)*((4*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(d^10 + d^2*e^8
 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(c*e^7*8i
 - c*d^2*e^5*12i + c*d^6*e*4i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*
d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (c*(d^2 + 2*e^2)*((e*((x - 1)^(1/2) - 1
i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*
e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*
d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^
4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2...
```

3.41 $\int (a+bx)^2 \sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)\,dx$

Optimal. Leaf size=1348

$$(de-cf)\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+9\right.$$

```
[Out] -1/20*(2*a*C*d*f-b*(4*B*d*f-3*C*(c*f+d*e)))*(b*x+a)^2*(d*x+c)^(3/2)*(f*x+e)
^(3/2)/b/d^2/f^2+1/6*C*(b*x+a)^3*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/960*(d
*x+c)^(3/2)*(f*x+e)^(3/2)*(64*a^3*C*d^3*f^3-8*a^2*b*d^2*f^2*(16*B*d*f-7*C*(
c*f+d*e))-8*a*b^2*d*f*(C*(35*c^2*f^2+38*c*d*e*f+35*d^2*e^2)+10*d*f*(8*A*d*f
-5*B*(c*f+d*e)))+b^3*(7*C*(15*c^3*f^3+17*c^2*d*e*f^2+17*c*d^2*e^2*f+15*d^3*
e^3)+4*d*f*(50*A*d*f*(c*f+d*e)-B*(35*c^2*f^2+38*c*d*e*f+35*d^2*e^2)))+6*b*d
*f*(10*b*d*f*(-4*A*b*d*f+C*a*c*f+C*a*d*e+2*C*b*c*e)+(4*a*d*f-7*b*(c*f+d*e))
*(2*a*C*d*f-b*(4*B*d*f-3*C*(c*f+d*e))))*x)/b/d^4/f^4-1/512*(-c*f+d*e)^2*(8*
a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))
-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*
d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^2*(C*(21*c^4*f^4+28*c^3
*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e^4)+4*d*f*(2*A*d*f*(5*c^
2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e
^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(11/2)/f^(11/
2)+1/256*(8*a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B
*(c*f+d*e)))-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)
+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^2*(C*(21*c^
4*f^4+28*c^3*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e^4)+4*d*f*(2
*A*d*f*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e
^2*f+7*d^3*e^3))))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^5/f^4+1/512*(-c*f+d*e)*(8*
a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))
-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*
d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^2*(C*(21*c^4*f^4+28*c^3
*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e^4)+4*d*f*(2*A*d*f*(5*c^
2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e
^3))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^5/f^5
```

Rubi [A]

time = 1.52, antiderivative size = 1345, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1629, 158, 152, 52, 65, 223, 212}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*
(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*
e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f
^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f +
 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(512*d^5*f^5) + ((8*a^2*d^2*f^2*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*
(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d
*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 +
28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*
(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f
 + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(256*d^5*f^
4) + ((4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f))*(a + b*x)^2*(c + d*x)^(3/
2)*(e + f*x)^(3/2))/(20*b*d^2*f^2) + (C*(a + b*x)^3*(c + d*x)^(3/2)*(e + f*
x)^(3/2))/(6*b*d*f) - ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(64*a^3*C*d^3*f^3 -
8*a^2*b*d^2*f^2*(16*B*d*f - 7*C*(d*e + c*f)) - 8*a*b^2*d*f*(C*(35*d^2*e^2 +
 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5*B*(d*e + c*f))) + b^3*(7*C*
(15*d^3*e^3 + 17*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 15*c^3*f^3) + 4*d*f*(50*A*d
*f*(d*e + c*f) - B*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2))) + 6*b*d*f*(10*b
*d*f*(2*b*c*C*e + a*C*d*e + a*c*C*f - 4*A*b*d*f) - (4*a*d*f - 7*b*(d*e + c*
f))*(4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f)))*x))/(960*b*d^4*f^4) - ((d*
e - c*f)^2*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2
*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*
d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*
f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2
 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5
*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*Ar
cTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(512*d^(11/2)*f^(11
/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
```

```
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int (a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx = \frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf} + \frac{\int (a+bx)^2\sqrt{c+}}{}$$

$$= \frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/}}{20bd^2f^2}$$

$$= \frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/}}{20bd^2f^2}$$

$$= \frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B}{}$$

$$= \frac{(de-cf)\,(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(}{}$$

$$= \frac{(de-cf)\,(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(}{}$$

$$= \frac{(de-cf)\,(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(}{}$$

$$= \frac{(de-cf)\,(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(}{}$$

Mathematica [A]
time = 6.93, size = 1253, normalized size = 0.93

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(40*a^2*d^2*f^2*(C*(15*c^3*f^3 - c^2*d*f^2*(7*
e + 10*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2
```

```
*f*x + 8*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) +
B*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))) +
 8*a*b*d*f*(C*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f^2*(-
17*e^2 + 11*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*e*f^2*
x^2 + 24*f^3*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48*e*f^3*
x^3 + 384*f^4*x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2
*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x)
 + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*
f^2*x^2 + 48*f^3*x^3)))) + b^2*(C*(315*c^5*f^5 - 105*c^4*d*f^4*(e + 2*f*x)
+ 2*c^3*d^2*f^3*(-41*e^2 + 28*e*f*x + 84*f^2*x^2) - 2*c^2*d^3*f^2*(41*e^3 -
 26*e^2*f*x + 20*e*f^2*x^2 + 72*f^3*x^3) + c*d^4*f*(-105*e^4 + 56*e^3*f*x -
 40*e^2*f^2*x^2 + 32*e*f^3*x^3 + 128*f^4*x^4) + d^5*(315*e^5 - 210*e^4*f*x
+ 168*e^3*f^2*x^2 - 144*e^2*f^3*x^3 + 128*e*f^4*x^4 + 1280*f^5*x^5)) + 4*d*
f*(10*A*d*f*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*d^2*f*(-7*e^2 + 4*e*
f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*x^2 + 48*f^3*x^3)) +
B*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f^2*(-17*e^2 + 11*
e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*e*f^2*x^2 + 24*f^3
*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48*e*f^3*x^3 + 384*f^
4*x^4))))))/(7680*d^5*f^5) - ((d*e - c*f)^2*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 +
6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7
*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e
 + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28
*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2
*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f +
 9*c^2*d*e*f^2 + 7*c^3*f^3))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqr
t[c + d*x])])/(512*d^(11/2)*f^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5733 vs. $2(1304) = 2608$.

time = 0.10, size = 5734, normalized size = 4.25

method	result	size
default	Expression too large to display	5734

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 4.19, size = 3091, normalized size = 2.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/30720*(15*(21*C*b^2*d^6*e^6 + (21*C*b^2*c^6 + 128*A*a^2*c^2*d^4 - 28*(2*
C*a*b + B*b^2)*c^5*d + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^4*d^2 - 64*(B*a^2 + 2
*A*a*b)*c^3*d^3)*f^6 - 2*(7*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b + B
*b^2)*c^4*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*b)
*c^2*d^4)*f^5*e - (5*C*b^2*c^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*a*b + B*b^2)*c^
3*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*
f^4*e^2 - 4*(C*b^2*c^3*d^3 - 2*(2*C*a*b + B*b^2)*c^2*d^4 + 8*(C*a^2 + 2*B*a
*b + A*b^2)*c*d^5 + 16*(B*a^2 + 2*A*a*b)*d^6)*f^3*e^3 - 5*(C*b^2*c^2*d^4 -
4*(2*C*a*b + B*b^2)*c*d^5 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^2*e^4 - 14*(
C*b^2*c*d^5 + 2*(2*C*a*b + B*b^2)*d^6)*f*e^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 +
 8*c*d*f^2*x + c^2*f^2 + d^2*e^2 - 4*(2*d*f*x + c*f + d*e)*sqrt(d*f)*sqrt(d
*x + c)*sqrt(f*x + e) + 2*(4*d^2*f*x + 3*c*d*f)*e) + 4*(1280*C*b^2*d^6*f^6*
x^5 + 128*(C*b^2*c*d^5 + 12*(2*C*a*b + B*b^2)*d^6)*f^6*x^4 + 315*C*b^2*d^6*
f*e^5 - 48*(3*C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*B*a
*b + A*b^2)*d^6)*f^6*x^3 + 8*(21*C*b^2*c^3*d^3 - 28*(2*C*a*b + B*b^2)*c^2*d
^4 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 320*(B*a^2 + 2*A*a*b)*d^6)*f^6*x^
2 - 10*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 28*(2*C*a*b + B*b^2)*c^3*d^3 + 4
0*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*f^6*x + 1
5*(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b + B*b^2)*c^4*d^2 + 40*(C*
a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*b)*c^2*d^4)*f^6 - 105*(2
*C*b^2*d^6*f^2*x + (C*b^2*c*d^5 + 4*(2*C*a*b + B*b^2)*d^6)*f^2)*e^4 + 2*(84
*C*b^2*d^6*f^3*x^2 + 28*(C*b^2*c*d^5 + 5*(2*C*a*b + B*b^2)*d^6)*f^3*x - (41
*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 300*(C*a^2 + 2*B*a*b + A*b^2)
*d^6)*f^3)*e^3 - 2*(72*C*b^2*d^6*f^4*x^3 + 4*(5*C*b^2*c*d^5 + 28*(2*C*a*b +
 B*b^2)*d^6)*f^4*x^2 - 2*(13*C*b^2*c^2*d^4 - 22*(2*C*a*b + B*b^2)*c*d^5 - 1
00*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^4*x + (41*C*b^2*c^3*d^3 - 68*(2*C*a*b +
 B*b^2)*c^2*d^4 + 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 480*(B*a^2 + 2*A*a*
b)*d^6)*f^4)*e^2 + (128*C*b^2*d^6*f^5*x^4 + 32*(C*b^2*c*d^5 + 6*(2*C*a*b +
B*b^2)*d^6)*f^5*x^3 - 8*(5*C*b^2*c^2*d^4 - 8*(2*C*a*b + B*b^2)*c*d^5 - 40*(
```

```
C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^5*x^2 + 8*(7*C*b^2*c^3*d^3 - 11*(2*C*a*b +
B*b^2)*c^2*d^4 + 20*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 80*(B*a^2 + 2*A*a*b)*
d^6)*f^5*x - 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 32*(2*C*a*b + B*b^2)*c^3
*d^3 + 56*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(B*a^2 + 2*A*a*b)*c*d^5)*
f^5)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(d^6*f^6), 1/15360*(15*(21*C*b^2*d^6*e
^6 + (21*C*b^2*c^6 + 128*A*a^2*c^2*d^4 - 28*(2*C*a*b + B*b^2)*c^5*d + 40*(C
*a^2 + 2*B*a*b + A*b^2)*c^4*d^2 - 64*(B*a^2 + 2*A*a*b)*c^3*d^3)*f^6 - 2*(7*
C*b^2*c^5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b + B*b^2)*c^4*d^2 + 16*(C*a^2 +
2*B*a*b + A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*b)*c^2*d^4)*f^5*e - (5*C*b^2*c
^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*a*b + B*b^2)*c^3*d^3 + 16*(C*a^2 + 2*B*a*b
+ A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*f^4*e^2 - 4*(C*b^2*c^3*d^3 -
 2*(2*C*a*b + B*b^2)*c^2*d^4 + 8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 16*(B*a^
2 + 2*A*a*b)*d^6)*f^3*e^3 - 5*(C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 -
8*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^2*e^4 - 14*(C*b^2*c*d^5 + 2*(2*C*a*b + B
*b^2)*d^6)*f*e^5)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + c*f + d*e)*sqrt(-d*f)*sq
rt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*f^2*x + (d^2*f*x + c*d*f)*e))
+ 2*(1280*C*b^2*d^6*f^6*x^5 + 128*(C*b^2*c*d^5 + 12*(2*C*a*b + B*b^2)*d^6)*
f^6*x^4 + 315*C*b^2*d^6*f*e^5 - 48*(3*C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c
*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^6*x^3 + 8*(21*C*b^2*c^3*d^3 - 28
*(2*C*a*b + B*b^2)*c^2*d^4 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 320*(B*a^
2 + 2*A*a*b)*d^6)*f^6*x^2 - 10*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 28*(2*C*
a*b + B*b^2)*c^3*d^3 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2
*A*a*b)*c*d^5)*f^6*x + 15*(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b +
 B*b^2)*c^4*d^2 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*
b)*c^2*d^4)*f^6 - 105*(2*C*b^2*d^6*f^2*x + (C*b^2*c*d^5 + 4*(2*C*a*b + B*b^
2)*d^6)*f^2)*e^4 + 2*(84*C*b^2*d^6*f^3*x^2 + 28*(C*b^2*c*d^5 + 5*(2*C*a*b +
 B*b^2)*d^6)*f^3*x - (41*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 300*(
C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^3)*e^3 - 2*(72*C*b^2*d^6*f^4*x^3 + 4*(5*C*b
^2*c*d^5 + 28*(2*C*a*b + B*b^2)*d^6)*f^4*x^2 - 2*(13*C*b^2*c^2*d^4 - 22*(2*
C*a*b + B*b^2)*c*d^5 - 100*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^4*x + (41*C*b^2
*c^3*d^3 - 68*(2*C*a*b + B*b^2)*c^2*d^4 + 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d
^5 + 480*(B*a^2 + 2*A*a*b)*d^6)*f^4)*e^2 + (128*C*b^2*d^6*f^5*x^4 + 32*(C*b
^2*c*d^5 + 6*(2*C*a*b + B*b^2)*d^6)*f^5*x^3 - 8*(5*C*b^2*c^2*d^4 - 8*(2*C*a
*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^5*x^2 + 8*(7*C*b^2*
c^3*d^3 - 11*(2*C*a*b + B*b^2)*c^2*d^4 + 20*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5
 + 80*(B*a^2 + 2*A*a*b)*d^6)*f^5*x - 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 -
32*(2*C*a*b + B*b^2)*c^3*d^3 + 56*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(
B*a^2 + 2*A*a*b)*c*d^5)*f^5)*e)*sqrt(d*x + c)*s...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4708 vs. $2(1313) = 2626$.
time = 1.22, size = 4708, normalized size = 3.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/7680*(7680*((c*d*f - d^2*e)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/sqrt(d*f) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)
*sqrt(d*x + c))*A*a^2*c*abs(d)/d^2 + 320*(sqrt((d*x + c)*d*f - c*d*f + d^2*
e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)
/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) +
 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*s
qrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*C*a
^2*c*abs(d)/d^2 + 640*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2
*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^7*f^4)) + 3*(11
*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(5*c^3*f^3 - 3*c
^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt
((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*B*a*b*c*abs(d)/d^2 + 8
0*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x +
c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*
c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^
12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3
*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e
^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e))
)/(sqrt(d*f)*d^2*f^3))*C*a*b*c*abs(d)/d^2 + 320*(sqrt((d*x + c)*d*f - c*d*f
 + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6
*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*
f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt
(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^
2))*A*b^2*c*abs(d)/d^2 + 40*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x +
c)*(4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6))
 + (163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(9
3*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^
14*f^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2
 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*B*b^2*c*abs(d)/d^2 + 4*(sq
rt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d*x + c)
/d^4 - (41*c*d^19*f^8 - d^20*f^7*e)/(d^23*f^8)) + (513*c^2*d^19*f^8 - 26*c*
```

```
d^20*f^7*e - 7*d^21*f^6*e^2)/(d^23*f^8)) - 5*(447*c^3*d^19*f^8 - 37*c^2*d^2
0*f^7*e - 19*c*d^21*f^6*e^2 - 7*d^22*f^5*e^3)/(d^23*f^8))*(d*x + c) + 15*(1
93*c^4*d^19*f^8 - 28*c^3*d^20*f^7*e - 18*c^2*d^21*f^6*e^2 - 12*c*d^22*f^5*e
^3 - 7*d^23*f^4*e^4)/(d^23*f^8))*sqrt(d*x + c) + 15*(63*c^5*f^5 - 35*c^4*d*
f^4*e - 10*c^3*d^2*f^3*e^2 - 6*c^2*d^3*f^2*e^3 - 5*c*d^4*f*e^4 - 7*d^5*e^5)
*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(
sqrt(d*f)*d^3*f^4))*C*b^2*c*abs(d)/d^2 + 320*(sqrt((d*x + c)*d*f - c*d*f +
d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^
3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4
)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*
f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))
*B*a^2*abs(d)/d + 40*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(
d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (163
*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^3*d
^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f^6)
)*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*
d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*
f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*C*a^2*abs(d)/d + 640*(sqrt((d*x +
 c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*
c*d^5*f^4 - d^6*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7
*f^2*e^2)/(d^7*f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3
)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/
(sqrt(d*f)*d*f^2))*A*a*b*abs(d)/d + 80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)
*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/
(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f
^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f
^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*
d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) +
 sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*B*a*b*abs(d)/d
+ 8*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d
*x + c)/d^4 - (41*c*d^19*f^8 - d^20*f^7*e)/(d^23*f^8)) + (513*c^2*d^19*f^8
- 26*c*d^20*f^7*e - 7*d^21*f^6*e^2)/(d^23*f^8)) - 5*(447*c^3*d^19*f^8 - 37*
c^2*d^20*f^7*e - 19*c*d^21*f^6*e^2 - 7*d^22*f^5*e^3)/(d^23*f^8))*(d*x + c)
+ 15*(193*c^4*d^19*f^8 - 28*c^3*d^20*f^7*e - 18*c^2*d^21*f^6*e^2 - 12*c*d^2
2*f^5*e^3 - 7*d^23*f^4*e^4)/(d^23*f^8))*sqrt(d*...
```

Mupad [F(-1)]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] \text{Hanged}
```

3.42 $\int (a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)\,dx$

Optimal. Leaf size=721

$$\frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)-b\left(C\left(7d^3e^3+9cd^2e^2f+9c^2def^2+\right.\right.}{128d^4f^4}$$

```
[Out] 1/5*C*(b*x+a)^2*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/240*(d*x+c)^(3/2)*(f*x+
e)^(3/2)*(48*a^2*C*d^2*f^2-10*a*b*d*f*(8*B*d*f-5*C*(c*f+d*e))-b^2*(C*(35*c^
2*f^2+38*c*d*e*f+35*d^2*e^2)+10*d*f*(8*A*d*f-5*B*(c*f+d*e)))+6*b*d*f*(6*a*C
*d*f-b*(10*B*d*f-7*C*(c*f+d*e)))*x)/b/d^3/f^3-1/128*(-c*f+d*e)^2*(2*a*d*f*(
C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*
f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^
2*f^2+6*c*d*e*f+5*d^2*e^2))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)
^(1/2))/d^(9/2)/f^(9/2)+1/64*(2*a*d*f*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*
d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^
3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2))))*(d*x+c
)^(3/2)*(f*x+e)^(1/2)/d^4/f^3+1/128*(-c*f+d*e)*(2*a*d*f*(C*(5*c^2*f^2+6*c*d
*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*c^2*d*e*f^2+
9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*
d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^4/f^4
```

Rubi [A]
time = 0.61, antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1629, 152, 52, 65, 223, 212}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d
*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*
c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^
2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(128*d^4*f^4) + ((2*a*d*f*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e
^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f
) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*(c + d*x)^(3/2)*Sqrt[e + f*x])
/(64*d^4*f^3) + (C*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*d*f) -
 ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(48*a^2*C*d^2*f^2 - 10*a*b*d*f*(8*B*d*f -
 5*C*(d*e + c*f)) - b^2*(C*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*
(8*A*d*f - 5*B*(d*e + c*f))) - 6*b*d*f*(10*b*B*d*f - 6*a*C*d*f - 7*b*C*(d*e
 + c*f))*x))/(240*b*d^3*f^3) - ((d*e - c*f)^2*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*
```

```
d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*
(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int (a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx = \frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf} + \frac{\int (a+bx)\sqrt{c+dx}$$

$$= \frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf} - \frac{(c+dx)^{3/2}(e+f$$

$$= \frac{(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de$$

$$= \frac{(de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Ad\right.}{}$$

$$= \frac{(de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Ad\right.}{}$$

$$= \frac{(de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Ad\right.}{}$$

$$= \frac{(de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Ad\right.}{}$$

Mathematica [A]
time = 3.41, size = 662, normalized size = 0.92

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(10*a*d*f*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10
*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x +
 8*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(-3*
c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))) + b*(C*
(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f^2*(-17*e^2 + 11*e*
f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*e*f^2*x^2 + 24*f^3*x
^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48*e*f^3*x^3 + 384*f^4*
x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e
*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*d^2*f*(-7
*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*x^2 + 48*f
^3*x^3))))))/(1920*d^4*f^4) + ((d*e - c*f)^2*(-2*a*d*f*(C*(5*d^2*e^2 + 6*c*
d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) + b*(C*(7*d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*
(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqr
t[f]*Sqrt[c + d*x])])/(128*d^(9/2)*f^(9/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3024 vs. $2(683) = 1366$.
time = 0.10, size = 3025, normalized size = 4.20

method	result	size
default	Expression too large to display	3025

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-32*C*b*c*d^3*e*f^3*x^2*(d*f)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)+44*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^2*d^2*e*
f^3*x-80*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^3*e*f^3*x-80*C*(d*f)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^3*e*f^3*x+44*C*(d*f)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*b*c*d^3*e^2*f^2*x-105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^5-105*C*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^5*f^5+480*A*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^5
*e^2*f^3-240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*
e)/(d*f)^(1/2))*b*c^3*d^2*f^5-240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^3*f^2-240*B*ln(1/2*(2*d*f*x+2*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^3*d^2*f^5-240
*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*a*d^5*e^3*f^2+480*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*f^5+150*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^5*e^4*f+150*B*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^4*d*f^
```

5+150*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^4*f+150*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^4*d*f^5+210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^4*e^4+210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^4*f^4-768*C*b*d^4*f^4*x^4*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-960*B*b*d^4*f^4*x^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-960*C*a*d^4*f^4*x^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-1280*A*b*d^4*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-1280*B*a*d^4*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-96*C*b*c*d^3*f^4*x^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-96*C*b*d^4*e*f^3*x^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-160*B*b*c*d^3*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-160*B*b*d^4*e*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-160*C*a*c*d^3*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-160*C*a*d^4*e*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-960*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^3*f^4-960*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^4*e*f^3-60*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e^2*f^3-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^3*f^2-960*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e*f^4+240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e*f^4+240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^2*f^3-1920*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^4*f^4*x+240*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*e*f^4+200*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^4*e^2*f^2*x-140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^3*d*f^4*x-140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^4*e^3*f*x-320*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^3*e*f^3-320*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^3*e*f^3+140*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^2*d^2*e*f^3+140*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^3*e^2*f^2+200*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c^2*d^2*f^4*x-68*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^2*d^2*e^2*f^2-300*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^3*d*f^4-300*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^4*e^3*f-300*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c^3*d*f^4+112*C*b*c^2*d^2*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+112*C*b*d^4*e^2*f^2*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-300*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^4*e^3*f+480*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c^2*d^2*f^4+480*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^4*e^2*f^2+30*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e^3*f^2+75*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^4*f-80*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^3*e^3*f-80*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^3*d*e*f^3+140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c^2*d^2*e*f^3+140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^3*e^2*f^2+480*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^2*d^2*f^4+480*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^4*e^2*f^2-120*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^3*d^2*e*f^4-60*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*e^2*f^3-120*C*ln(1/

```
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*
d^4*e^3*f^2+75*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*b*c^4*d*e*f^4+30*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*e^2*f^3-320*A*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)*b*c*d^3*f^4*x-320*A*(d*f...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 1.84, size = 1621, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/7680*(15*(7*C*b*d^5*e^5 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c
^4*d + 16*(B*a + A*b)*c^3*d^2)*f^5 - (5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a +
 B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^4*e - 2*(C*b*c^3*d^2 + 16*A*a*d^5
 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*f^3*e^2 - 2*(C*b*c^2*d^3 -
4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*f^2*e^3 - 5*(C*b*c*d^4 + 2*(C*a +
B*b)*d^5)*f*e^4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + 8*c*d*f^2*x + c^2*f^2 + d^2*
e^2 + 4*(2*d*f*x + c*f + d*e)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 2*(4*
d^2*f*x + 3*c*d*f)*e) + 4*(384*C*b*d^5*f^5*x^4 + 48*(C*b*c*d^4 + 10*(C*a +
B*b)*d^5)*f^5*x^3 - 105*C*b*d^5*f*e^4 - 8*(7*C*b*c^2*d^3 - 10*(C*a + B*b)*c
*d^4 - 80*(B*a + A*b)*d^5)*f^5*x^2 + 10*(7*C*b*c^3*d^2 + 96*A*a*d^5 - 10*(C
*a + B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*f^5*x - 15*(7*C*b*c^4*d - 32*A*a*
c*d^4 - 10*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 + 10*(7*C*b*d^
5*f^2*x + (4*C*b*c*d^4 + 15*(C*a + B*b)*d^5)*f^2)*e^3 - 2*(28*C*b*d^5*f^3*x
^2 + (11*C*b*c*d^4 + 50*(C*a + B*b)*d^5)*f^3*x - (17*C*b*c^2*d^3 - 35*(C*a
+ B*b)*c*d^4 - 120*(B*a + A*b)*d^5)*f^3)*e^2 + 2*(24*C*b*d^5*f^4*x^3 + 8*(C
*b*c*d^4 + 5*(C*a + B*b)*d^5)*f^4*x^2 - (11*C*b*c^2*d^3 - 20*(C*a + B*b)*c*
d^4 - 80*(B*a + A*b)*d^5)*f^4*x + 5*(4*C*b*c^3*d^2 + 48*A*a*d^5 - 7*(C*a +
```

```
B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*f^4)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(
d^5*f^5), -1/3840*(15*(7*C*b*d^5*e^5 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*
a + B*b)*c^4*d + 16*(B*a + A*b)*c^3*d^2)*f^5 - (5*C*b*c^4*d - 64*A*a*c*d^4
- 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^4*e - 2*(C*b*c^3*d^2 +
16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*f^3*e^2 - 2*(C*b*
c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*f^2*e^3 - 5*(C*b*c*d^4 +
 2*(C*a + B*b)*d^5)*f*e^4)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + c*f + d*e)*sqrt
(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*f^2*x + (d^2*f*x + c*
d*f)*e)) - 2*(384*C*b*d^5*f^5*x^4 + 48*(C*b*c*d^4 + 10*(C*a + B*b)*d^5)*f^5
*x^3 - 105*C*b*d^5*f*e^4 - 8*(7*C*b*c^2*d^3 - 10*(C*a + B*b)*c*d^4 - 80*(B*
a + A*b)*d^5)*f^5*x^2 + 10*(7*C*b*c^3*d^2 + 96*A*a*d^5 - 10*(C*a + B*b)*c^2
*d^3 + 16*(B*a + A*b)*c*d^4)*f^5*x - 15*(7*C*b*c^4*d - 32*A*a*c*d^4 - 10*(C
*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 + 10*(7*C*b*d^5*f^2*x + (4*
C*b*c*d^4 + 15*(C*a + B*b)*d^5)*f^2)*e^3 - 2*(28*C*b*d^5*f^3*x^2 + (11*C*b*
c*d^4 + 50*(C*a + B*b)*d^5)*f^3*x - (17*C*b*c^2*d^3 - 35*(C*a + B*b)*c*d^4
- 120*(B*a + A*b)*d^5)*f^3)*e^2 + 2*(24*C*b*d^5*f^4*x^3 + 8*(C*b*c*d^4 + 5*
(C*a + B*b)*d^5)*f^4*x^2 - (11*C*b*c^2*d^3 - 20*(C*a + B*b)*c*d^4 - 80*(B*a
 + A*b)*d^5)*f^4*x + 5*(4*C*b*c^3*d^2 + 48*A*a*d^5 - 7*(C*a + B*b)*c^2*d^3
+ 16*(B*a + A*b)*c*d^4)*f^4)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2643 vs. 2(696) = 1392.
time = 0.96, size = 2643, normalized size = 3.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/1920*(1920*((c*d*f - d^2*e)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/sqrt(d*f) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)
*sqrt(d*x + c))*A*a*c*abs(d)/d^2 + 80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*
sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d
^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*
```

```
(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt
(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*C*a*c*
abs(d)/d^2 + 80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x
+ c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d
^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f
^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*B*b*c*abs(d)/d^2 + 10*(sqrt(
(d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 -
 (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*c*d^12*f
^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e
 - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3*(35*c^4
*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(
abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(
d*f)*d^2*f^3))*C*b*c*abs(d)/d^2 + 80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*s
qrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^
7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(
5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(
d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*B*a*abs
(d)/d + 10*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(
6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*
f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 -
15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x
 + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3
- 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f
+ d^2*e)))/(sqrt(d*f)*d^2*f^3))*C*a*abs(d)/d + 80*(sqrt((d*x + c)*d*f - c*d
*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d
^6*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^
7*f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sq
rt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*
f^2))*A*b*abs(d)/d + 10*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(
4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (
163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^
3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f
^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4
*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)
*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*B*b*abs(d)/d + (sqrt((d*x + c)
*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d*x + c)/d^4 - (41*c
*d^19*f^8 - d^20*f^7*e)/(d^23*f^8)) + (513*c^2*d^19*f^8 - 26*c*d^20*f^7*e -
 7*d^21*f^6*e^2)/(d^23*f^8)) - 5*(447*c^3*d^19*f^8 - 37*c^2*d^20*f^7*e - 19
*c*d^21*f^6*e^2 - 7*d^22*f^5*e^3)/(d^23*f^8))*(d*x + c) + 15*(193*c^4*d^19*
f^8 - 28*c^3*d^20*f^7*e - 18*c^2*d^21*f^6*e^2 - 12*c*d^22*f^5*e^3 - 7*d^23*
f^4*e^4)/(d^23*f^8))*sqrt(d*x + c) + 15*(63*c^5*f^5 - 35*c^4*d*f^4*e - 10*c
^3*d^2*f^3*e^2 - 6*c^2*d^3*f^2*e^3 - 5*c*d^4*f*e^4 - 7*d^5*e^5)*log(abs(-sq
rt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^
3*f^4))*C*b*abs(d)/d + 480*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2*
```

```
c - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^3
*e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e
)))/(sqrt(d*f)*f))*B*a*c*abs(d)/d^3 + 480*(sqrt((d*x + c)*d*f - c*d*f + d^2
*e)*(2*d*x + 2*c - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*
c*d^2*f*e - d^3*e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f
- c*d*f + d^2*e)))/(sqrt(d*f)*f))*A*b*c*abs(d)/d^3 + 480*(sqrt((d*x + c)*d*
f - c*d*f + d^2*e)*(2*d*x + 2*c - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3
*c^2*d*f^2 - 2*c*d^2*f*e - d^3*e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt
((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*f))*A*a*abs(d)/d^2)/d
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] \text{Hanged}
```

3.43 $\int \sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)\,dx$

Optimal. Leaf size=330

$$\frac{(de-cf)\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\,\sqrt{e+fx}}{64d^3f^3}+\frac{(C(5d^2e^2+6cdef\ ...}{}$$

```
[Out] -1/24*(-8*B*d*f+11*C*c*f+5*C*d*e)*(d*x+c)^(3/2)*(f*x+e)^(3/2)/d^2/f^2+1/4*C
*(d*x+c)^(5/2)*(f*x+e)^(3/2)/d^2/f-1/64*(-c*f+d*e)^2*(C*(5*c^2*f^2+6*c*d*e*
f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(7/2)+1/32*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)
+8*d*f*(2*A*d*f-B*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^3/f^2+1/64*(-c*
f+d*e)*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))*(d*x
+c)^(1/2)*(f*x+e)^(1/2)/d^3/f^3
```

Rubi [A]

time = 0.20, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {965, 81, 52, 65, 223, 212}

$$-\frac{(de-cf)^2\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)\left(8df(2Adf-B(cf+de))+C(5c^2f^2+6cdef+5d^2e^2)\right)}{64d^{7/2}f^{7/2}}+\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(8df(2Adf-B(cf+de))+C(5c^2f^2+6cdef+5d^2e^2)\right)}{32d^3f^2}+\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(de-cf)\left(8df(2Adf-B(cf+de))+C(5c^2f^2+6cdef+5d^2e^2)\right)}{64d^3f^3}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{24d^2f^2}+\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(64*d^3*f^3) + ((C*(5*d^2*e^2 + 6*
c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f)))*(c + d*x)^(3/2)*Sqr
t[e + f*x])/(32*d^3*f^2) - ((5*C*d*e + 11*c*C*f - 8*B*d*f)*(c + d*x)^(3/2)*
(e + f*x)^(3/2))/(24*d^2*f^2) + (C*(c + d*x)^(5/2)*(e + f*x)^(3/2))/(4*d^2*
f) - ((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f
 - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])]
)/(64*d^(7/2)*f^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx = \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} + \frac{\int \sqrt{c+dx}\,\sqrt{e+fx}\,\left(\frac{1}{2}(-5cCde\right.}{}$$

$$= -\frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+d}{}$$

$$= \frac{\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)(c+}{32d^3f^2}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+\right.}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+\right.}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+\right.}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+\right.}{64d^3f^3}$$

Mathematica [A]
time = 1.00, size = 283, normalized size = 0.86

$$\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(C\left(15c^3f^3-c^2df^2(7e+10fx)+cd^2f\left(-7e^2+4efx+8f^2x^2\right)+d^3\left(15e^3-10e^2fx+8ef^2x^2+48f^3x^3\right)\right)+8df\left(6Adf(cf+d(e+2fx))+B\left(-3c^2f^2+2cdf(e+fx)+d^2\left(-3e^2+2efx+8f^2x^2\right)\right)\right)\right)}{192d^3f^3} - \frac{(de-cf)^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{e+fx}}{\sqrt{f}\,\sqrt{c+dx}}\right)}{64d^{7/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*
d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*x
^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(-3*c^2*f^2 +
2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))))/(192*d^3*f^3) -
((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*
(d*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(64
*d^(7/2)*f^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(292) = 584$.
time = 0.10, size = 1207, normalized size = 3.66

method	result
default	$-\frac{\sqrt{dx+c}\,\sqrt{fx+e}\,\left(20C\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,c^2d\,f^3x+20C\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,d^3e^2fx-32\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c^2*d*f^3*x+20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e^2*f*x-32*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e*f^2*x+14*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*e^2*f-32*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*f^3*x-32*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*e*f^2+14*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c^2*d*e*f^2-96*C*d^3*f^3*x^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-128*B*d^3*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^3*d*e*f^3-6*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*e^2*f^2-96*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e*f^3-12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e^3*f-192*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*f^3*x+24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*e*f^3+24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e^2*f^2-96*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*f^3-96*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e*f^2+48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c^2*d*f^3+48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e^2*f+15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^4*f^4+15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^4-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^3*f-30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c^3*f^3-30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e^3+48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*f^4+48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^2*f^2-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^3*d*f^4-8*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*e*f^2*x-16*C*c*d^2*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-16*C*d^3*e*f^2*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2))/((d*x+c)*(f*x+e))^(1/2)/d^3/f^3/(d*f)^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 1.20, size = 845, normalized size = 2.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*C*d^4*e^4 + (5*C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4 - 4*(C*c^
3*d - 2*B*c^2*d^2 + 8*A*c*d^3)*f^3*e - 2*(C*c^2*d^2 - 4*B*c*d^3 - 8*A*d^4)*
f^2*e^2 - 4*(C*c*d^3 + 2*B*d^4)*f*e^3)*sqrt(d*f)*log(8*d^2*f^2*x^2 + 8*c*d*
f^2*x + c^2*f^2 + d^2*e^2 - 4*(2*d*f*x + c*f + d*e)*sqrt(d*f)*sqrt(d*x + c)
*sqrt(f*x + e) + 2*(4*d^2*f*x + 3*c*d*f)*e) + 4*(48*C*d^4*f^4*x^3 + 8*(C*c*
d^3 + 8*B*d^4)*f^4*x^2 + 15*C*d^4*f*e^3 - 2*(5*C*c^2*d^2 - 8*B*c*d^3 - 48*A
*d^4)*f^4*x + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 - (10*C*d^4*f^2*
x + (7*C*c*d^3 + 24*B*d^4)*f^2)*e^2 + (8*C*d^4*f^3*x^2 + 4*(C*c*d^3 + 4*B*d
^4)*f^3*x - (7*C*c^2*d^2 - 16*B*c*d^3 - 48*A*d^4)*f^3)*e)*sqrt(d*x + c)*sqr
t(f*x + e))/(d^4*f^4), 1/384*(3*(5*C*d^4*e^4 + (5*C*c^4 - 8*B*c^3*d + 16*A*
c^2*d^2)*f^4 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d^3)*f^3*e - 2*(C*c^2*d^2 -
 4*B*c*d^3 - 8*A*d^4)*f^2*e^2 - 4*(C*c*d^3 + 2*B*d^4)*f*e^3)*sqrt(-d*f)*arc
tan(1/2*(2*d*f*x + c*f + d*e)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f
^2*x^2 + c*d*f^2*x + (d^2*f*x + c*d*f)*e)) + 2*(48*C*d^4*f^4*x^3 + 8*(C*c*d
^3 + 8*B*d^4)*f^4*x^2 + 15*C*d^4*f*e^3 - 2*(5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*
d^4)*f^4*x + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 - (10*C*d^4*f^2*x
 + (7*C*c*d^3 + 24*B*d^4)*f^2)*e^2 + (8*C*d^4*f^3*x^2 + 4*(C*c*d^3 + 4*B*d^
4)*f^3*x - (7*C*c^2*d^2 - 16*B*c*d^3 - 48*A*d^4)*f^3)*e)*sqrt(d*x + c)*sqrt
(f*x + e))/(d^4*f^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(303) = 606$.
time = 1.00, size = 1103, normalized size = 3.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(192*((c*d*f - d^2*e)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x +
c)*d*f - c*d*f + d^2*e)))/sqrt(d*f) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)*s
qrt(d*x + c))*A*c*abs(d)/d^2 + 8*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(
d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^7*f^
4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(5*c^
3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x
+ c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*C*c*abs(d)/
d^2 + 8*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*
(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 -
 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c
*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f
 - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*B*abs(d)/d + (sqrt((d*x + c)*d*f - c
*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 -
 d^12*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4
*e^2)/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e
^2 - 5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f
^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sq
rt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*C*
abs(d)/d + 48*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2*c - (5*c*f^2
- d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^3*e^2)*log(abs
(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f
)*f))*B*c*abs(d)/d^3 + 48*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2*c
 - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^3*
e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)
))/(sqrt(d*f)*f))*A*abs(d)/d^2)/d
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] \text{Hanged}
```

3.44 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{a+bx}\,dx$

Optimal. Leaf size=450

$$\frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))\sqrt{c+dx}\,\sqrt{e+fx}}{8b^3d^2f^2} -$$

```
[Out] 1/3*C*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/8*(16*a^3*C*d^3*f^3-8*a^2*b*d^2*f
^2*(2*B*d*f+C*c*f+C*d*e)-2*a*b^2*d*f*(C*(-c*f+d*e)^2-4*d*f*(2*A*d*f+B*c*f+B
*d*e))-b^3*(C*(-c*f+d*e)^2*(c*f+d*e)-2*d*f*(B*(-c*f+d*e)^2-4*A*d*f*(c*f+d*e
))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^4/d^(5/2)/f^(5/
2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^
(1/2)/(f*x+e)^(1/2))*(-a*d+b*c)^(1/2)*(-a*f+b*e)^(1/2)/b^4-1/4*(2*a*C*d*f+b
*(-2*B*d*f+C*c*f+C*d*e))*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/d/f^2+1/8*(4*b*d*f
*(2*A*b*d*f-a*C*(c*f+d*e))+(4*a*d*f-b*c*f+b*d*e)*(2*a*C*d*f+b*(-2*B*d*f+C*c
*f+C*d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d^2/f^2
```

Rubi [A]
time = 0.91, antiderivative size = 453, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1629, 159, 163, 65, 223, 212, 95, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df(C(de-cf)^2-4df(2Adf+Bcf+Bde))-(b^3(C(de-cf)^2(cf+de)-2df(B(de-cf)^2-4Adf(cf+de)))))}{8b^4d^{5/2}f^{5/2}}+\frac{\sqrt{c+dx}\,\sqrt{e+fx}\left(\frac{(4adf-bcf+bde)(2aCdf+b(-2Bdf+cCf+Cde))}{bdf}-4aC(cf+de)+8Abdf\right)}{8b^2df}-\frac{2\sqrt{bc-ad}\,\sqrt{be-af}\,(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{be-af}}{\sqrt{e+fx}\,\sqrt{bc-ad}}\right)}{b^4}-\frac{\sqrt{c+dx}\,(e+fx)^{3/2}(2aCdf+b(-2Bdf+cCf+Cde))}{4b^2df^2}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

```
[Out] ((8*A*b*d*f - 4*a*C*(d*e + c*f) + ((b*d*e - b*c*f + 4*a*d*f)*(2*a*C*d*f + b
*(C*d*e + c*C*f - 2*B*d*f)))/(b*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(8*b^2*d
*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^(3
/2))/(4*b^2*d*f^2) + (C*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*d*f) - ((16*a
^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) - 2*a*b^2*d*f*(C*(
d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) - b^3*(C*(d*e - c*f)^2*(d*e
 + c*f) - 2*d*f*(B*(d*e - c*f)^2 - 4*A*d*f*(d*e + c*f))))*ArcTanh[(Sqrt[f]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*b^4*d^(5/2)*f^(5/2)) - (2*(A*b^
2 - a*(b*B - a*C))*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]
*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/b^4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{a+bx}\,dx = \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf} + \frac{\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(\frac{3}{2}b(2Abdf-aC(\ldots\right.}{a\ldots}}{3\ldots}$$

$$= -\frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}\,(e+fx)^{3/2}}{4b^2df^2} + \ldots$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+\ldots}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+\ldots}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+\ldots}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+\ldots}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+\ldots}{8b^3d^2f^2}$$

Mathematica [A]

time = 2.01, size = 404, normalized size = 0.90

$$\frac{\frac{b\sqrt{c+dx}\,\sqrt{e+fx}\,\left(24a^2Cd^2f^2-6abdf(cCf+4Bdf+Cd(e+2fx))+b^2\left(6df(Bcf+4Adf+Bd(e+2fx))+C\left(-3c^2f^2+2cdf(e+fx)+d^2\left(-3e^2+2efx+8f^2x^2\right)\right)\right)\right)}{d^2f^2}-48(Ab^2+a(-bB+aC))\sqrt{bc-ad}\,\sqrt{-be+af}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,\sqrt{e+fx}}{\sqrt{-be+af}\,\sqrt{c+dx}}\right)+\frac{3\left(-16a^3Cd^3f^3+8a^2bd^2f^2(Cde+cCf+2Bdf)+2ab^2df\left(C(de-cf)^2-4df(Bde+Bcf+2Adf)\right)+b^3\left(C(de-cf)^2(de+cf)+2df\left(-B(de-cf)^2+4Adf(de+cf)\right)\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{e+fx}}{\sqrt{f}\,\sqrt{c+dx}}\right)}{d^{5/2}f^{5/2}}}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

```
[Out] ((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 - 6*a*b*d*f*(c*C*f + 4*B*
d*f + C*d*(e + 2*f*x)) + b^2*(6*d*f*(B*c*f + 4*A*d*f + B*d*(e + 2*f*x)) + C
*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))))/(
d^2*f^2) - 48*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]
*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])]
 + (3*(-16*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) + 2*a*
b^2*d*f*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) + b^3*(C*(d*e -
 c*f)^2*(d*e + c*f) + 2*d*f*(-(B*(d*e - c*f)^2) + 4*A*d*f*(d*e + c*f))))*Ar
cTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(5/2)*f^(5/2)))/
(24*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3897 vs. $2(406) = 812$.
time = 0.10, size = 3898, normalized size = 8.66

method	result	size
default	Expression too large to display	3898

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-24*A*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)
/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*b^4*d^3*e*f^2+48*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(
f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2
*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b^2*d^3*f^3-48*B*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d^3*f^3+6*C*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*b^4*c^2*f^2+6*C*((d*x+c)*(
f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*b^4
*d^2*e^2+6*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+
2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c^2*d*f^3+6
*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d^3*e^2*f+48*A*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^3*f^3-24*A*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*d^2*f^3-48*B*ln((-2*a*d*f*x+b*c*f*x+b
*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^3*b*d^3*f^3+48*C*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
```

```
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d^3*f^3-48*A*((d*x+c)*(f*x+e))
^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*b^4*d^2*f^
2+12*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*d^2*e*f^2+24*
C*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*
f)^(1/2)*a*b^3*d^2*f^2*x-4*C*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*b^4*c*d*f^2*x-4*C*((d*x+c)*(f*x+e))^(1/
2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*b^4*d^2*e*f*x-
48*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b
*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2
)*a*b^3*c*d^2*e*f^2+48*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))
^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)
/(b*x+a))*(d*f)^(1/2)*a^2*b^2*c*d^2*e*f^2+12*C*((d*x+c)*(f*x+e))^(1/2)*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*a*b^3*c*d*f^2+12*C*((
d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(
1/2)*a*b^3*d^2*e*f-4*C*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*b^4*c*d*e*f-3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*b^4*c^3*f^3+48*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c
)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*
e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^4*d^3*f^3-3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*b^4*d^3*e^3-48*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x
+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*
d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a*b^3*c*d^2*f^3-48*A*ln((-2*a*d*f*x+b*c*f
*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a*b^3*d^3*e*f^2+48*A*ln
((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*b^4*c
*d^2*e*f^2-48*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))
*(d*f)^(1/2)*a^3*b*d^3*e*f^2+48*B*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*a*b^3*d^2*f^2-12*B*((d*x+c)*(f*x+e
))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*b^4*c*d*
f^2+3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*d^2*e^2*f-6*C*
((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^3*e^2*f+3*C*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c^2*d*e*f^2-16*C*b^4*d^2*f^2*x^2
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)-12*B*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*(d*f)^(1/2)*b^4*d^2*e*f-48*C*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*a^2*b^2*d^2*f^2+48*B*ln((-2*a*d
*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^
```

```
2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b^2*c*d^2
*f^3+48*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*...
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?` fo
r more
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x),x)
```

```
[Out] \text{Hanged}
```

3.45 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^2}\,dx$

Optimal. Leaf size=521

$$\frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf)))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^3df(be-af)}+\frac{(3a^2Cdf}{}$$

```
[Out] -(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b
*x+a)+1/4*(24*a^2*C*d^2*f^2-8*a*b*d*f*(2*B*d*f+C*c*f+C*d*e)-b^2*(C*(-c*f+d*
e)^2-4*d*f*(2*A*d*f+B*c*f+B*d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f
*x+e)^(1/2))/b^4/d^(3/2)/f^(3/2)+(6*a^3*C*d*f-b^3*(A*c*f+A*d*e+2*B*c*e)+a*b
^2*(2*A*d*f+3*B*c*f+3*B*d*e+4*C*c*e)-a^2*b*(4*B*d*f+5*C*(c*f+d*e)))*arctanh
((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(-a*d+b
*c)^(1/2)/(-a*f+b*e)^(1/2)+1/2*(3*a^2*C*d*f+b^2*(2*A*d*f+C*c*e)-a*b*(2*B*d*
f+C*c*f+C*d*e))*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)+1/4
*(12*a^2*C*d*f^2-a*b*f*(8*B*d*f+C*c*f+7*C*d*e)+b^2*(4*d*f*(A*f+B*e)-C*e*(-c
*f+d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d/f/(-a*f+b*e)
```

Rubi [A]

time = 1.14, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1627, 159, 163, 65, 223, 212, 95, 214}

$$\frac{\sqrt{c+dx}\,(e+fx)^{3/2}\,(3a^2Cdf-ab(2Bdf+cCf+Cde)+b^2(2Adf+cCe))}{2b^2f(bc-ad)(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)-(b^2(C(de-cf)^2-4df(2Adf+Bcf+Bde))))}{4b^4d^{3/2}f^{3/2}}+\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(12a^2Cdf^2-abf(8Bdf+cCf+7Cde)+b^2(4df(Af+Be)-Ce(de-cf)))}{4b^3df(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{be-af}}{\sqrt{e+fx}\,\sqrt{bc-ad}}\right)(6a^3Cdf-a^2b(4Bdf+5C(cf+de))+ab^2(2Adf+3Bcf+3Bde+4cCe)-b^3(Acf+Ade+2Bce))}{b^4\sqrt{bc-ad}\,\sqrt{be-af}}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\,(Ab^2-a(bB-aC))}{b(a+bx)(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

```
[Out] ((12*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 8*B*d*f) + b^2*(4*d*f*(B*e + A*
f) - C*e*(d*e - c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b^3*d*f*(b*e - a*f))
 + ((3*a^2*C*d*f + b^2*(c*C*e + 2*A*d*f) - a*b*(C*d*e + c*C*f + 2*B*d*f))*S
qrt[c + d*x]*(e + f*x)^(3/2))/(2*b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 -
 a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)
*(a + b*x)) + ((24*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 2*B*d*f) - b^
2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)))*ArcTanh[(Sqrt[f]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*b^4*d^(3/2)*f^(3/2)) + ((6*a^3*C*d
*f - b^3*(2*B*c*e + A*d*e + A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2
*A*d*f) - a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[
c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^4*Sqrt[b*c - a*d]*Sqrt[b*e -
 a*f])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^2}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)} - \int \frac{\sqrt{c+dx}\,\sqrt{}}{}$$

$$= \frac{(3a^2Cdf+b^2(cCe+2Adf)-ab(Cde+cCf+2Bdf))\,\sqrt{c+d}}{2b^2(bc-ad)f(be-af)}$$

$$= \frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-}{4b^3df(be-af)}$$

$$= \frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-}{4b^3df(be-af)}$$

$$= \frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-}{4b^3df(be-af)}$$

$$= \frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-}{4b^3df(be-af)}$$

$$= \frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-}{4b^3df(be-af)}$$

Mathematica [**A**]

time = 2.84, size = 358, normalized size = 0.69

$$\frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}\,(-12a^2Cdf+ab(cCf+8Bdf+Cd(e-6fx))+b^2(-4Adf+x(cCf+4Bdf+Cd(e+2fx))))}{df(a+bx)}+\frac{4(-6a^3Cdf+b^3(2Bce+Ade+Acf)-ab^2(4cCe+3Bde+3Bcf+2Adf)+a^2b(4Bdf+5C(de+cf)))\tan^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{e+fx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{\sqrt{bc-ad}\sqrt{-be+af}}-\frac{(-24a^2Cd^2f^2+8abdf(Cde+cCf+2Bdf)+b^2(C(de-cf)^2-4df(Bde+Bcf+2Adf)))\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{d^{3/2}f^{3/2}}}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

```
[Out] ((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(-12*a^2*C*d*f + a*b*(c*C*f + 8*B*d*f + C*d
*(e - 6*f*x)) + b^2*(-4*A*d*f + x*(c*C*f + 4*B*d*f + C*d*(e + 2*f*x)))))/(d
*f*(a + b*x)) + (4*(-6*a^3*C*d*f + b^3*(2*B*c*e + A*d*e + A*c*f) - a*b^2*(4
*c*C*e + 3*B*d*e + 3*B*c*f + 2*A*d*f) + a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*
ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])
/(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) - ((-24*a^2*C*d^2*f^2 + 8*a*b*d*f*(C*
d*e + c*C*f + 2*B*d*f) + b^2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*
d*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(3/2)*f
^(3/2)))/(4*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4679 vs. $2(479) = 958$.
time = 0.10, size = 4680, normalized size = 8.98

method	result	size
default	Expression too large to display	4680

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/8*(-4*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3
*d^2*e*f*(d*f)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e)
)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e
)/(b*x+a))*a^2*b^2*c*d*f^2*(d*f)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+
2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a
*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d^2*e*f*(d*f)^(1/2)+4*B*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*d*f^2
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+4*B*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^2*e*f*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-20*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*
((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c
*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*c*d*f^2*(d*f)^(1/2)-20*C*ln((-2*a*d*f*x+b*
c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d^2*e*f*(d*f)^(1/2)-8*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a
```

^2*b^2*c*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-8*C*ln(1/2*(2*
d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d
^2*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+24*C*ln((-2*a*d*f*x+b*
c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^4*d^2*f^2*(d*f)^(1/2)-12*C*a*b
^3*d*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/
2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*
x+a))*a*b^3*c*d*f^2*x*(d*f)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d
x+c)(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-
a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d^2*e*f*x*(d*f)^(1/2)-8*B*ln((-2*a*d*f*x+b*c*
f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*d*e*f*x*(d*f)^(1/2)-20*C*ln(
(-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*d*f^2*x*
(d*f)^(1/2)-20*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a)
)*a^2*b^2*d^2*e*f*x*(d*f)^(1/2)-8*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*d*f^2*x*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)-8*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^2*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*b^4*c*d*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^
(1/2)-8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3
*c*d*e*f*(d*f)^(1/2)+16*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e)
)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e
)/(b*x+a))*a^2*b^2*c*d*e*f*(d*f)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*d*e*f*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)+16*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)
*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e
+2*b*c*e)/(b*x+a))*a*b^3*c*d*e*f*x*(d*f)^(1/2)+4*C*b^4*d*f*x^2*((a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+8*B*b
^4*d*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)+2*C*b^4*c*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+2*C*b^4*d*e*x*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+16*B*a*b^3*d*f*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)-24*C*a^2*b^2*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+2*C*a*b^3*c*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+2*C*a*b^3*d*e*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+8*A*
ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d^2*f^2*x
*(d*f)^(1/2)-4*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*(

```
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a)
)*b^4*c*d*f^2*x*(d*f)^(1/2)-4*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(
f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2
*b*c*e)/(b*x+a))*b^4*d^2*e*f*x*(d*f)^(1/2)-16*B*ln((-2*a*d*f*x+b*c*f*x+b*d*
e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d^2*f^...
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the ’assume’ command before evaluation *may* h
elp (example of legal syntax is ’assume(2*a*d*f-b*c*f>0)’, see ‘assume?‘ fo
r more
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1585 vs. $2(500) = 1000$.
time = 2.27, size = 1585, normalized size = 3.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"giac")
```

```
[Out] 1/4*sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*C*abs(d)
/(b^2*d^3) - (C*b^7*c*d^3*f^2*abs(d) + 8*C*a*b^6*d^4*f^2*abs(d) - 4*B*b^7*d
^4*f^2*abs(d) - C*b^7*d^4*f*abs(d)*e)/(b^9*d^6*f^2)) - (5*sqrt(d*f)*C*a^2*b
*c*f*abs(d) - 3*sqrt(d*f)*B*a*b^2*c*f*abs(d) + sqrt(d*f)*A*b^3*c*f*abs(d) -
 6*sqrt(d*f)*C*a^3*d*f*abs(d) + 4*sqrt(d*f)*B*a^2*b*d*f*abs(d) - 2*sqrt(d*f
)*A*a*b^2*d*f*abs(d) - 4*sqrt(d*f)*C*a*b^2*c*abs(d)*e + 2*sqrt(d*f)*B*b^3*c
*abs(d)*e + 5*sqrt(d*f)*C*a^2*b*d*abs(d)*e - 3*sqrt(d*f)*B*a*b^2*d*abs(d)*e
 + sqrt(d*f)*A*b^3*d*abs(d)*e)*arctan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e -
 (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt
(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d))/(sqrt(a*b*c*d*f
^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*b^4*d) - 2*(sqrt(d*f)*C*a^2*b
*c^2*d*f^2*abs(d) - sqrt(d*f)*B*a*b^2*c^2*d*f^2*abs(d) + sqrt(d*f)*A*b^3*c^
2*d*f^2*abs(d) - 2*sqrt(d*f)*C*a^2*b*c*d^2*f*abs(d)*e + 2*sqrt(d*f)*B*a*b^2
*c*d^2*f*abs(d)*e - 2*sqrt(d*f)*A*b^3*c*d^2*f*abs(d)*e - sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*c*f*abs(d
) + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^2*B*a*b^2*c*f*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^2*A*b^3*c*f*abs(d) + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*d*f*abs(d) - 2*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^
2*b*d*f*abs(d) + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*A*a*b^2*d*f*abs(d) + sqrt(d*f)*C*a^2*b*d^3*abs(d)*e^2 -
 sqrt(d*f)*B*a*b^2*d^3*abs(d)*e^2 + sqrt(d*f)*A*b^3*d^3*abs(d)*e^2 - sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2
*b*d*abs(d)*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^2*B*a*b^2*d*abs(d)*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*d*abs(d)*e)/((b*c^2*d^2*f^2 -
2*b*c*d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d
^2*e))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*b)*b^4) + 1/8*(sqrt(d*f)*C*b^2*c^2*f^2*abs(d) +
 8*sqrt(d*f)*C*a*b*c*d*f^2*abs(d) - 4*sqrt(d*f)*B*b^2*c*d*f^2*abs(d) - 24*s
qrt(d*f)*C*a^2*d^2*f^2*abs(d) + 16*sqrt(d*f)*B*a*b*d^2*f^2*abs(d) - 8*sqrt(
d*f)*A*b^2*d^2*f^2*abs(d) - 2*sqrt(d*f)*C*b^2*c*d*f*abs(d)*e + 8*sqrt(d*f)*
C*a*b*d^2*f*abs(d)*e - 4*sqrt(d*f)*B*b^2*d^2*f*abs(d)*e + sqrt(d*f)*C*b^2*d
^2*abs(d)*e^2)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^2)/(b^4*d^3*f^2)
```

Mupad [**F(-1)**]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^2,x)
[Out] \text{Hanged}
```

3.46 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^3}\,dx$

Optimal. Leaf size=658

$$-\frac{(12a^3Cdf^2-a^2bf(17Cde+11cCf+4Bdf)+ab^2(Bf(5de+3cf)+4Ce(de+4cf))-b^3(Adef+c(4Ce^2}{4b^3(bc-ad)(be-af)^2}$$

```
[Out] -1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^2-1/4*(24*a^4*C*d^2*f^2-3*a*b^3*(B*d^2*e^2+c^2*f*(B*f+8*C*e)+2*c*
d*e*(3*B*f+4*C*e))-8*a^3*b*d*f*(B*d*f+5*C*(c*f+d*e))-b^4*(A*d^2*e^2-2*c*d*e
*(A*f+2*B*e)-c^2*(-A*f^2+4*B*e*f+8*C*e^2))+3*a^2*b^2*(4*B*d*f*(c*f+d*e)+C*(
5*c^2*f^2+22*c*d*e*f+5*d^2*e^2)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-
a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(3/2)-(6*a*C*
d*f-b*(2*B*d*f+C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^
(1/2))/b^4/d^(1/2)/f^(1/2)+1/4*(6*a^3*C*d*f-b^3*(-A*c*f-A*d*e+4*B*c*e)+a*b^
2*(-2*A*d*f+3*B*c*f+3*B*d*e+8*C*c*e)-a^2*b*(2*B*d*f+7*C*(c*f+d*e)))*(f*x+e)
^(3/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)-1/4*(12*a^3*C*d*f^
2-a^2*b*f*(4*B*d*f+11*C*c*f+17*C*d*e)+a*b^2*(B*f*(3*c*f+5*d*e)+4*C*e*(4*c*f
+d*e))-b^3*(A*d*e*f+c*(-A*f^2+4*B*e*f+4*C*e^2)))*(d*x+c)^(1/2)*(f*x+e)^(1/2
)/b^3/(-a*d+b*c)/(-a*f+b*e)^2
```

Rubi [A]
time = 1.75, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1627, 154, 159, 163, 65, 223, 212, 95, 214}

$$-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(12a^3Cdf^2-a^2bf(4Bdf+11cCf+17Cde)+ab^2(Bf(3cf+5de)+4Ce(4cf+de))-b^3(cf(4Be-Af)+Adef+4cCe^2))}{4b^3(bc-ad)(be-af)^2}+\frac{\sqrt{c+dx}\,(e+fx)^{3/2}\,(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce))}{4b^2(a+bx)(bc-ad)(be-af)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{be-af}}{\sqrt{e+fx}\,\sqrt{bc-ad}}\right)(24a^4Cd^2f^2-8a^3bdf(Bdf+5C(cf+de))+3a^2b^2(4Bdf(cf+de)+C(5c^2f^2+22cdef+5d^2e^2))-3ab^3(c^2f(Bf+8Ce)+2cde(3Bf+4Ce)+Bd^2e^2)-b^4(-(c^2(-Af^2+4Bef+8Ce^2))-2cde(Af+2Be)+Ad^2e^2))}{4b^4(bc-ad)^{3/2}(be-af)^{3/2}}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\,(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)(6aCdf-b(2Bdf+cCf+Cde))}{b^4\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

```
[Out] -1/4*((12*a^3*C*d*f^2 - a^2*b*f*(17*C*d*e + 11*c*C*f + 4*B*d*f) - b^3*(4*c*
C*e^2 + A*d*e*f + c*f*(4*B*e - A*f)) + a*b^2*(B*f*(5*d*e + 3*c*f) + 4*C*e*(
d*e + 4*c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b^3*(b*c - a*d)*(b*e - a*f)^2)
 + ((6*a^3*C*d*f - b^3*(4*B*c*e - A*d*e - A*c*f) + a*b^2*(8*c*C*e + 3*B*d*e
 + 3*B*c*f - 2*A*d*f) - a^2*b*(2*B*d*f + 7*C*(d*e + c*f)))*Sqrt[c + d*x]*(e
 + f*x)^(3/2))/(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b
*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(2*b*(b*c - a*d)*(b*e - a*f)*(a
 + b*x)^2) - ((6*a*C*d*f - b*(C*d*e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[f]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^4*Sqrt[d]*Sqrt[f]) - ((24*a^4*C*d
^2*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) + 2*c*d*e*(4*C*e + 3*B*f)
) - 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) - b^4*(A*d^2*e^2 - 2*c*d*e*(2*B*e
```

```
 + A*f) - c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a^2*b^2*(4*B*d*f*(d*e + c*f)
 + C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^4*(b*c - a*d)^(3/2)*(b*e -
a*f)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^3}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2} - \int \frac{\sqrt{c+dx}\,\sqrt{\ }}{}$$

$$= \frac{(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3B}{4b^2(bc-ad)(be}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + }{}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + }{}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + }{}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + }{}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + }{}$$

Mathematica [A]
time = 7.46, size = 536, normalized size = 0.81

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

```
[Out] ((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(12*a^4*C*d*f + 4*b^4*c*e*x*(-B + C*x) + A*
b^3*(a*c*f + a*d*(e + 2*f*x) - b*(2*c*e + d*e*x + c*f*x)) + a*b^3*(-4*C*x*(
-4*c*e + d*e*x + c*f*x) + B*(-2*c*e + 5*d*e*x + 5*c*f*x)) + a^2*b^2*(3*B*d*
(e - 2*f*x) + C*d*x*(-17*e + 4*f*x) + c*(10*C*e + 3*B*f - 17*C*f*x)) - a^3*
b*(4*B*d*f + C*(11*d*e + 11*c*f - 18*d*f*x))))/((b*c - a*d)*(b*e - a*f)*(a
+ b*x)^2) - ((24*a^4*C*d^2*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) +
```

```
 2*c*d*e*(4*C*e + 3*B*f)) - 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) + b^4*(-(
A*d^2*e^2) + 2*c*d*e*(2*B*e + A*f) + c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a
^2*b^2*(4*B*d*f*(d*e + c*f) + C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcT
an[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/((b
*c - a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) + (4*(-6*a*C*d*f + b*(C*d*e + c*C*f +
 2*B*d*f))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(Sqrt[
d]*Sqrt[f]))/(4*b^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11203 vs. $2(614) = 1228$.
time = 0.11, size = 11204, normalized size = 17.03

method	result	size
default	Expression too large to display	11204

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8347 vs. 2(634) = 1268.
time = 7.15, size = 8347, normalized size = 12.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"giac")
```

```
[Out] 1/4*(15*sqrt(d*f)*C*a^2*b^2*c^2*f^2*abs(d) - 3*sqrt(d*f)*B*a*b^3*c^2*f^2*ab
s(d) - sqrt(d*f)*A*b^4*c^2*f^2*abs(d) - 40*sqrt(d*f)*C*a^3*b*c*d*f^2*abs(d)
 + 12*sqrt(d*f)*B*a^2*b^2*c*d*f^2*abs(d) + 24*sqrt(d*f)*C*a^4*d^2*f^2*abs(d
) - 8*sqrt(d*f)*B*a^3*b*d^2*f^2*abs(d) - 24*sqrt(d*f)*C*a*b^3*c^2*f*abs(d)*
e + 4*sqrt(d*f)*B*b^4*c^2*f*abs(d)*e + 66*sqrt(d*f)*C*a^2*b^2*c*d*f*abs(d)*
e - 18*sqrt(d*f)*B*a*b^3*c*d*f*abs(d)*e + 2*sqrt(d*f)*A*b^4*c*d*f*abs(d)*e
- 40*sqrt(d*f)*C*a^3*b*d^2*f*abs(d)*e + 12*sqrt(d*f)*B*a^2*b^2*d^2*f*abs(d)
*e + 8*sqrt(d*f)*C*b^4*c^2*abs(d)*e^2 - 24*sqrt(d*f)*C*a*b^3*c*d*abs(d)*e^2
 + 4*sqrt(d*f)*B*b^4*c*d*abs(d)*e^2 + 15*sqrt(d*f)*C*a^2*b^2*d^2*abs(d)*e^2
 - 3*sqrt(d*f)*B*a*b^3*d^2*abs(d)*e^2 - sqrt(d*f)*A*b^4*d^2*abs(d)*e^2)*arc
tan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c
*d*f*e + a*b*d^2*f*e)*d))/((a*b^5*c*f - a^2*b^4*d*f - b^6*c*e + a*b^5*d*e)*
sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d) + 1/2*(9*sqr
t(d*f)*C*a^2*b^3*c^5*d^3*f^5*abs(d) - 5*sqrt(d*f)*B*a*b^4*c^5*d^3*f^5*abs(d
) + sqrt(d*f)*A*b^5*c^5*d^3*f^5*abs(d) - 10*sqrt(d*f)*C*a^3*b^2*c^4*d^4*f^5
*abs(d) + 6*sqrt(d*f)*B*a^2*b^3*c^4*d^4*f^5*abs(d) - 2*sqrt(d*f)*A*a*b^4*c^
4*d^4*f^5*abs(d) - 8*sqrt(d*f)*C*a*b^4*c^5*d^3*f^4*abs(d)*e + 4*sqrt(d*f)*B
*b^5*c^5*d^3*f^4*abs(d)*e - 27*sqrt(d*f)*C*a^2*b^3*c^4*d^4*f^4*abs(d)*e + 1
5*sqrt(d*f)*B*a*b^4*c^4*d^4*f^4*abs(d)*e - 3*sqrt(d*f)*A*b^5*c^4*d^4*f^4*ab
s(d)*e + 40*sqrt(d*f)*C*a^3*b^2*c^3*d^5*f^4*abs(d)*e - 24*sqrt(d*f)*B*a^2*b
^3*c^3*d^5*f^4*abs(d)*e + 8*sqrt(d*f)*A*a*b^4*c^3*d^5*f^4*abs(d)*e - 27*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*
a^2*b^3*c^4*d^2*f^4*abs(d) + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^4*d^2*f^4*abs(d) - 3*sqrt(d*f)*(
```

```
sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^4*
d^2*f^4*abs(d) + 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*C*a^3*b^2*c^3*d^3*f^4*abs(d) - 44*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^3*d^3*f
^4*abs(d) + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d
*f + d^2*e))^2*A*a*b^4*c^3*d^3*f^4*abs(d) - 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c^2*d^4*f^4*abs(d)
+ 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^2*B*a^3*b^2*c^2*d^4*f^4*abs(d) - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*c^2*d^4*f^4*abs(d) + 32*s
qrt(d*f)*C*a*b^4*c^4*d^4*f^3*abs(d)*e^2 - 16*sqrt(d*f)*B*b^5*c^4*d^4*f^3*ab
s(d)*e^2 + 18*sqrt(d*f)*C*a^2*b^3*c^3*d^5*f^3*abs(d)*e^2 - 10*sqrt(d*f)*B*a
*b^4*c^3*d^5*f^3*abs(d)*e^2 + 2*sqrt(d*f)*A*b^5*c^3*d^5*f^3*abs(d)*e^2 - 60
*sqrt(d*f)*C*a^3*b^2*c^2*d^6*f^3*abs(d)*e^2 + 36*sqrt(d*f)*B*a^2*b^3*c^2*d^
6*f^3*abs(d)*e^2 - 12*sqrt(d*f)*A*a*b^4*c^2*d^6*f^3*abs(d)*e^2 + 24*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^
4*c^4*d^2*f^3*abs(d)*e - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
+ c)*d*f - c*d*f + d^2*e))^2*B*b^5*c^4*d^2*f^3*abs(d)*e - 44*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^3
*d^3*f^3*abs(d)*e + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*
d*f - c*d*f + d^2*e))^2*B*a*b^4*c^3*d^3*f^3*abs(d)*e + 4*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^3*d^3*f^3
*abs(d)*e - 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^2*C*a^3*b^2*c^2*d^4*f^3*abs(d)*e + 44*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^2*d^4*f^3*
abs(d)*e - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*
f + d^2*e))^2*A*a*b^4*c^2*d^4*f^3*abs(d)*e + 112*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c*d^5*f^3*abs(d)*
e - 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^
2*e))^2*B*a^3*b^2*c*d^5*f^3*abs(d)*e + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*c*d^5*f^3*abs(d)*e + 2
7*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^4*C*a^2*b^3*c^3*d*f^3*abs(d) - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*b^4*c^3*d*f^3*abs(d) + 3*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*b^5*c^3
*d*f^3*abs(d) - 102*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^4*C*a^3*b^2*c^2*d^2*f^3*abs(d) + 58*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a^2*b^3*c^2*d^2*f
^3*abs(d) - 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^4*A*a*b^4*c^2*d^2*f^3*abs(d) + 152*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^4*b*c*d^3*f^3*abs(d)
- 88*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(...
```

Mupad [**F(-1)**]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^3,x)
[Out] \text{Hanged}
```

3.47 $\int \frac{(a+bx)^2\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1032

$$-\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e^2f+9c^2def}{}$$

```
[Out] 1/128*(-c*f+d*e)*(16*a^2*d^2*f^2*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2
+2*c*d*e*f+5*d^2*e^2))+4*a*b*d*f*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f
+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)))-b
^2*(C*(7*c^4*f^4+12*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+63*d^4*e^
4)+2*d*f*(8*A*d*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)-B*(5*c^3*f^3+9*c^2*d*e*f^2+
15*c*d^2*e^2*f+35*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)
^(1/2))/d^(9/2)/f^(11/2)-1/40*(4*a*C*d*f+b*(-10*B*d*f+7*C*c*f+9*C*d*e))*(b*
x+a)^2*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d^2/f^2+1/5*C*(b*x+a)^3*(d*x+c)^(3/2)*
(f*x+e)^(1/2)/b/d/f-1/960*(d*x+c)^(3/2)*(96*a^3*C*d^3*f^3+8*a^2*b*d^2*f^2*(
-30*B*d*f+9*C*c*f+23*C*d*e)+20*a*b^2*d*f*(8*d*f*(-6*A*d*f+3*B*c*f+5*B*d*e)-
C*(15*c^2*f^2+22*c*d*e*f+35*d^2*e^2))+b^3*(C*(105*c^3*f^3+145*c^2*d*e*f^2+2
03*c*d^2*e^2*f+315*d^3*e^3)+10*d*f*(8*A*d*f*(3*c*f+5*d*e)-B*(15*c^2*f^2+22*
c*d*e*f+35*d^2*e^2)))+4*b*d*f*(8*b*d*f*(-10*A*b*d*f+C*a*c*f+3*C*a*d*e+6*C*b
*c*e)-(-4*a*d*f+5*b*c*f+7*b*d*e)*(4*a*C*d*f+b*(-10*B*d*f+7*C*c*f+9*C*d*e)))
*x)*(f*x+e)^(1/2)/b/d^4/f^4-1/128*(16*a^2*d^2*f^2*(2*d*f*(-4*A*d*f+B*c*f+3*
B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))+4*a*b*d*f*(C*(5*c^3*f^3+9*c^2*d*e*f
^2+15*c*d^2*e^2*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e
*f+5*d^2*e^2)))-b^2*(C*(7*c^4*f^4+12*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+28*c*d^
3*e^3*f+63*d^4*e^4)+2*d*f*(8*A*d*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)-B*(5*c^3*f
^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/
d^4/f^5
```

Rubi [A]
time = 1.13, antiderivative size = 1032, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1629, 158, 152, 52, 65, 223, 212}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] -1/128*((16*a^2*d^2*f^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 +
 2*c*d*e*f + c^2*f^2)) + 4*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*
d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*
```

```
e*f + c^2*f^2))) - b^2*(C*(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2
 + 12*c^3*d*e*f^3 + 7*c^4*f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^
2*f^2) - B*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*Sqr
t[c + d*x]*Sqrt[e + f*x])/(d^4*f^5) - ((4*a*C*d*f + b*(9*C*d*e + 7*c*C*f -
10*B*d*f))*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x])/(40*b*d^2*f^2) + (C*(
a + b*x)^3*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - ((c + d*x)^(3/2)*Sqrt
[e + f*x]*(96*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(23*C*d*e + 9*c*C*f - 30*B*d*
f) + 20*a*b^2*d*f*(8*d*f*(5*B*d*e + 3*B*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22
*c*d*e*f + 15*c^2*f^2)) + b^3*(C*(315*d^3*e^3 + 203*c*d^2*e^2*f + 145*c^2*d
*e*f^2 + 105*c^3*f^3) + 10*d*f*(8*A*d*f*(5*d*e + 3*c*f) - B*(35*d^2*e^2 + 2
2*c*d*e*f + 15*c^2*f^2))) + 4*b*d*f*(8*b*d*f*(6*b*c*C*e + 3*a*C*d*e + a*c*C
*f - 10*A*b*d*f) - (7*b*d*e + 5*b*c*f - 4*a*d*f)*(4*a*C*d*f + b*(9*C*d*e +
7*c*C*f - 10*B*d*f)))*x))/(960*b*d^4*f^4) + ((d*e - c*f)*(16*a^2*d^2*f^2*(2
*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) + 4
*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d
*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))) - b^2*(C*
(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*c^3*d*e*f^3 + 7*c^4*
f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) - B*(35*d^3*e^3 + 1
5*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(11/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
```

```
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{(a+bx)^2\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{\int \frac{(a+bx)^2\sqrt{c+dx}\,(-\frac{1}{2}b(6bcC\cdots}{}}{\cdots}$$

$$= -\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{\cdots}}{40bd^2f^2}$$

$$= -\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{\cdots}}{40bd^2f^2}$$

$$= -\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+\cdots}{}$$

$$= -\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+\cdots}{}$$

$$= -\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+\cdots}{}$$

$$= -\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+\cdots}{}$$

Mathematica [A]
time = 7.44, size = 888, normalized size = 0.86

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (d*Sqrt[c + d*x]*Sqrt[e + f*x]*(80*a^2*d^2*f^2*(6*d*f*(4*A*d*f + B*(-3*d*e
+ c*f + 2*d*f*x)) + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10
*e*f*x + 8*f^2*x^2))) + 20*a*b*d*f*(C*(15*c^3*f^3 + c^2*d*f^2*(17*e - 10*f*
x) + c*d^2*f*(25*e^2 - 12*e*f*x + 8*f^2*x^2) + d^3*(-105*e^3 + 70*e^2*f*x -
 56*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(-3*d*e + c*f + 2*d*f*x) + B*
(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)))
 + b^2*(C*(-105*c^4*f^4 + 10*c^3*d*f^3*(-11*e + 7*f*x) - 2*c^2*d^2*f^2*(68*
```

```
e^2 - 39*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(-105*e^3 + 49*e^2*f*x - 32*e*f^2*
x^2 + 24*f^3*x^3) + d^4*(945*e^4 - 630*e^3*f*x + 504*e^2*f^2*x^2 - 432*e*f^
3*x^3 + 384*f^4*x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x)
+ d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 + c^2*d*f^2*(17*e -
10*f*x) + c*d^2*f*(25*e^2 - 12*e*f*x + 8*f^2*x^2) + d^3*(-105*e^3 + 70*e^2*
f*x - 56*e*f^2*x^2 + 48*f^3*x^3))))) + 15*Sqrt[d/f]*(d*e - c*f)*(16*a^2*d^2
*f^2*(2*d*f*(-3*B*d*e - B*c*f + 4*A*d*f) + C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f
^2)) - 4*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^
3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))) +
 b^2*(C*(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*c^3*d*e*f^3
+ 7*c^4*f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) - B*(35*d^3
*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*Log[Sqrt[c + d*x] - S
qrt[d/f]*Sqrt[e + f*x]])/(1920*d^5*f^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3957 vs. $2(994) = 1988$.
time = 0.11, size = 3958, normalized size = 3.84

method	result	size
default	Expression too large to display	3958

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(320*C*a*b*c*d^3*f^4*x^2*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)-2240*C*a*b*d^4*e*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)-128*C*b^2*c*d^3*e*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-945*C*
ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*b^2*d^5*e^5+2880*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*a*b*d^5*e^2*f^3+720*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^2*f^3-3200*B*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*d^4*e*f^3*x+640*B*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)*a*b*c*d^3*f^4*x+1000*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b
*c*d^3*e^2*f^2+680*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c^2*d^2*e*f^3-
1280*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^3*e*f^3+196*C*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^3*e^2*f^2*x-400*C*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)*a*b*c^2*d^2*f^4*x+2800*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a
*b*d^4*e^2*f^2*x+156*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d^2*e*f^
3*x-240*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^3*e*f^3*x+240*A*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*
c^3*d^2*f^5+240*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f
+d*e)/(d*f)^(1/2))*a^2*c^3*d^2*f^5-480*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^2*d^3*f^5-150*B*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^4*d*
```

f^5-210*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^4*f^4+1890*C*(d*f)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^4*e^4-1200*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e^3*f^2+3840*A*(d*f)^
(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^4*f^4+1050*B*ln(1/2*(2*d*f*x+2*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^5*e^4*f+1440*B*ln(1
/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2
*d^5*e^2*f^3+1920*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*f^5-1920*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e*f^4-1200*A*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^5*e^
3*f^2-480*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^3*e*f^3*x+340*B*((d
x+c)(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d^2*e*f^3+500*B*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)*b^2*c*d^3*e^2*f^2-360*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*e^2*f^3-640*A*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^3*e*f^3-200*B*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)*b^2*c^2*d^2*f^4*x+1400*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b
^2*d^4*e^2*f^2*x+480*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*e*f^4+1920*C*a*b*d^4*f^4*x^3*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)+96*C*b^2*c*d^3*f^4*x^3*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)-864*C*b^2*d^4*e*f^3*x^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+2560*
B*a*b*d^4*f^4*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+160*B*b^2*c*d^3*f^4*x
^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-1120*B*b^2*d^4*e*f^3*x^2*((d*x+c)*(f
x+e))^(1/2)(d*f)^(1/2)-112*C*b^2*c^2*d^2*f^4*x^2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+1008*C*b^2*d^4*e^2*f^2*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-
960*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*a*b*c^2*d^3*f^5+240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^3*e*f^4+1920*B*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)*a^2*d^4*f^4*x+480*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^3*d^2*f^5-120*B*ln(1/2*(2*d*f*x+
2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*e*f
^4-1200*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*a*b*c*d^4*e^3*f^2-960*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*
c^2*d^2*f^4-480*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d^2*f^4-240*C
ln(1/2(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2)
)*a*b*c^3*d^2*e*f^4-4200*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^4*e^3*
f+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f
)^(1/2))*b^2*c^5*f^5+1440*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^4*e^2*f^3-2880*B*(d*f)^(1/2)*((d*x+c)*
(f*x+e))^(1/2)*a^2*d^4*e*f^3-2100*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2
*d^4*e^3*f-272*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d^2*e^2*f^2-42
0*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^3*e^3*f+3840*A*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)*a*b*d^4*f^4*x+2400*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)*b^2*d^4*e^2*f^2-5760*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^4*e*
f^3+1920*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^3*f^4+600*C*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c^3*d*f^4-640*C*((d*x+c)*(f*x+e))^(1/2)*(d*

```
f)^(1/2)*a^2*c*d^3*e*f^3-220*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^3*
d*e*f^3+4800*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 2.65, size = 2173, normalized size = 2.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/7680*(15*(63*C*b^2*d^5*e^5 - (7*C*b^2*c^5 + 128*A*a^2*c*d^4 - 10*(2*C*a*
b + B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^2 - 32*(B*a^2 + 2*A*a
*b)*c^2*d^3)*f^5 - (5*C*b^2*c^4*d - 128*A*a^2*d^5 - 8*(2*C*a*b + B*b^2)*c^3
*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 64*(B*a^2 + 2*A*a*b)*c*d^4)*f
^4*e - 6*(C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3 + 8*(C*a^2 + 2*B*a*b
+ A*b^2)*c*d^4 + 16*(B*a^2 + 2*A*a*b)*d^5)*f^3*e^2 - 10*(C*b^2*c^2*d^3 - 4*
(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*f^2*e^3 - 35*(C*
b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5)*f*e^4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + 8
*c*d*f^2*x + c^2*f^2 + d^2*e^2 - 4*(2*d*f*x + c*f + d*e)*sqrt(d*f)*sqrt(d*x
 + c)*sqrt(f*x + e) + 2*(4*d^2*f*x + 3*c*d*f)*e) + 4*(384*C*b^2*d^5*f^5*x^4
 + 945*C*b^2*d^5*f*e^4 + 48*(C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2)*d^5)*f^5*x^
3 - 8*(7*C*b^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C*a^2 + 2*B*a*b +
 A*b^2)*d^5)*f^5*x^2 + 10*(7*C*b^2*c^3*d^2 - 10*(2*C*a*b + B*b^2)*c^2*d^3 +
 16*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 96*(B*a^2 + 2*A*a*b)*d^5)*f^5*x - 15*
(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 +
 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a*b)*c*d^4)*f^5 - 210*(3*C*b^2*
d^5*f^2*x + (C*b^2*c*d^4 + 5*(2*C*a*b + B*b^2)*d^5)*f^2)*e^3 + 2*(252*C*b^2
*d^5*f^3*x^2 + 7*(7*C*b^2*c*d^4 + 50*(2*C*a*b + B*b^2)*d^5)*f^3*x - (68*C*b
^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*d^4 - 600*(C*a^2 + 2*B*a*b + A*b^2)*d^
5)*f^3)*e^2 - 2*(216*C*b^2*d^5*f^4*x^3 + 8*(4*C*b^2*c*d^4 + 35*(2*C*a*b + B
```

```
*b^2)*d^5)*f^4*x^2 - (39*C*b^2*c^2*d^3 - 60*(2*C*a*b + B*b^2)*c*d^4 - 400*(
C*a^2 + 2*B*a*b + A*b^2)*d^5)*f^4*x + 5*(11*C*b^2*c^3*d^2 - 17*(2*C*a*b + B
*b^2)*c^2*d^3 + 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a^2 + 2*A*a*b)*
d^5)*f^4)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^6), 1/3840*(15*(63*C*b^2*d
^5*e^5 - (7*C*b^2*c^5 + 128*A*a^2*c*d^4 - 10*(2*C*a*b + B*b^2)*c^4*d + 16*(
C*a^2 + 2*B*a*b + A*b^2)*c^3*d^2 - 32*(B*a^2 + 2*A*a*b)*c^2*d^3)*f^5 - (5*C
*b^2*c^4*d - 128*A*a^2*d^5 - 8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*
a*b + A*b^2)*c^2*d^3 - 64*(B*a^2 + 2*A*a*b)*c*d^4)*f^4*e - 6*(C*b^2*c^3*d^2
 - 2*(2*C*a*b + B*b^2)*c^2*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 16*(B*
a^2 + 2*A*a*b)*d^5)*f^3*e^2 - 10*(C*b^2*c^2*d^3 - 4*(2*C*a*b + B*b^2)*c*d^4
 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*f^2*e^3 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b
+ B*b^2)*d^5)*f*e^4)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + c*f + d*e)*sqrt(-d*f)
*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*f^2*x + (d^2*f*x + c*d*f)*e
)) + 2*(384*C*b^2*d^5*f^5*x^4 + 945*C*b^2*d^5*f*e^4 + 48*(C*b^2*c*d^4 + 10*
(2*C*a*b + B*b^2)*d^5)*f^5*x^3 - 8*(7*C*b^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*
c*d^4 - 80*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*f^5*x^2 + 10*(7*C*b^2*c^3*d^2 - 1
0*(2*C*a*b + B*b^2)*c^2*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 96*(B*a^
2 + 2*A*a*b)*d^5)*f^5*x - 15*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b +
 B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a*
b)*c*d^4)*f^5 - 210*(3*C*b^2*d^5*f^2*x + (C*b^2*c*d^4 + 5*(2*C*a*b + B*b^2)
*d^5)*f^2)*e^3 + 2*(252*C*b^2*d^5*f^3*x^2 + 7*(7*C*b^2*c*d^4 + 50*(2*C*a*b
+ B*b^2)*d^5)*f^3*x - (68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*d^4 - 600
*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*f^3)*e^2 - 2*(216*C*b^2*d^5*f^4*x^3 + 8*(4*
C*b^2*c*d^4 + 35*(2*C*a*b + B*b^2)*d^5)*f^4*x^2 - (39*C*b^2*c^2*d^3 - 60*(2
*C*a*b + B*b^2)*c*d^4 - 400*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*f^4*x + 5*(11*C*
b^2*c^3*d^2 - 17*(2*C*a*b + B*b^2)*c^2*d^3 + 32*(C*a^2 + 2*B*a*b + A*b^2)*c
*d^4 + 144*(B*a^2 + 2*A*a*b)*d^5)*f^4)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5
*f^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.17, size = 1505, normalized size = 1.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/1920*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8
*(d*x + c)*C*b^2/(d^5*f) - (31*C*b^2*c*d^20*f^8 - 20*C*a*b*d^21*f^8 - 10*B*
b^2*d^21*f^8 + 9*C*b^2*d^21*f^7*e)/(d^25*f^9)) + (263*C*b^2*c^2*d^20*f^8 -
340*C*a*b*c*d^21*f^8 - 170*B*b^2*c*d^21*f^8 + 80*C*a^2*d^22*f^8 + 160*B*a*b
*d^22*f^8 + 80*A*b^2*d^22*f^8 + 154*C*b^2*c*d^21*f^7*e - 140*C*a*b*d^22*f^7
*e - 70*B*b^2*d^22*f^7*e + 63*C*b^2*d^22*f^6*e^2)/(d^25*f^9)) - 5*(121*C*b^
2*c^3*d^20*f^8 - 236*C*a*b*c^2*d^21*f^8 - 118*B*b^2*c^2*d^21*f^8 + 112*C*a^
2*c*d^22*f^8 + 224*B*a*b*c*d^22*f^8 + 112*A*b^2*c*d^22*f^8 - 96*B*a^2*d^23*
f^8 - 192*A*a*b*d^23*f^8 + 109*C*b^2*c^2*d^21*f^7*e - 200*C*a*b*c*d^22*f^7*
e - 100*B*b^2*c*d^22*f^7*e + 80*C*a^2*d^23*f^7*e + 160*B*a*b*d^23*f^7*e + 8
0*A*b^2*d^23*f^7*e + 91*C*b^2*c*d^22*f^6*e^2 - 140*C*a*b*d^23*f^6*e^2 - 70*
B*b^2*d^23*f^6*e^2 + 63*C*b^2*d^23*f^5*e^3)/(d^25*f^9))*(d*x + c) + 15*(7*C
*b^2*c^4*d^20*f^8 - 20*C*a*b*c^3*d^21*f^8 - 10*B*b^2*c^3*d^21*f^8 + 16*C*a^
2*c^2*d^22*f^8 + 32*B*a*b*c^2*d^22*f^8 + 16*A*b^2*c^2*d^22*f^8 - 32*B*a^2*c
*d^23*f^8 - 64*A*a*b*c*d^23*f^8 + 128*A*a^2*d^24*f^8 + 12*C*b^2*c^3*d^21*f^
7*e - 36*C*a*b*c^2*d^22*f^7*e - 18*B*b^2*c^2*d^22*f^7*e + 32*C*a^2*c*d^23*f
^7*e + 64*B*a*b*c*d^23*f^7*e + 32*A*b^2*c*d^23*f^7*e - 96*B*a^2*d^24*f^7*e
- 192*A*a*b*d^24*f^7*e + 18*C*b^2*c^2*d^22*f^6*e^2 - 60*C*a*b*c*d^23*f^6*e^
2 - 30*B*b^2*c*d^23*f^6*e^2 + 80*C*a^2*d^24*f^6*e^2 + 160*B*a*b*d^24*f^6*e^
2 + 80*A*b^2*d^24*f^6*e^2 + 28*C*b^2*c*d^23*f^5*e^3 - 140*C*a*b*d^24*f^5*e^
3 - 70*B*b^2*d^24*f^5*e^3 + 63*C*b^2*d^24*f^4*e^4)/(d^25*f^9))*sqrt(d*x + c
) - 15*(7*C*b^2*c^5*f^5 - 20*C*a*b*c^4*d*f^5 - 10*B*b^2*c^4*d*f^5 + 16*C*a^
2*c^3*d^2*f^5 + 32*B*a*b*c^3*d^2*f^5 + 16*A*b^2*c^3*d^2*f^5 - 32*B*a^2*c^2*
d^3*f^5 - 64*A*a*b*c^2*d^3*f^5 + 128*A*a^2*c*d^4*f^5 + 5*C*b^2*c^4*d*f^4*e
- 16*C*a*b*c^3*d^2*f^4*e - 8*B*b^2*c^3*d^2*f^4*e + 16*C*a^2*c^2*d^3*f^4*e +
 32*B*a*b*c^2*d^3*f^4*e + 16*A*b^2*c^2*d^3*f^4*e - 64*B*a^2*c*d^4*f^4*e - 1
28*A*a*b*c*d^4*f^4*e - 128*A*a^2*d^5*f^4*e + 6*C*b^2*c^3*d^2*f^3*e^2 - 24*C
*a*b*c^2*d^3*f^3*e^2 - 12*B*b^2*c^2*d^3*f^3*e^2 + 48*C*a^2*c*d^4*f^3*e^2 +
96*B*a*b*c*d^4*f^3*e^2 + 48*A*b^2*c*d^4*f^3*e^2 + 96*B*a^2*d^5*f^3*e^2 + 19
2*A*a*b*d^5*f^3*e^2 + 10*C*b^2*c^2*d^3*f^2*e^3 - 80*C*a*b*c*d^4*f^2*e^3 - 4
0*B*b^2*c*d^4*f^2*e^3 - 80*C*a^2*d^5*f^2*e^3 - 160*B*a*b*d^5*f^2*e^3 - 80*A
*b^2*d^5*f^2*e^3 + 35*C*b^2*c*d^4*f*e^4 + 140*C*a*b*d^5*f*e^4 + 70*B*b^2*d^
5*f*e^4 - 63*C*b^2*d^5*e^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x +
c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^4*f^5))*d/abs(d)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.48 $\int \frac{(a+bx)\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=540

$$-\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2def^2+5c^3 f}{64d^3f^4}$$

```
[Out] 1/64*(-c*f+d*e)*(8*a*d*f*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e
*f+5*d^2*e^2))+b*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3)+8*d
*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))))*arctanh(f^(1/2)*
(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(9/2)+1/4*C*(b*x+a)^2*(d*x+c
)^(3/2)*(f*x+e)^(1/2)/b/d/f-1/96*(d*x+c)^(3/2)*(24*a^2*C*d^2*f^2+8*a*b*d*f*
(-6*B*d*f+3*C*c*f+5*C*d*e)+b^2*(8*d*f*(-6*A*d*f+3*B*c*f+5*B*d*e)-C*(15*c^2*
f^2+22*c*d*e*f+35*d^2*e^2))+4*b*d*f*(4*a*C*d*f+b*(-8*B*d*f+5*C*c*f+7*C*d*e)
)*x)*(f*x+e)^(1/2)/b/d^3/f^3-1/64*(8*a*d*f*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-
C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))+b*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2
*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)))
)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^3/f^4
```

Rubi [A]

time = 0.45, antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1629, 152, 52, 65, 223, 212}

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}\,(24a^2Cd^2f^2+4bdfx(4aCdf+b(-8Bdf+5cCf+7Cde))+8abdf(-6Bdf+3cCf+5Cde)+b^2(8df(-6Adf+3Bcf+5Bde)-C(15c^2f^2+22cdef+35d^2e^2)))}{96bd^3f^3}-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2Adf(cf+3de)-B(c^2f^2+2cdef+5d^2e^2))+C(5c^3f^3+9c^2def^2+15cd^2e^2f+35d^3e^3)))}{64d^3f^4}+\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2Adf(cf+3de)-B(c^2f^2+2cdef+5d^2e^2))+C(5c^3f^3+9c^2def^2+15cd^2e^2f+35d^3e^3)))}{64d^{7/2}f^{9/2}}+\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] -1/64*((8*a*d*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e
*f + c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*
f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)))
)*Sqrt[c + d*x]*Sqrt[e + f*x])/(d^3*f^4) + (C*(a + b*x)^2*(c + d*x)^(3/2)*S
qrt[e + f*x])/(4*b*d*f) - ((c + d*x)^(3/2)*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2
+ 8*a*b*d*f*(5*C*d*e + 3*c*C*f - 6*B*d*f) + b^2*(8*d*f*(5*B*d*e + 3*B*c*f -
 6*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2)) + 4*b*d*f*(4*a*C*d*f
+ b*(7*C*d*e + 5*c*C*f - 8*B*d*f))*x))/(96*b*d^3*f^3) + ((d*e - c*f)*(8*a*d
*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)
) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*
(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*ArcTanh[(Sq
rt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(7/2)*f^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
```

```
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{(a+bx)\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf} + \frac{\int \frac{(a+bx)\sqrt{c+dx}\,\left(-\frac{1}{2}b(4bcCe+ \cdots}{\cdots}$$

$$= \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf} - \frac{(c+dx)^{3/2}\sqrt{e+fx}\,(24a^2C \cdots}{\cdots}$$

$$= -\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2)) \cdots}{\cdots}$$

$$= -\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2)) \cdots}{\cdots}$$

$$= -\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2)) \cdots}{\cdots}$$

$$= -\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2)) \cdots}{\cdots}$$

Mathematica [A]
time = 2.87, size = 454, normalized size = 0.84

$$\frac{d\sqrt{c+dx}\,\sqrt{e+fx}\,(8adf(6df(4Adf+B(-3de+cf+2dfx))+C(-3c^2f^2+2cdf(-2e+fx)+d^2(15e^2-10efx+8f^2x^2)))+b(C(15c^3f^3+c^2df^2(17e-10fx)+cd^2f(25e^2-12efx+8f^2x^2)+d^3(-105e^3+70e^2fx-56ef^2x^2+48f^3x^3))+8df(6Adf(-3de+cf+2dfx)+B(-3c^2f^2+2cdf(-2e+fx)+d^2(15e^2-10efx+8f^2x^2)))))+3\sqrt{\frac{d}{f}}\,(de-cf)\,(8adf(2df(-3Bde-Bcf+4Adf)+C(5d^2e^2+2cdef+c^2f^2))-b(C(35d^3e^3+15cd^2e^2f+9c^2def^2+5c^3f^3)+8df(2Adf(3de+cf)-B(5d^2e^2+2cdef+c^2f^2))))\log\left(\sqrt{c+dx}-\sqrt{\frac{d}{f}}\,\sqrt{e+fx}\right)}{192d^4f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (d*Sqrt[c + d*x]*Sqrt[e + f*x]*(8*a*d*f*(6*d*f*(4*A*d*f + B*(-3*d*e + c*f +
 2*d*f*x)) + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x
+ 8*f^2*x^2))) + b*(C*(15*c^3*f^3 + c^2*d*f^2*(17*e - 10*f*x) + c*d^2*f*(25
*e^2 - 12*e*f*x + 8*f^2*x^2) + d^3*(-105*e^3 + 70*e^2*f*x - 56*e*f^2*x^2 +
```

```
48*f^3*x^3)) + 8*d*f*(6*A*d*f*(-3*d*e + c*f + 2*d*f*x) + B*(-3*c^2*f^2 + 2*
c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2))))) + 3*Sqrt[d/f]*
(d*e - c*f)*(8*a*d*f*(2*d*f*(-3*B*d*e - B*c*f + 4*A*d*f) + C*(5*d^2*e^2 + 2
*c*d*e*f + c^2*f^2)) - b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 +
5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*
f^2))))*Log[Sqrt[c + d*x] - Sqrt[d/f]*Sqrt[e + f*x]])/(192*d^4*f^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(508) = 1016$.
time = 0.10, size = 2002, normalized size = 3.71

method	result	size
default	Expression too large to display	2002

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-24*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*b*c*d^2*e*f^2*x+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^4-48*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*f^4-48*A*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*f^4+
24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*b*c^3*d*f^4+24*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*a*c^3*d*f^4+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*b*c^3*f^3-210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*e^3-120*C*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^
4*e^3*f+384*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^3*f^3+144*B*ln(1/2*(2
*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^4*e^
2*f^2-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*b*d^4*e^3*f+192*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*f^4-192*A*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^4*e*f^3+144*A*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4
*e^2*f^2-288*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^3*e*f^2+240*B*(d*f)^
(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*e^2*f+240*C*(d*f)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*a*d^3*e^2*f-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*e*f^2+
72*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*a*c*d^3*e^2*f^2-60*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3*e^3*f-96*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*e*f^3+72*B*ln(1/2*(
2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3
*e^2*f^2-96*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*b*c*d^3*e*f^3-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)
```

```
*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^4*f^4+24*B*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*e*f^3+192*B*(d
*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^3*f^3*x+24*C*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*e*f^3-48*C*(d
*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c^2*d*f^3+32*B*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*b*c*d^2*f^3*x-160*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*e
*f^2*x+32*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^2*f^3*x-160*C*(d*f)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^3*e*f^2*x-20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b*c^2*d*f^3*x+140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*e^2*
f*x-64*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^2*e*f^2-64*C*(d*f)^(1/2)
*((d*x+c)*(f*x+e))^(1/2)*a*c*d^2*e*f^2+34*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(
1/2)*b*c^2*d*e*f^2+50*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^2*e^2*f-1
12*C*b*d^3*e*f^2*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-12*C*ln(1/2*(2*d*f
*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d*e*f^
3-18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*b*c^2*d^2*e^2*f^2+96*C*b*d^3*f^3*x^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)+128*B*b*d^3*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+128*C*a*d^3*f
^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+96*A*(d*f)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b*c*d^2*f^3+192*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*f^3*x+
96*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^2*f^3-48*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*b*c^2*d*f^3+16*C*b*c*d^2*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f
*x+e))^(1/2))/f^4/((d*x+c)*(f*x+e))^(1/2)/d^3/(d*f)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 1.63, size = 1117, normalized size = 2.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/768*(3*(35*C*b*d^4*e^4 - (5*C*b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3*d
 + 16*(B*a + A*b)*c^2*d^2)*f^4 - 4*(C*b*c^3*d + 16*A*a*d^4 - 2*(C*a + B*b)*
c^2*d^2 + 8*(B*a + A*b)*c*d^3)*f^3*e - 6*(C*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3
 - 8*(B*a + A*b)*d^4)*f^2*e^2 - 20*(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*f*e^3)*s
qrt(d*f)*log(8*d^2*f^2*x^2 + 8*c*d*f^2*x + c^2*f^2 + d^2*e^2 + 4*(2*d*f*x +
 c*f + d*e)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 2*(4*d^2*f*x + 3*c*d*f)
*e) + 4*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*f*e^3 + 8*(C*b*c*d^3 + 8*(C*a + B
*b)*d^4)*f^4*x^2 - 2*(5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*
d^4)*f^4*x + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^2*d^2 + 16*(B*a
+ A*b)*c*d^3)*f^4 + 5*(14*C*b*d^4*f^2*x + (5*C*b*c*d^3 + 24*(C*a + B*b)*d^4
)*f^2)*e^2 - (56*C*b*d^4*f^3*x^2 + 4*(3*C*b*c*d^3 + 20*(C*a + B*b)*d^4)*f^3
*x - (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 - 144*(B*a + A*b)*d^4)*f^3)*e)*
sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f^5), -1/384*(3*(35*C*b*d^4*e^4 - (5*C*b*
c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3*d + 16*(B*a + A*b)*c^2*d^2)*f^4 - 4*
(C*b*c^3*d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 + 8*(B*a + A*b)*c*d^3)*f^3*
e - 6*(C*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*a + A*b)*d^4)*f^2*e^2 - 20*
(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*f*e^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + c*f
 + d*e)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*f^2*x + (
d^2*f*x + c*d*f)*e)) - 2*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*f*e^3 + 8*(C*b*c
*d^3 + 8*(C*a + B*b)*d^4)*f^4*x^2 - 2*(5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3
- 48*(B*a + A*b)*d^4)*f^4*x + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c
^2*d^2 + 16*(B*a + A*b)*c*d^3)*f^4 + 5*(14*C*b*d^4*f^2*x + (5*C*b*c*d^3 + 2
4*(C*a + B*b)*d^4)*f^2)*e^2 - (56*C*b*d^4*f^3*x^2 + 4*(3*C*b*c*d^3 + 20*(C*
a + B*b)*d^4)*f^3*x - (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 - 144*(B*a + A
*b)*d^4)*f^3)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.83, size = 736, normalized size = 1.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/192*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*
x + c)*C*b/(d^4*f) - (17*C*b*c*d^12*f^6 - 8*C*a*d^13*f^6 - 8*B*b*d^13*f^6 +
 7*C*b*d^13*f^5*e)/(d^16*f^7)) + (59*C*b*c^2*d^12*f^6 - 56*C*a*c*d^13*f^6 -
 56*B*b*c*d^13*f^6 + 48*B*a*d^14*f^6 + 48*A*b*d^14*f^6 + 50*C*b*c*d^13*f^5*
e - 40*C*a*d^14*f^5*e - 40*B*b*d^14*f^5*e + 35*C*b*d^14*f^4*e^2)/(d^16*f^7)
) - 3*(5*C*b*c^3*d^12*f^6 - 8*C*a*c^2*d^13*f^6 - 8*B*b*c^2*d^13*f^6 + 16*B*
a*c*d^14*f^6 + 16*A*b*c*d^14*f^6 - 64*A*a*d^15*f^6 + 9*C*b*c^2*d^13*f^5*e -
 16*C*a*c*d^14*f^5*e - 16*B*b*c*d^14*f^5*e + 48*B*a*d^15*f^5*e + 48*A*b*d^1
5*f^5*e + 15*C*b*c*d^14*f^4*e^2 - 40*C*a*d^15*f^4*e^2 - 40*B*b*d^15*f^4*e^2
 + 35*C*b*d^15*f^3*e^3)/(d^16*f^7))*sqrt(d*x + c) + 3*(5*C*b*c^4*f^4 - 8*C*
a*c^3*d*f^4 - 8*B*b*c^3*d*f^4 + 16*B*a*c^2*d^2*f^4 + 16*A*b*c^2*d^2*f^4 - 6
4*A*a*c*d^3*f^4 + 4*C*b*c^3*d*f^3*e - 8*C*a*c^2*d^2*f^3*e - 8*B*b*c^2*d^2*f
^3*e + 32*B*a*c*d^3*f^3*e + 32*A*b*c*d^3*f^3*e + 64*A*a*d^4*f^3*e + 6*C*b*c
^2*d^2*f^2*e^2 - 24*C*a*c*d^3*f^2*e^2 - 24*B*b*c*d^3*f^2*e^2 - 48*B*a*d^4*f
^2*e^2 - 48*A*b*d^4*f^2*e^2 + 20*C*b*c*d^3*f*e^3 + 40*C*a*d^4*f*e^3 + 40*B*
b*d^4*f*e^3 - 35*C*b*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^3*f^4))*d/abs(d)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.49 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=246

$$\frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\,\sqrt{e+fx}}{8d^2f^3}-\frac{(5Cde+7cCf-6Bdf)(c+d\ldots}{12d^2f^2}$$

```
[Out] -1/8*(-c*f+d*e)*(C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+2*d*f*(4*A*d*f-B*(c*f+3*d*
e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(5/2)/f^(7/2)-1
/12*(-6*B*d*f+7*C*c*f+5*C*d*e)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^2/f^2+1/3*C*(d
*x+c)^(5/2)*(f*x+e)^(1/2)/d^2/f+1/8*(C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+2*d*f*
(4*A*d*f-B*(c*f+3*d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^2/f^3
```

Rubi [A]
time = 0.15, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {965, 81, 52, 65, 223, 212}

$$\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(2df(4Adf-B(cf+3de))+C(c^2f^2+2cdef+5d^2e^2))}{8d^2f^3}-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)(2df(4Adf-B(cf+3de))+C(c^2f^2+2cdef+5d^2e^2))}{8d^{5/2}f^{7/2}}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\,(-6Bdf+7cCf+5Cde)}{12d^2f^2}+\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] ((C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(8*d^2*f^3) - ((5*C*d*e + 7*c*C*f - 6*B*d*f)*(
c + d*x)^(3/2)*Sqrt[e + f*x])/(12*d^2*f^2) + (C*(c + d*x)^(5/2)*Sqrt[e + f*
x])/(3*d^2*f) - ((d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(
4*A*d*f - B*(3*d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
 + f*x])])/(8*d^(5/2)*f^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} + \frac{\int \frac{\sqrt{c+dx}\,\left(\frac{1}{2}(-5cCde-c^2Cf+6Ad^2f)-\frac{1}{2}d(5Cde+7cCf-}{\sqrt{e+fx}}}{3d^2f}$$

$$= -\frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2} + \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}$$

$$= \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\,\sqrt{e+}}{8d^2f^3}$$

$$= \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\,\sqrt{e+}}{8d^2f^3}$$

$$= \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\,\sqrt{e+}}{8d^2f^3}$$

$$= \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\,\sqrt{e+}}{8d^2f^3}$$

Mathematica [A]
time = 0.89, size = 199, normalized size = 0.81

$$\frac{d\sqrt{c+dx}\,\sqrt{e+fx}\,(6df(4Adf+B(-3de+cf+2dfx))+C(-3c^2f^2+2cdf(-2e+fx)+d^2(15e^2-10efx+8f^2x^2)))+3\sqrt{\frac{d}{f}}\,(de-cf)\,(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\log\left(\sqrt{c+dx}-\sqrt{\frac{d}{f}}\,\sqrt{e+fx}\right)}{24d^3f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (d*Sqrt[c + d*x]*Sqrt[e + f*x]*(6*d*f*(4*A*d*f + B*(-3*d*e + c*f + 2*d*f*x)
) + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x
^2))) + 3*Sqrt[d/f]*(d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*
f*(4*A*d*f - B*(3*d*e + c*f)))*Log[Sqrt[c + d*x] - Sqrt[d/f]*Sqrt[e + f*x]]
)/(24*d^3*f^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(214) = 428.
time = 0.10, size = 763, normalized size = 3.10

method	result
default	$\frac{\sqrt{dx+c}\,\sqrt{fx+e}\,\left(16C\,d^2f^2x^2\sqrt{df}\,\sqrt{(dx+c)\,(fx+e)}+24A\ln\left(\frac{2dfx+2\sqrt{(dx+c)\,(fx+e)}\,\sqrt{df}+}{2\sqrt{df}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(16*C*d^2*f^2*x^2*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)+24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e*f^2-6*B*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d*f^3-12*B*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^2*e*f^
2+18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*d^3*e^2*f+24*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*f^2*x+3*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c
^3*f^3+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*c^2*d*e*f^2+9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^2*e^2*f-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e^3+4*C*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*c*d*f^2*x-20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*
e*f*x+48*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*f^2+12*B*(d*f)^(1/2)*((d
*x+c)*(f*x+e))^(1/2)*c*d*f^2-36*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*e
*f-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c^2*f^2-8*C*(d*f)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*c*d*e*f+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*e^2)/f
^3/((d*x+c)*(f*x+e))^(1/2)/d^2/(d*f)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 1.77, size = 581, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*C*d^3*e^3 - (C*c^3 - 2*B*c^2*d + 8*A*c*d^2)*f^3 - (C*c^2*d - 4*
B*c*d^2 - 8*A*d^3)*f^2*e - 3*(C*c*d^2 + 2*B*d^3)*f*e^2)*sqrt(d*f)*log(8*d^2
*f^2*x^2 + 8*c*d*f^2*x + c^2*f^2 + d^2*e^2 - 4*(2*d*f*x + c*f + d*e)*sqrt(d
*f)*sqrt(d*x + c)*sqrt(f*x + e) + 2*(4*d^2*f*x + 3*c*d*f)*e) + 4*(8*C*d^3*f
^3*x^2 + 15*C*d^3*f*e^2 + 2*(C*c*d^2 + 6*B*d^3)*f^3*x - 3*(C*c^2*d - 2*B*c*
d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*f^2*x + (2*C*c*d^2 + 9*B*d^3)*f^2)*e)*sqrt(
d*x + c)*sqrt(f*x + e))/(d^3*f^4), 1/48*(3*(5*C*d^3*e^3 - (C*c^3 - 2*B*c^2*
d + 8*A*c*d^2)*f^3 - (C*c^2*d - 4*B*c*d^2 - 8*A*d^3)*f^2*e - 3*(C*c*d^2 + 2
*B*d^3)*f*e^2)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + c*f + d*e)*sqrt(-d*f)*sqrt(
d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*f^2*x + (d^2*f*x + c*d*f)*e)) + 2
*(8*C*d^3*f^3*x^2 + 15*C*d^3*f*e^2 + 2*(C*c*d^2 + 6*B*d^3)*f^3*x - 3*(C*c^2
*d - 2*B*c*d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*f^2*x + (2*C*c*d^2 + 9*B*d^3)*f^
2)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.83, size = 315, normalized size = 1.28

$$\frac{\left(\sqrt{(dx+c)df-cdf+d^2e}\,\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)C}{d^3f}-\frac{7\,Ccd^6f^4-6\,Bd^7f^4+5\,Cd^7f^3e}{d^9f^5}\right)+\frac{3\,(Cc^2d^6f^4-2\,Bcd^7f^4+8\,Ad^8f^4+2\,Ccd^7f^3e-6\,Bd^8f^3e+5\,Cd^8f^2e^2)}{d^9f^5}\right)-\frac{3\,(Cc^3f^3-2\,Bc^2df^3+8\,Acd^2f^3+Cc^2df^2e-4\,Bcd^2f^2e-8\,Ad^3f^2e+3\,Ccd^2fe^2+6\,Bd^3fe^2-5\,Cd^3e^3)\log\left(\left|-\sqrt{df}\,\sqrt{dx+c}+\sqrt{(dx+c)df-cdf+d^2e}\right|\right)}{\sqrt{df}\,d^2f^3}\right)d}{24\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)*C/(d^3*f) - (7*C*c*d^6*f^4 - 6*B*d^7*f^4 + 5*C*d^7*f^3*e)/(d^9*f^5))
 + 3*(C*c^2*d^6*f^4 - 2*B*c*d^7*f^4 + 8*A*d^8*f^4 + 2*C*c*d^7*f^3*e - 6*B*d
^8*f^3*e + 5*C*d^8*f^2*e^2)/(d^9*f^5)) - 3*(C*c^3*f^3 - 2*B*c^2*d*f^3 + 8*A
*c*d^2*f^3 + C*c^2*d*f^2*e - 4*B*c*d^2*f^2*e - 8*A*d^3*f^2*e + 3*C*c*d^2*f*
e^2 + 6*B*d^3*f*e^2 - 5*C*d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(
(d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*d/abs(d)
```

Mupad [B]
time = 90.55, size = 1832, normalized size = 7.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] ((((c + d*x)^(1/2) - c^(1/2))*(2*A*d^2*e + 2*A*c*d*f))/(f^3*((e + f*x)^(1/2
) - e^(1/2))) + ((2*A*c*f + 2*A*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(f^2*((
e + f*x)^(1/2) - e^(1/2))^3) - (8*A*c^(1/2)*d*e^(1/2)*((c + d*x)^(1/2) - c^
(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))
^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/2
))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/2))*((
C*c^3*d^3*f^3)/4 - (5*C*d^6*e^3)/4 + (C*c^2*d^4*e*f^2)/4 + (3*C*c*d^5*e^2*f
)/4))/(f^9*((e + f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^5*((
33*C*d^4*e^3)/2 + (19*C*c^3*d*f^3)/2 + (275*C*c^2*d^2*e*f^2)/2 + (313*C*c*d
^3*e^2*f)/2))/(f^7*((e + f*x)^(1/2) - e^(1/2))^5) - (((c + d*x)^(1/2) - c^(
1/2))^7*((19*C*c^3*f^3)/2 + (33*C*d^3*e^3)/2 + (313*C*c*d^2*e^2*f)/2 + (275
*C*c^2*d*e*f^2)/2))/(f^6*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2)
 - c^(1/2))^3*((17*C*c^3*d^2*f^3)/12 - (85*C*d^5*e^3)/12 + (91*C*c^2*d^3*e*
f^2)/4 + (17*C*c*d^4*e^2*f)/4))/(f^8*((e + f*x)^(1/2) - e^(1/2))^3) + (((c
+ d*x)^(1/2) - c^(1/2))^11*((C*c^3*f^3)/4 - (5*C*d^3*e^3)/4 + (3*C*c*d^2*e^
2*f)/4 + (C*c^2*d*e*f^2)/4))/(d^2*f^4*((e + f*x)^(1/2) - e^(1/2))^11) - (((
c + d*x)^(1/2) - c^(1/2))^9*((17*C*c^3*f^3)/12 - (85*C*d^3*e^3)/12 + (17*C*
c*d^2*e^2*f)/4 + (91*C*c^2*d*e*f^2)/4))/(d*f^5*((e + f*x)^(1/2) - e^(1/2))^
9) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^8*(32*C*c^2*f + 96*C*c*d*
e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^8) + (c^(1/2)*e^(1/2)*(96*C*c*d^3*e +
32*C*c^2*d^2*f)*((c + d*x)^(1/2) - c^(1/2))^4)/(f^6*((e + f*x)^(1/2) - e^(1
/2))^4) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6*(128*C*d^3*e^2 + 6
4*C*c^2*d*f^2 + (704*C*c*d^2*e*f)/3))/(f^6*((e + f*x)^(1/2) - e^(1/2))^6))/
(((c + d*x)^(1/2) - c^(1/2))^12/((e + f*x)^(1/2) - e^(1/2))^12 + d^6/f^6 -
(6*d*((c + d*x)^(1/2) - c^(1/2))^10)/(f*((e + f*x)^(1/2) - e^(1/2))^10) - (
6*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/(f^5*((e + f*x)^(1/2) - e^(1/2))^2) +
(15*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(f^4*((e + f*x)^(1/2) - e^(1/2))^4)
- (20*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/(f^3*((e + f*x)^(1/2) - e^(1/2))^6
) + (15*d^2*((c + d*x)^(1/2) - c^(1/2))^8)/(f^2*((e + f*x)^(1/2) - e^(1/2))
^8)) + ((((c + d*x)^(1/2) - c^(1/2))*((B*c^2*d^2*f^2)/2 - (3*B*d^4*e^2)/2 +
 B*c*d^3*e*f))/(f^6*((e + f*x)^(1/2) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1
/2))^3*((11*B*d^3*e^2)/2 + (7*B*c^2*d*f^2)/2 + 23*B*c*d^2*e*f))/(f^5*((e +
f*x)^(1/2) - e^(1/2))^3) + (((c + d*x)^(1/2) - c^(1/2))^5*((7*B*c^2*f^2)/2
+ (11*B*d^2*e^2)/2 + 23*B*c*d*e*f))/(f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (
((c + d*x)^(1/2) - c^(1/2))^7*((B*c^2*f^2)/2 - (3*B*d^2*e^2)/2 + B*c*d*e*f)
)/(d*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (c^(1/2)*e^(1/2)*((c + d*x)^(1/2)
 - c^(1/2))^4*(32*B*d^2*e + 16*B*c*d*f))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4
) - (8*B*c^(3/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6)/(f^2*((e + f*x)^(1/
2) - e^(1/2))^6) - (8*B*c^(3/2)*d^2*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/
(f^4*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f
*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*(
```

```
(e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*
((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2
*((e + f*x)^(1/2) - e^(1/2))^4)) + (2*A*atanh((f^(1/2)*((c + d*x)^(1/2) - c
^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c*f - d*e))/(d^(1/2)*f^(3/
2)) + (C*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1
/2) - e^(1/2))))*(c*f - d*e)*(c^2*f^2 + 5*d^2*e^2 + 2*c*d*e*f))/(4*d^(5/2)*
f^(7/2)) - (B*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*
x)^(1/2) - e^(1/2))))*(c*f - d*e)*(c*f + 3*d*e))/(2*d^(3/2)*f^(5/2))
```

3.50 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$

Optimal. Leaf size=290

$$-\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}+\frac{(2bdf(4Abdf-aC(3de$$

```
[Out] 1/4*(2*b*d*f*(4*A*b*d*f-a*C*(c*f+3*d*e))+(2*a*d*f-b*c*f+b*d*e)*(4*a*C*d*f+b
*(-4*B*d*f+C*c*f+3*C*d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(
1/2))/b^3/d^(3/2)/f^(5/2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e)^(1/2)*(d
*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))*(-a*d+b*c)^(1/2)/b^3/(-a*f+b*e)
^(1/2)+1/2*C*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f-1/4*(4*a*C*d*f+b*(-4*B*d*f+C
*c*f+3*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d/f^2
```

Rubi [A]
time = 0.46, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1629, 159, 163, 65, 223, 212, 95, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))}{4b^3d^{3/2}f^{5/2}}-\frac{2\sqrt{bc-ad}\,(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^3\sqrt{be-af}}-\frac{\sqrt{c+dx}\sqrt{e+fx}\,(4aCdf+b(-4Bdf+cCf+3Cde))}{4b^2df^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

```
[Out] -1/4*((4*a*C*d*f + b*(3*C*d*e + c*C*f - 4*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*
x])/(b^2*d*f^2) + (C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*d*f) + ((2*b*d*f*(
4*A*b*d*f - a*C*(3*d*e + c*f)) + (b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(
3*C*d*e + c*C*f - 4*B*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
e + f*x])])/(4*b^3*d^(3/2)*f^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x
])])/(b^3*Sqrt[b*e - a*f])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
```

```
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\int \frac{\sqrt{c+dx}\,\left(\frac{1}{2}b(4Abdf-aC(3de+cf))-\frac{1}{2}b(4aCdf+b(3Cd}{(a+bx)\sqrt{e+fx}}}{2b^2df}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/}}{2b}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/}}{2b}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/}}{2b}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/}}{2b}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/}}{2b}$$

Mathematica [A]
time = 10.43, size = 367, normalized size = 1.27

$$\frac{\frac{2b\sqrt{c+dx}\,\sqrt{e+fx}\,(4bBdf-4aCdf+bC(-3de+cf+2dfx))}{df^2}+\frac{8(Ab^2+a(-bB+aC))\sqrt{bc-ad}\,\log(a+bx)}{\sqrt{be-af}}+\frac{(8a^2Cd^2f^2-4abdf(-Cde+cCf+2Bdf)+b^2(4df(-Bde+Bcf+2Adf)+C(3d^2e^2-2cdef-c^2f^2)))\log\left(de+cf+2dfx+2\sqrt{d}\,\sqrt{f}\,\sqrt{c+dx}\,\sqrt{e+fx}\right)}{d^{3/2}f^{5/2}}-\frac{8(Ab^2+a(-bB+aC))\sqrt{bc-ad}\,\log\left(2\sqrt{bc-ad}\,\sqrt{be-af}\,\sqrt{c+dx}\,\sqrt{e+fx}+b(2ce+dex+cfx)-a(de+cf+2dfx)\right)}{\sqrt{be-af}}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

```
[Out] ((2*b*Sqrt[c + d*x]*Sqrt[e + f*x]*(4*b*B*d*f - 4*a*C*d*f + b*C*(-3*d*e + c*
f + 2*d*f*x)))/(d*f^2) + (8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[b*c - a*d]*Log[
a + b*x])/Sqrt[b*e - a*f] + ((8*a^2*C*d^2*f^2 - 4*a*b*d*f*(-(C*d*e) + c*C*f
 + 2*B*d*f) + b^2*(4*d*f*(-(B*d*e) + B*c*f + 2*A*d*f) + C*(3*d^2*e^2 - 2*c*
```

```
d*e*f - c^2*f^2)))*Log[d*e + c*f + 2*d*f*x + 2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x
]*Sqrt[e + f*x]])/(d^(3/2)*f^(5/2)) - (8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[b*
c - a*d]*Log[2*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*Sqrt[c + d*x]*Sqrt[e + f*x]
+ b*(2*c*e + d*e*x + c*f*x) - a*(d*e + c*f + 2*d*f*x)])/Sqrt[b*e - a*f])/(8
*b^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1821 vs. $2(252) = 504$.

time = 0.10, size = 1822, normalized size = 6.28

method	result	size
default	Expression too large to display	1822

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/8*(8*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/
2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*
x+a))*a*b^2*d^2*f^2-8*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+
c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d
*e+2*b*c*e)/(b*x+a))*b^3*c*d*f^2+8*A*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f
)^(1/2))*b^3*d^2*f^2-8*B*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x
+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*
d*e+2*b*c*e)/(b*x+a))*a^2*b*d^2*f^2+8*B*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+
b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^2*c*d*f^2-8*B*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^2*d^2*f^2+4*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*b^3*c*d*f^2-4*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*b^3*d^2*e*f+4*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)*b^3*d*f*x+8*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*
x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)
^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*d^2*f^2-8*C*(d*f)^(1/2)*ln((-2*a
*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+
b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b*c*d*f^2+8*C*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b*d^2*f^2-4*C*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^2*c*d*f^2+4*C*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*a*b^2*d^2*e*f-C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
```

```
2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*b^3*c^2*f^2-2*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(
1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^
3*c*d*e*f+3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^3*d^2*e^2+8*
B*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b^3*d*f-8*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)*a*b^2*d*f+2*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^3*c*f-6*C*(d*f)^(1/2)*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^3*d*e
)*(f*x+e)^(1/2)*(d*x+c)^(1/2)/(d*f)^(1/2)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)/((d*x+c)*(f*x+e))^(1/2)/b^4/d/f^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)*sqrt(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

3.51 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$

Optimal. Leaf size=364

$$\frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{b^2(bc-ad)f(be-af)}-\frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}}{b(bc-ad)(be-af)(a+b}$$

```
[Out] -(4*a*C*d*f+b*(-2*B*d*f-C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/(f*x+e)^(1/2))/b^3/f^(3/2)/d^(1/2)+(4*a^3*C*d*f-b^3*(-A*c*f+A*d*e+2*B*c*e)
+a*b^2*(B*c*f+3*B*d*e+4*C*c*e)-a^2*b*(2*B*d*f+3*C*c*f+5*C*d*e))*arctanh((-a
*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^3/(-a*f+b*e)^
(3/2)/(-a*d+b*c)^(1/2)-(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-
a*d+b*c)/(-a*f+b*e)/(b*x+a)+(2*a^2*C*d*f+b^2*(A*d*f+C*c*e)-a*b*(B*d*f+C*c*f
+C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)
```

Rubi [A]

time = 0.73, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1627, 159, 163, 65, 223, 212, 95, 214}

$$\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe))}{b^2f(bc-ad)(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{be-af}}{\sqrt{e+fx}\,\sqrt{bc-ad}}\right)(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf+Ade+2Bce))}{b^3\sqrt{bc-ad}\,(be-af)^{3/2}}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\,(Ab^2-a(bB-aC))}{b(a+bx)(bc-ad)(be-af)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)(4aCdf+b(-2Bdf-cCf+Cde))}{b^3\sqrt{d}\,f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

```
[Out] ((2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*Sqrt[c +
 d*x]*Sqrt[e + f*x])/(b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 - a*(b*B - a
*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e - a*f)*(a + b*x)) -
 ((4*a*C*d*f + b*(C*d*e - c*C*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqrt[d]*f^(3/2)) + ((4*a^3*C*d*f - b^3*(2*B
*c*e + A*d*e - A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + B*c*f) - a^2*b*(5*C*d*e
+ 3*c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a
*d]*Sqrt[e + f*x])])/(b^3*Sqrt[b*c - a*d]*(b*e - a*f)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
```

```
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{\sqrt{c+dx}\,\left(-\frac{a^2C(3de+cf)+\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\,\sqrt{e+fx}\cdots}{b^2(bc-ad)f(be-af)}$$

$$= \frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\,\sqrt{e+fx}\cdots}{b^2(bc-ad)f(be-af)}$$

$$= \frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\,\sqrt{e+fx}\cdots}{b^2(bc-ad)f(be-af)}$$

$$= \frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\,\sqrt{e+fx}\cdots}{b^2(bc-ad)f(be-af)}$$

$$= \frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\,\sqrt{e+fx}\cdots}{b^2(bc-ad)f(be-af)}$$

Mathematica [A]
time = 10.70, size = 421, normalized size = 1.16

$$\frac{2b\sqrt{c+dx}\,\sqrt{e+fx}\left(\frac{C}{f}+\frac{-Ab^2+a(bB-aC)}{(be-af)(a+bx)}\right)+\frac{(-4a^3Cdf+b^3(2Bce+Ade-Acf)-ab^2(4cCe+3Bde+Bcf)+a^2b(5Cde+3cCf+2Bdf))\log(a+bx)}{\sqrt{bc-ad}\,(be-af)^{3/2}}+\frac{(-4aCdf+b(-Cde+cCf+2Bdf))\log\left(de+cf+2dfx+2\sqrt{d}\,\sqrt{f}\,\sqrt{c+dx}\,\sqrt{e+fx}\right)}{\sqrt{d}\,f^{3/2}}-\frac{(-4a^3Cdf+b^3(2Bce+Ade-Acf)-ab^2(4cCe+3Bde+Bcf)+a^2b(5Cde+3cCf+2Bdf))\log\left(2\sqrt{bc-ad}\,\sqrt{be-af}\,\sqrt{c+dx}\,\sqrt{e+fx}+b(2ce+dex+cfx)-a(de+cf+2dfx)\right)}{\sqrt{bc-ad}\,(be-af)^{3/2}}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

```
[Out] (2*b*Sqrt[c + d*x]*Sqrt[e + f*x]*(C/f + (-(A*b^2) + a*(b*B - a*C))/((b*e -
a*f)*(a + b*x))) + ((-4*a^3*C*d*f + b^3*(2*B*c*e + A*d*e - A*c*f) - a*b^2*(
```

```
4*c*C*e + 3*B*d*e + B*c*f) + a^2*b*(5*C*d*e + 3*c*C*f + 2*B*d*f))*Log[a + b
*x])/(Sqrt[b*c - a*d]*(b*e - a*f)^(3/2)) + ((-4*a*C*d*f + b*(-(C*d*e) + c*C
*f + 2*B*d*f))*Log[d*e + c*f + 2*d*f*x + 2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*Sq
rt[e + f*x]])/(Sqrt[d]*f^(3/2)) - ((-4*a^3*C*d*f + b^3*(2*B*c*e + A*d*e - A
*c*f) - a*b^2*(4*c*C*e + 3*B*d*e + B*c*f) + a^2*b*(5*C*d*e + 3*c*C*f + 2*B*
d*f))*Log[2*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*Sqrt[c + d*x]*Sqrt[e + f*x] + b
*(2*c*e + d*e*x + c*f*x) - a*(d*e + c*f + 2*d*f*x)])/(Sqrt[b*c - a*d]*(b*e
- a*f)^(3/2)))/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3669 vs. $2(332) = 664$.
time = 0.10, size = 3670, normalized size = 10.08

method	result	size
default	Expression too large to display	3670

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+
c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d
*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*f^2*(d*f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b
*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^4*d*f^2*(d*f)^(1/2)-2*A*b^4*f*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+
3*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*e*f
*x*(d*f)^(1/2)-3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*a*b^3*d*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)-5*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^
2*d*e*f*x*(d*f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e)
)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e
)/(b*x+a))*a*b^3*c*e*f*x*(d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d
*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-
a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*e*f*(d*f)^(1/2)+2*B*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*e*f*((a^2*d*f-
a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+3*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f
-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*e*f*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*f
*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*f*(d*f)^(1/2)-3*C*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^
2*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x+2*(
```

```
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*e*f*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-5*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+
2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a
*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*e*f*(d*f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c
*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b
^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*e*f*(d*f)^(1/2)-2*C*a*b
^3*f*x*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))
*b^4*d*e*f*x*(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)/b^2)^(1/2)+2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*b^4*d*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)-2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*
d*f^2*x*(d*f)^(1/2)+B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1
/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b
*x+a))*a*b^3*c*f^2*x*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x
+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*
d*e+2*b*c*e)/(b*x+a))*b^4*c*e*f*x*(d*f)^(1/2)+4*C*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d*f^2*x*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*f^2*x*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)
/b^2)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))
*a^3*b*d*f^2*x*(d*f)^(1/2)-3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f
*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*
b*c*e)/(b*x+a))*a^2*b^2*c*f^2*x*(d*f)^(1/2)+4*C*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d*f^2*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*c*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+
b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*e^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^
(1/2)-3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b
*c*f^2*(d*f)^(1/2)+2*C*b^4*e*x*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*B*a*b^3*f*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-4*C*a^2*b^2*f*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)+2*C*a*b^3*e*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**2/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**2*sqrt(e + f*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. 2(347) = 694.
time = 2.22, size = 1388, normalized size = 3.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] (3*sqrt(d*f)*C*a^2*b*c*d^2*f - sqrt(d*f)*B*a*b^2*c*d^2*f - sqrt(d*f)*A*b^3*
c*d^2*f - 4*sqrt(d*f)*C*a^3*d^3*f + 2*sqrt(d*f)*B*a^2*b*d^3*f - 4*sqrt(d*f)
*C*a*b^2*c*d^2*e + 2*sqrt(d*f)*B*b^3*c*d^2*e + 5*sqrt(d*f)*C*a^2*b*d^3*e -
3*sqrt(d*f)*B*a*b^2*d^3*e + sqrt(d*f)*A*b^3*d^3*e)*arctan(-1/2*(b*c*d*f - 2
*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)
*d))/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*(a*b^3*f*
abs(d) - b^4*abs(d)*e)*d) + 2*(sqrt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*
a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*c^2*d^3*f^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4
*f*e + 2*sqrt(d*f)*B*a*b^2*c*d^4*f*e - 2*sqrt(d*f)*A*b^3*c*d^4*f*e - sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2
*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*
f + d^2*e))^2*B*a*b^2*c*d^2*f - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*d^3*f - 2*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*
b*d^3*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
 + d^2*e))^2*A*a*b^2*d^3*f + sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*
d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^2*d^3*e - sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^
3*d^3*e)/((b*c^2*d^2*f^2 - 2*b*c*d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*b)*(a*b^3*f*abs(d) -
 b^4*abs(d)*e)) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*C*abs(d
)/(b^2*d^2*f) - 1/2*(sqrt(d*f)*C*b*c*f - 4*sqrt(d*f)*C*a*d*f + 2*sqrt(d*f)*
B*b*d*f - sqrt(d*f)*C*b*d*e)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*
d*f - c*d*f + d^2*e))^2)/(b^3*f^2*abs(d))
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^2),x)
[Out] \text{Hanged}
```

3.52 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$

Optimal. Leaf size=484

$$\frac{(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-4Adf))\sqrt{c+dx}\,\sqrt{e+}}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

```
[Out] -1/4*(8*a^4*C*d^2*f^2-4*a^3*b*C*d*f*(3*c*f+5*d*e)+3*a^2*b^2*C*(c^2*f^2+10*c
*d*e*f+5*d^2*e^2)-a*b^3*(d^2*e*(-4*A*f+3*B*e)+c^2*f*(-B*f+8*C*e)+2*c*d*(2*A
*f^2-B*e*f+12*C*e^2))-b^4*(A*d^2*e^2-2*c*d*e*(-A*f+2*B*e)-c^2*(3*A*f^2-4*B*
e*f+8*C*e^2)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x
+e)^(1/2))/b^3/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(5/2)+2*C*arctanh(f^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/(f*x+e)^(1/2))*d^(1/2)/b^3/f^(1/2)-1/2*(A*b^2-a*(B*b-C*a))*
(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2+1/4*(4*a^3*C*
d*f-a^2*b*C*(5*c*f+7*d*e)-b^3*(-3*A*c*f-A*d*e+4*B*c*e)+a*b^2*(-4*A*d*f+B*c*
f+3*B*d*e+8*C*c*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2
/(b*x+a)
```

Rubi [A]

time = 1.05, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1627, 154, 163, 65, 223, 212, 95, 214}

$$\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{4b^2(a+bx)(bc-ad)(be-af)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{be-af}}{\sqrt{e+fx}\,\sqrt{bc-ad}}\right)(8a^4Cd^2f^2-4a^3bCdf(3cf+5de)+3a^2b^2C(c^2f^2+10cdef+5d^2e^2)-ab^3(2cd(2Af^2-Bef+12Ce^2)+d^2e(3Be-4Af)+c^2f(8Ce-Bf))-b^4(-(c^2(3Af^2-4Bef+8Ce^2))-2cde(2Be-Af)+Ad^2e^2))}{4b^3(bc-ad)^{3/2}(be-af)^{5/2}}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\,(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}+\frac{2C\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)}{b^3\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

```
[Out] ((4*a^3*C*d*f - a^2*b*C*(7*d*e + 5*c*f) - b^3*(4*B*c*e - A*d*e - 3*A*c*f) +
 a*b^2*(8*c*C*e + 3*B*d*e + B*c*f - 4*A*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/
(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c +
 d*x)^(3/2)*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (2*C
*Sqrt[d]*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqr
t[f]) - ((8*a^4*C*d^2*f^2 - 4*a^3*b*C*d*f*(5*d*e + 3*c*f) + 3*a^2*b^2*C*(5*
d^2*e^2 + 10*c*d*e*f + c^2*f^2) - a*b^3*(d^2*e*(3*B*e - 4*A*f) + c^2*f*(8*C
*e - B*f) + 2*c*d*(12*C*e^2 - B*e*f + 2*A*f^2)) - b^4*(A*d^2*e^2 - 2*c*d*e*
(2*B*e - A*f) - c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)))*ArcTanh[(Sqrt[b*e - a*f
]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^3*(b*c - a*d)^(3/2)
*(b*e - a*f)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2}{}\right.}{}}{}$$

$$= \frac{(4a^3Cdf - a^2bC(7de+5cf) - b^3(4Bce - Ade - 3Acf) + ab^2(8cCe + 3B}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{(4a^3Cdf - a^2bC(7de+5cf) - b^3(4Bce - Ade - 3Acf) + ab^2(8cCe + 3B}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{(4a^3Cdf - a^2bC(7de+5cf) - b^3(4Bce - Ade - 3Acf) + ab^2(8cCe + 3B}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{(4a^3Cdf - a^2bC(7de+5cf) - b^3(4Bce - Ade - 3Acf) + ab^2(8cCe + 3B}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{(4a^3Cdf - a^2bC(7de+5cf) - b^3(4Bce - Ade - 3Acf) + ab^2(8cCe + 3B}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

Mathematica [A]
time = 11.95, size = 693, normalized size = 1.43

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*b*Sqrt[c + d*x]*Sqrt[e + f*x]*(2*(A*b^2 + a*(-(b*B) + a*C))*(b*c - a*d
)*(b*e - a*f) + (-6*a^3*C*d*f + b^3*(4*B*c*e + A*d*e - 3*A*c*f) - a*b^2*(8*
c*C*e + 5*B*d*e + B*c*f - 2*A*d*f) + a^2*b*(9*C*d*e + 5*c*C*f + 2*B*d*f))*(
a + b*x)))/((b*c - a*d)*(b*e - a*f)^2*(a + b*x)^2) + ((8*a^4*C*d^2*f^2 - 4*
a^3*b*C*d*f*(5*d*e + 3*c*f) + 3*a^2*b^2*C*(5*d^2*e^2 + 10*c*d*e*f + c^2*f^2
) + a*b^3*(d^2*e*(-3*B*e + 4*A*f) + c^2*f*(-8*C*e + B*f) + 2*c*d*(-12*C*e^2
 + B*e*f - 2*A*f^2)) + b^4*(-(A*d^2*e^2) + 2*c*d*e*(2*B*e - A*f) + c^2*(8*C
*e^2 - 4*B*e*f + 3*A*f^2)))*Log[a + b*x])/((b*c - a*d)^(3/2)*(b*e - a*f)^(5
/2)) + (8*C*Sqrt[d]*Log[d*e + c*f + 2*d*f*x + 2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*
x]*Sqrt[e + f*x]])/Sqrt[f] - ((8*a^4*C*d^2*f^2 - 4*a^3*b*C*d*f*(5*d*e + 3*c
*f) + 3*a^2*b^2*C*(5*d^2*e^2 + 10*c*d*e*f + c^2*f^2) + a*b^3*(d^2*e*(-3*B*e
 + 4*A*f) + c^2*f*(-8*C*e + B*f) + 2*c*d*(-12*C*e^2 + B*e*f - 2*A*f^2)) + b
^4*(-(A*d^2*e^2) + 2*c*d*e*(2*B*e - A*f) + c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2
)))*Log[2*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*Sqrt[c + d*x]*Sqrt[e + f*x] + b*(
2*c*e + d*e*x + c*f*x) - a*(d*e + c*f + 2*d*f*x)])/((b*c - a*d)^(3/2)*(b*e
- a*f)^(5/2)))/(8*b^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9099 vs. $2(446) = 892$.
time = 0.10, size = 9100, normalized size = 18.80

method	result	size
default	Expression too large to display	9100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**3/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8004 vs. 2(458) = 916.
time = 26.35, size = 8004, normalized size = 16.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] -1/4*(3*sqrt(d*f)*C*a^2*b^2*c^2*d^2*f^2 + sqrt(d*f)*B*a*b^3*c^2*d^2*f^2 + 3
*sqrt(d*f)*A*b^4*c^2*d^2*f^2 - 12*sqrt(d*f)*C*a^3*b*c*d^3*f^2 - 4*sqrt(d*f)
*A*a*b^3*c*d^3*f^2 + 8*sqrt(d*f)*C*a^4*d^4*f^2 - 8*sqrt(d*f)*C*a*b^3*c^2*d^
2*f*e - 4*sqrt(d*f)*B*b^4*c^2*d^2*f*e + 30*sqrt(d*f)*C*a^2*b^2*c*d^3*f*e +
2*sqrt(d*f)*B*a*b^3*c*d^3*f*e - 2*sqrt(d*f)*A*b^4*c*d^3*f*e - 20*sqrt(d*f)*
C*a^3*b*d^4*f*e + 4*sqrt(d*f)*A*a*b^3*d^4*f*e + 8*sqrt(d*f)*C*b^4*c^2*d^2*e
^2 - 24*sqrt(d*f)*C*a*b^3*c*d^3*e^2 + 4*sqrt(d*f)*B*b^4*c*d^3*e^2 + 15*sqrt
(d*f)*C*a^2*b^2*d^4*e^2 - 3*sqrt(d*f)*B*a*b^3*d^4*e^2 - sqrt(d*f)*A*b^4*d^4
*e^2)*arctan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c)
 - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^
2 - b^2*c*d*f*e + a*b*d^2*f*e)*d))/((a^2*b^4*c*f^2*abs(d) - a^3*b^3*d*f^2*a
bs(d) - 2*a*b^5*c*f*abs(d)*e + 2*a^2*b^4*d*f*abs(d)*e + b^6*c*abs(d)*e^2 -
a*b^5*d*abs(d)*e^2)*sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*
f*e)*d) - sqrt(d*f)*C*d*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
```

c*d*f + d^2*e))^2)/(b^3*f*abs(d)) - 1/2*(5*sqrt(d*f)*C*a^2*b^3*c^5*d^5*f^5
- sqrt(d*f)*B*a*b^4*c^5*d^5*f^5 - 3*sqrt(d*f)*A*b^5*c^5*d^5*f^5 - 6*sqrt(d
*f)*C*a^3*b^2*c^4*d^6*f^5 + 2*sqrt(d*f)*B*a^2*b^3*c^4*d^6*f^5 + 2*sqrt(d*f)
*A*a*b^4*c^4*d^6*f^5 - 8*sqrt(d*f)*C*a*b^4*c^5*d^5*f^4*e + 4*sqrt(d*f)*B*b^
5*c^5*d^5*f^4*e - 11*sqrt(d*f)*C*a^2*b^3*c^4*d^6*f^4*e - sqrt(d*f)*B*a*b^4*
c^4*d^6*f^4*e + 13*sqrt(d*f)*A*b^5*c^4*d^6*f^4*e + 24*sqrt(d*f)*C*a^3*b^2*c
^3*d^7*f^4*e - 8*sqrt(d*f)*B*a^2*b^3*c^3*d^7*f^4*e - 8*sqrt(d*f)*A*a*b^4*c^
3*d^7*f^4*e - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^2*C*a^2*b^3*c^4*d^4*f^4 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^4*d^4*f^4 + 9*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5
*c^4*d^4*f^4 + 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^2*C*a^3*b^2*c^3*d^5*f^4 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^3*d^5*f^4 - 28*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A
*a*b^4*c^3*d^5*f^4 - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)
*d*f - c*d*f + d^2*e))^2*C*a^4*b*c^2*d^6*f^4 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3*b^2*c^2*d^6*f^4 + 1
6*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^2*A*a^2*b^3*c^2*d^6*f^4 + 32*sqrt(d*f)*C*a*b^4*c^4*d^6*f^3*e^2 - 16*sqrt(d
*f)*B*b^5*c^4*d^6*f^3*e^2 - 6*sqrt(d*f)*C*a^2*b^3*c^3*d^7*f^3*e^2 + 14*sqrt
(d*f)*B*a*b^4*c^3*d^7*f^3*e^2 - 22*sqrt(d*f)*A*b^5*c^3*d^7*f^3*e^2 - 36*sqr
t(d*f)*C*a^3*b^2*c^2*d^8*f^3*e^2 + 12*sqrt(d*f)*B*a^2*b^3*c^2*d^8*f^3*e^2 +
12*sqrt(d*f)*A*a*b^4*c^2*d^8*f^3*e^2 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^4*c^4*d^4*f^3*e - 12*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b
^5*c^4*d^4*f^3*e - 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d
*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^3*d^5*f^3*e + 32*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^3*d^5*f^3*e
- 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^2*A*b^5*c^3*d^5*f^3*e - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*
x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^2*c^2*d^6*f^3*e - 16*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^2*
d^6*f^3*e + 52*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^2*A*a*b^4*c^2*d^6*f^3*e + 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c*d^7*f^3*e - 16*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3
*b^2*c*d^7*f^3*e - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d
*f - c*d*f + d^2*e))^2*A*a^2*b^3*c*d^7*f^3*e + 15*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*c^3*d^3*f^3 -
3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^4*B*a*b^4*c^3*d^3*f^3 - 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^4*A*b^5*c^3*d^3*f^3 - 58*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*c^2*d^4*f^3 +
14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e

```
))^4*B*a^2*b^3*c^2*d^4*f^3 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*c^2*d^4*f^3 + 88*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^4*b*c*d^5*f^
3 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^
2*e))^4*B*a^3*b^2*c*d^5*f^3 - 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^2*b^3*c*d^5*f^3 - 48*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^5*d^6*f^3 +
 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((...
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^3),x)
[Out] \text{Hanged}
```

3.53 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx$

Optimal. Leaf size=685

$$\frac{(4a^3Cdf - b^3(6Bce - 3Ade - 5Acf) + ab^2(12cCe + 3Bde + Bcf - 8Adf) - a^2b(9Cde + 7cCf - 2Bdf))\ \sqrt{}}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

```
[Out] -1/8*(-c*f+d*e)*(b^2*(A*d^2*e^2-2*c*d*e*(-A*f+B*e)+c^2*(5*A*f^2-6*B*e*f+8*C
*e^2))+a*b*(d^2*e*(-4*A*f+B*e)-c^2*f*(-B*f+4*C*e)-2*c*d*(6*A*f^2-7*B*e*f+6*
C*e^2))-a^2*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)
))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(
-a*d+b*c)^(5/2)/(-a*f+b*e)^(7/2)-1/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x
+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^3+1/12*(4*a^3*C*d*f-b^3*(-5*A*c*f
-3*A*d*e+6*B*c*e)+a*b^2*(-8*A*d*f+B*c*f+3*B*d*e+12*C*c*e)-a^2*b*(-2*B*d*f+7
*C*c*f+9*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b
*x+a)^2-1/24*(8*a^4*C*d^2*f^2-2*a^3*b*d*f*(-2*B*d*f+7*C*c*f+13*C*d*e)-b^4*(
3*A*d^2*e^2-2*c*d*e*(-2*A*f+3*B*e)-3*c^2*(5*A*f^2-6*B*e*f+8*C*e^2))-a*b^3*(
d^2*e*(-10*A*f+3*B*e)+3*c^2*f*(-B*f+4*C*e)+2*c*d*(13*A*f^2-14*B*e*f+30*C*e^
2))-a^2*b^2*(4*d*f*(-2*A*d*f+B*c*f+4*B*d*e)-C*(3*c^2*f^2+44*c*d*e*f+33*d^2*
e^2)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^2/(-a*f+b*e)^3/(b*x+a)
```

Rubi [A]

time = 1.22, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1627, 154, 156, 12, 95, 214}

$$-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-\left(a^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)\right)+ab\left(-2cd\left(6Af^2-7Bef+6Ce^2\right)+d^2e(Be-4Af)+c^2(-f)(4Ce-Bf)\right)+b^2\left(c^2\left(5Af^2-6Bef+8Ce^2\right)-2cde(Be-Af)+Ad^2e^2\right)\right)}{8(bc-ad)^{5/2}(be-af)^{7/2}}+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce)\right)}{12b^2(a+bx)^2(bc-ad)(be-af)^2}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^4Cd^2f^2-2a^3bdf(-2Bdf+7cCf+13Cde)-a^2b^2\left(4df(-2Adf+Bcf+4Bde)-C\left(3c^2f^2+44cdef+33d^2e^2\right)\right)-ab^3\left(2cd\left(13Af^2-14Bef+30Ce^2\right)+d^2e(3Be-10Af)+3c^2f(4Ce-Bf)\right)-b^4\left(-3c^2\left(5Af^2-6Bef+8Ce^2\right)-2cde(3Be-2Af)+3Ad^2e^2\right)\right)}{24b^2(a+bx)(bc-ad)^2(be-af)^3}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

```
[Out] ((4*a^3*C*d*f - b^3*(6*B*c*e - 3*A*d*e - 5*A*c*f) + a*b^2*(12*c*C*e + 3*B*d
*e + B*c*f - 8*A*d*f) - a^2*b*(9*C*d*e + 7*c*C*f - 2*B*d*f))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(12*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^2) - ((8*a^4*C*d
^2*f^2 - 2*a^3*b*d*f*(13*C*d*e + 7*c*C*f - 2*B*d*f) - b^4*(3*A*d^2*e^2 - 2*
c*d*e*(3*B*e - 2*A*f) - 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) - a*b^3*(d^2*e
*(3*B*e - 10*A*f) + 3*c^2*f*(4*C*e - B*f) + 2*c*d*(30*C*e^2 - 14*B*e*f + 13
*A*f^2)) - a^2*b^2*(4*d*f*(4*B*d*e + B*c*f - 2*A*d*f) - C*(33*d^2*e^2 + 44*
c*d*e*f + 3*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b^2*(b*c - a*d)^2*(
b*e - a*f)^3*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e +
 f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) - ((d*e - c*f)*(b^2*(A*d^2
*e^2 - 2*c*d*e*(B*e - A*f) + c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) + a*b*(d^2*
```

```
e*(B*e - 4*A*f) - c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 - 7*B*e*f + 6*A*f^2)
) - a^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2
*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f
*x])])/(8*(b*c - a*d)^(5/2)*(b*e - a*f)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
```

```
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+}{}\right.}{}}{}$$

$$= \frac{(4a^3Cdf - b^3(6Bce-3Ade-5Acf) + ab^2(12cCe+3Bde+Bcf-8A}{12b^2(bc-ad)(be-af)^2(}$$

$$= \frac{(4a^3Cdf - b^3(6Bce-3Ade-5Acf) + ab^2(12cCe+3Bde+Bcf-8A}{12b^2(bc-ad)(be-af)^2(}$$

$$= \frac{(4a^3Cdf - b^3(6Bce-3Ade-5Acf) + ab^2(12cCe+3Bde+Bcf-8A}{12b^2(bc-ad)(be-af)^2(}$$

$$= \frac{(4a^3Cdf - b^3(6Bce-3Ade-5Acf) + ab^2(12cCe+3Bde+Bcf-8A}{12b^2(bc-ad)(be-af)^2(}$$

$$= \frac{(4a^3Cdf - b^3(6Bce-3Ade-5Acf) + ab^2(12cCe+3Bde+Bcf-8A}{12b^2(bc-ad)(be-af)^2(}$$

Mathematica [A]
time = 11.90, size = 847, normalized size = 1.24

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

```
[Out] (-2*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*Sqrt[c + d*x]*Sqrt[e + f*x]*(8*(A*b^2 +
 a*(-(b*B) + a*C))*(b*c - a*d)^2*(b*e - a*f)^2 + 2*(b*c - a*d)*(b*e - a*f)*
(-8*a^3*C*d*f + b^3*(6*B*c*e + A*d*e - 5*A*c*f) - a*b^2*(12*c*C*e + 7*B*d*e
```

```
 + B*c*f - 4*A*d*f) + a^2*b*(13*C*d*e + 7*c*C*f + 2*B*d*f))*(a + b*x) + (8*
a^4*C*d^2*f^2 + 2*a^3*b*d*f*(-13*C*d*e - 7*c*C*f + 2*B*d*f) + b^4*(-3*A*d^2
*e^2 + 2*c*d*e*(3*B*e - 2*A*f) + 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) + a*b
^3*(d^2*e*(-3*B*e + 10*A*f) + 3*c^2*f*(-4*C*e + B*f) - 2*c*d*(30*C*e^2 - 14
*B*e*f + 13*A*f^2)) + a^2*b^2*(4*d*f*(-4*B*d*e - B*c*f + 2*A*d*f) + C*(33*d
^2*e^2 + 44*c*d*e*f + 3*c^2*f^2)))*(a + b*x)^2) + 3*b^2*(d*e - c*f)*(b^2*(A
*d^2*e^2 + 2*c*d*e*(-(B*e) + A*f) + c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) + a*
b*(d^2*e*(B*e - 4*A*f) + c^2*f*(-4*C*e + B*f) - 2*c*d*(6*C*e^2 - 7*B*e*f +
6*A*f^2)) + a^2*(2*d*f*(-3*B*d*e - B*c*f + 4*A*d*f) + C*(5*d^2*e^2 + 2*c*d*
e*f + c^2*f^2)))*(a + b*x)^3*Log[a + b*x] - 3*b^2*(d*e - c*f)*(b^2*(A*d^2*e
^2 + 2*c*d*e*(-(B*e) + A*f) + c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) + a*b*(d^2
*e*(B*e - 4*A*f) + c^2*f*(-4*C*e + B*f) - 2*c*d*(6*C*e^2 - 7*B*e*f + 6*A*f^
2)) + a^2*(2*d*f*(-3*B*d*e - B*c*f + 4*A*d*f) + C*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2)))*(a + b*x)^3*Log[2*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*Sqrt[c + d*x]*
Sqrt[e + f*x] + b*(2*c*e + d*e*x + c*f*x) - a*(d*e + c*f + 2*d*f*x)])/(48*b
^2*(b*c - a*d)^(5/2)*(b*e - a*f)^(7/2)*(a + b*x)^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 15989 vs. $2(653) = 1306$.
time = 0.10, size = 15990, normalized size = 23.34

method	result	size
default	Expression too large to display	15990

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**4/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 25485 vs. $2(676) = 1352$.
time = 67.23, size = 25485, normalized size = 37.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] -1/8*(sqrt(d*f)*C*a^2*c^3*d^2*f^3 + sqrt(d*f)*B*a*b*c^3*d^2*f^3 + 5*sqrt(d*
f)*A*b^2*c^3*d^2*f^3 - 2*sqrt(d*f)*B*a^2*c^2*d^3*f^3 - 12*sqrt(d*f)*A*a*b*c
^2*d^3*f^3 + 8*sqrt(d*f)*A*a^2*c*d^4*f^3 - 4*sqrt(d*f)*C*a*b*c^3*d^2*f^2*e
 - 6*sqrt(d*f)*B*b^2*c^3*d^2*f^2*e + sqrt(d*f)*C*a^2*c^2*d^3*f^2*e + 13*sqrt
(d*f)*B*a*b*c^2*d^3*f^2*e - 3*sqrt(d*f)*A*b^2*c^2*d^3*f^2*e - 4*sqrt(d*f)*B
*a^2*c*d^4*f^2*e + 8*sqrt(d*f)*A*a*b*c*d^4*f^2*e - 8*sqrt(d*f)*A*a^2*d^5*f^
2*e + 8*sqrt(d*f)*C*b^2*c^3*d^2*f*e^2 - 8*sqrt(d*f)*C*a*b*c^2*d^3*f*e^2 + 4
*sqrt(d*f)*B*b^2*c^2*d^3*f*e^2 + 3*sqrt(d*f)*C*a^2*c*d^4*f*e^2 - 13*sqrt(d*
f)*B*a*b*c*d^4*f*e^2 - sqrt(d*f)*A*b^2*c*d^4*f*e^2 + 6*sqrt(d*f)*B*a^2*d^5*
f*e^2 + 4*sqrt(d*f)*A*a*b*d^5*f*e^2 - 8*sqrt(d*f)*C*b^2*c^2*d^3*e^3 + 12*sq
rt(d*f)*C*a*b*c*d^4*e^3 + 2*sqrt(d*f)*B*b^2*c*d^4*e^3 - 5*sqrt(d*f)*C*a^2*d
^5*e^3 - sqrt(d*f)*B*a*b*d^5*e^3 - sqrt(d*f)*A*b^2*d^5*e^3)*arctan(-1/2*(b*
c*d*f - 2*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b
*d^2*f*e)*d))/((a^3*b^2*c^2*f^3*abs(d) - 2*a^4*b*c*d*f^3*abs(d) + a^5*d^2*f
^3*abs(d) - 3*a^2*b^3*c^2*f^2*abs(d)*e + 6*a^3*b^2*c*d*f^2*abs(d)*e - 3*a^4
*b*d^2*f^2*abs(d)*e + 3*a*b^4*c^2*f*abs(d)*e^2 - 6*a^2*b^3*c*d*f*abs(d)*e^2
 + 3*a^3*b^2*d^2*f*abs(d)*e^2 - b^5*c^2*abs(d)*e^3 + 2*a*b^4*c*d*abs(d)*e^3
 - a^2*b^3*d^2*abs(d)*e^3)*sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a
```

*b*d^2*f*e)*d) + 1/12*(3*sqrt(d*f)*C*a^2*b^5*c^8*d^7*f^8 + 3*sqrt(d*f)*B*a*
b^6*c^8*d^7*f^8 + 15*sqrt(d*f)*A*b^7*c^8*d^7*f^8 - 14*sqrt(d*f)*C*a^3*b^4*c
^7*d^8*f^8 - 4*sqrt(d*f)*B*a^2*b^5*c^7*d^8*f^8 - 26*sqrt(d*f)*A*a*b^6*c^7*d
^8*f^8 + 8*sqrt(d*f)*C*a^4*b^3*c^6*d^9*f^8 + 4*sqrt(d*f)*B*a^3*b^4*c^6*d^9*
f^8 + 8*sqrt(d*f)*A*a^2*b^5*c^6*d^9*f^8 - 12*sqrt(d*f)*C*a*b^6*c^8*d^7*f^7*
e - 18*sqrt(d*f)*B*b^7*c^8*d^7*f^7*e + 26*sqrt(d*f)*C*a^2*b^5*c^7*d^8*f^7*e
 + 10*sqrt(d*f)*B*a*b^6*c^7*d^8*f^7*e - 94*sqrt(d*f)*A*b^7*c^7*d^8*f^7*e +
58*sqrt(d*f)*C*a^3*b^4*c^6*d^9*f^7*e + 8*sqrt(d*f)*B*a^2*b^5*c^6*d^9*f^7*e
+ 166*sqrt(d*f)*A*a*b^6*c^6*d^9*f^7*e - 48*sqrt(d*f)*C*a^4*b^3*c^5*d^10*f^7
*e - 24*sqrt(d*f)*B*a^3*b^4*c^5*d^10*f^7*e - 48*sqrt(d*f)*A*a^2*b^5*c^5*d^1
0*f^7*e - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*
f + d^2*e))^2*C*a^2*b^5*c^7*d^6*f^7 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^6*c^7*d^6*f^7 - 75*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^7*c
^7*d^6*f^7 + 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^2*C*a^3*b^4*c^6*d^7*f^7 + 54*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^5*c^6*d^7*f^7 + 300*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A
*a*b^6*c^6*d^7*f^7 - 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c
)*d*f - c*d*f + d^2*e))^2*C*a^4*b^3*c^5*d^8*f^7 - 72*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3*b^4*c^5*d^8*f^7
 - 336*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^
2*e))^2*A*a^2*b^5*c^5*d^8*f^7 + 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^5*b^2*c^4*d^9*f^7 + 48*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^4*b^3*c
^4*d^9*f^7 + 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^2*A*a^3*b^4*c^4*d^9*f^7 + 24*sqrt(d*f)*C*b^7*c^8*d^7*f^6*e^2
 + 12*sqrt(d*f)*C*a*b^6*c^7*d^8*f^6*e^2 + 114*sqrt(d*f)*B*b^7*c^7*d^8*f^6*e
^2 - 186*sqrt(d*f)*C*a^2*b^5*c^6*d^9*f^6*e^2 - 126*sqrt(d*f)*B*a*b^6*c^6*d^
9*f^6*e^2 + 246*sqrt(d*f)*A*b^7*c^6*d^9*f^6*e^2 - 54*sqrt(d*f)*C*a^3*b^4*c^
5*d^10*f^6*e^2 + 36*sqrt(d*f)*B*a^2*b^5*c^5*d^10*f^6*e^2 - 450*sqrt(d*f)*A*
a*b^6*c^5*d^10*f^6*e^2 + 120*sqrt(d*f)*C*a^4*b^3*c^4*d^11*f^6*e^2 + 60*sqrt
(d*f)*B*a^3*b^4*c^4*d^11*f^6*e^2 + 120*sqrt(d*f)*A*a^2*b^5*c^4*d^11*f^6*e^2
 + 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2
*e))^2*C*a*b^6*c^7*d^6*f^6*e + 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b^7*c^7*d^6*f^6*e - 219*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^5*c^6
*d^7*f^6*e - 303*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^2*B*a*b^6*c^6*d^7*f^6*e + 225*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^7*c^6*d^7*f^6*e + 192*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C
*a^3*b^4*c^5*d^8*f^6*e + 228*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
 + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^5*c^5*d^8*f^6*e - 1128*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^6*c^5*d
^8*f^6*e + 264*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*

```
d*f + d^2*e))^2*C*a^4*b^3*c^4*d^9*f^6*e + 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3*b^4*c^4*d^9*f^6*e + 139
2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*...
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^4),x)
[Out] \text{Hanged}
```

3.54 $\int \frac{(a+bx)^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=718

$$-\frac{(2aCdf-b(8Bdf-7C(de+cf)))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2}+\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-\sqrt{c+d$$

```
[Out] 1/64*(16*a^2*d^2*f^2*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c
*f+d*e)))-16*a*b*d*f*(C*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3)+2
*d*f*(4*A*d*f*(c*f+d*e)-B*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)))+b^2*(C*(35*c^4*
f^4+20*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+20*c*d^3*e^3*f+35*d^4*e^4)+8*d*f*(2*A
*d*f*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)-B*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2
*f+5*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(9/
2)/f^(9/2)-1/24*(2*a*C*d*f-b*(8*B*d*f-7*C*(c*f+d*e)))*(b*x+a)^2*(d*x+c)^(1/
2)*(f*x+e)^(1/2)/b/d^2/f^2+1/4*C*(b*x+a)^3*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/
f-1/192*(32*a^3*C*d^3*f^3-8*a^2*b*d^2*f^2*(16*B*d*f-11*C*(c*f+d*e))-16*a*b^
2*d*f*(C*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)+6*d*f*(4*A*d*f-3*B*(c*f+d*e)))+
b^3*(5*C*(21*c^3*f^3+19*c^2*d*e*f^2+19*c*d^2*e^2*f+21*d^3*e^3)+8*d*f*(18*A*
d*f*(c*f+d*e)-B*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)))+2*b*d*f*(6*b*d*f*(-8*A
*b*d*f+C*a*c*f+C*a*d*e+6*C*b*c*e)+(4*a*d*f-5*b*(c*f+d*e))*(2*a*C*d*f-b*(8*B
*d*f-7*C*(c*f+d*e))))*x)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^4/f^4
```

Rubi [A]
time = 0.86, antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1629, 158, 152, 65, 223, 212}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((8*b*B*d*f - 2*a*C*d*f - 7*b*C*(d*e + c*f))*(a + b*x)^2*Sqrt[c + d*x]*Sqrt
[e + f*x])/(24*b*d^2*f^2) + (C*(a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*
b*d*f) - (Sqrt[c + d*x]*Sqrt[e + f*x]*(32*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(
16*B*d*f - 11*C*(d*e + c*f)) - 16*a*b^2*d*f*(C*(15*d^2*e^2 + 14*c*d*e*f + 1
5*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))) + b^3*(5*C*(21*d^3*e^3 + 19
*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 21*c^3*f^3) + 8*d*f*(18*A*d*f*(d*e + c*f) -
 B*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))) + 2*b*d*f*(6*b*d*f*(6*b*c*C*e +
 a*C*d*e + a*c*C*f - 8*A*b*d*f) - (4*a*d*f - 5*b*(d*e + c*f))*(8*b*B*d*f -
2*a*C*d*f - 7*b*C*(d*e + c*f)))*x))/(192*b*d^4*f^4) + ((16*a^2*d^2*f^2*(C*(
3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) - 16*
a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f*
```

```
(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))) + b^2*(C*(35
*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 + 35*c^4*f^
4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) - B*(5*d^3*e^3 + 3*
c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(9/2)*f^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = \frac{C(a+bx)^3\sqrt{c+dx}\,\sqrt{e+fx}}{4bdf} + \frac{\int \frac{(a+bx)^2\left(-\frac{1}{2}b(6bcCe+aCde+acCf-8Abdf)+\frac{1}{2}b(\cdots}{\sqrt{c+dx}\,\sqrt{e+\cdots}}}{4b^2df}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+\cdots}{\cdots}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+\cdots}{\cdots}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+\cdots}{\cdots}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+\cdots}{\cdots}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+\cdots}{\cdots}$$

Mathematica [A]
time = 2.56, size = 632, normalized size = 0.88

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*Sqrt[e + f*x]*(48*a^2*d^2*f^2*(4*B*d*f + C*(
-3*d*e - 3*c*f + 2*d*f*x)) + 16*a*b*d*f*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c*f
 + 2*d*f*x)) + C*(15*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f
*x + 8*f^2*x^2))) + b^2*(-(C*(105*c^3*f^3 + 5*c^2*d*f^2*(19*e - 14*f*x) + c
*d^2*f*(95*e^2 - 68*e*f*x + 56*f^2*x^2) + d^3*(105*e^3 - 70*e^2*f*x + 56*e*
f^2*x^2 - 48*f^3*x^3))) + 8*d*f*(6*A*d*f*(-3*d*e - 3*c*f + 2*d*f*x) + B*(15
*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)))))
+ 3*(16*a^2*d^2*f^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f
 - B*(d*e + c*f))) - 16*a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f
^2 + 5*c^3*f^3) + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3
*c^2*f^2))) + b^2*(C*(35*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20
*c^3*d*e*f^3 + 35*c^4*f^4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*
f^2) - B*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh
[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(192*d^(9/2)*f^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(686) = 1372$.
time = 0.10, size = 2528, normalized size = 3.52

method	result	size
default	Expression too large to display	2528

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/384*(96*C*b^2*d^3*f^3*x^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+128*B*b^2*d
^3*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+54*C*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^2*e^2*f^2+60
*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*b^2*c*d^3*e^3*f-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^3*f+144*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^2*d^2*f^4+144*C*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*
d^4*e^2*f^2+384*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^3*f^3-210*C*(d*
f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^3*f^3-210*C*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*b^2*d^3*e^3+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^2*f^4+144*A*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^2*f^2-192*B
*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2)
)*a^2*c*d^3*f^4-192*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*a^2*d^4*e*f^3-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d*f^4-320*C*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^2*f^3*x-320*C*((d*x+c)*(f*x+e))^(1/2)*(d*f
```

)^(1/2)*a*b*d^3*e*f^2*x+136*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^2
*e*f^2*x+192*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*d^3*f^3*x+96*A*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*
c*d^3*e*f^3-72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*b^2*c^2*d^2*e*f^3-72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e^2*f^2+192*C*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)*a^2*d^3*f^3*x+96*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
x+e))^(1/2)(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^3*e*f^3+60*C*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^
3*d*e*f^3+448*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d^2*e*f^2-384*A*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
a*b*c*d^3*f^4-384*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*a*b*d^4*e*f^3+288*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^2*f^4+288*B*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^4*e^
2*f^2+480*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c^2*d*f^3+480*C*(d*f)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^3*e^2*f-190*C*(d*f)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)*b^2*c^2*d*e*f^2-190*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d
^2*e^2*f+140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*d^3*e^2*f*x-144*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a
*b*c^2*d^2*e*f^3-144*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^3*e^2*f^2+384*B*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)*a*b*d^3*f^3*x-240*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^3*d*f^4-240*C*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^4*e^3*f+768*A*(d*f
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^3*f^3-288*A*(d*f)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*b^2*c*d^2*f^3-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^3*
e*f^2+240*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^2*d*f^3+240*B*(d*f)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^3*e^2*f-288*C*(d*f)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)*a^2*c*d^2*f^3-288*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^3*e
*f^2-160*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^2*f^3*x-160*B*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*d^3*e*f^2*x+192*B*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^3*e*f^3+384*A*
ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*a^2*d^4*f^4+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*b^2*c^4*f^4+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^4-576*B*(d*f)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)*a*b*d^3*e*f^2+224*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b
^2*c*d^2*e*f^2+140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d*f^3*x+25
6*C*a*b*d^3*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-112*C*b^2*c*d^2*f^3
x^2((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-112*C*b^2*d^3*e*f^2*x^2*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)-576*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d
^2*f^3)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/f^4/d^4/(d*f)^(1/2)/((d*x+c)*(f*x+e))^(
1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 3.73, size = 1441, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/768*(3*(35*C*b^2*d^4*e^4 + (35*C*b^2*c^4 + 128*A*a^2*d^4 - 40*(2*C*a*b +
 B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - 64*(B*a^2 + 2*A*a*b)
*c*d^3)*f^4 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*(C*a^2 + 2
*B*a*b + A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*f^3*e + 6*(3*C*b^2*c^2*d^
2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*f^2*e^2 +
20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*f*e^3)*sqrt(d*f)*log(8*d^2*f^2*x
^2 + 8*c*d*f^2*x + c^2*f^2 + d^2*e^2 + 4*(2*d*f*x + c*f + d*e)*sqrt(d*f)*sq
rt(d*x + c)*sqrt(f*x + e) + 2*(4*d^2*f*x + 3*c*d*f)*e) + 4*(48*C*b^2*d^4*f^
4*x^3 - 105*C*b^2*d^4*f*e^3 - 8*(7*C*b^2*c*d^3 - 8*(2*C*a*b + B*b^2)*d^4)*f
^4*x^2 + 2*(35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*b^2)*c*d^3 + 48*(C*a^2 + 2*B
*a*b + A*b^2)*d^4)*f^4*x - 3*(35*C*b^2*c^3*d - 40*(2*C*a*b + B*b^2)*c^2*d^2
 + 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 64*(B*a^2 + 2*A*a*b)*d^4)*f^4 + 5*(
14*C*b^2*d^4*f^2*x - (19*C*b^2*c*d^3 - 24*(2*C*a*b + B*b^2)*d^4)*f^2)*e^2 -
 (56*C*b^2*d^4*f^3*x^2 - 4*(17*C*b^2*c*d^3 - 20*(2*C*a*b + B*b^2)*d^4)*f^3*
x + (95*C*b^2*c^2*d^2 - 112*(2*C*a*b + B*b^2)*c*d^3 + 144*(C*a^2 + 2*B*a*b
+ A*b^2)*d^4)*f^3)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5), -1/384*(3*(35
*C*b^2*d^4*e^4 + (35*C*b^2*c^4 + 128*A*a^2*d^4 - 40*(2*C*a*b + B*b^2)*c^3*d
 + 48*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - 64*(B*a^2 + 2*A*a*b)*c*d^3)*f^4 +
 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*(C*a^2 + 2*B*a*b + A*b^
2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*f^3*e + 6*(3*C*b^2*c^2*d^2 - 4*(2*C*a*
b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*f^2*e^2 + 20*(C*b^2*c*d
^3 - 2*(2*C*a*b + B*b^2)*d^4)*f*e^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + c*f +
```

```
d*e)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*f^2*x + (d^
2*f*x + c*d*f)*e)) - 2*(48*C*b^2*d^4*f^4*x^3 - 105*C*b^2*d^4*f*e^3 - 8*(7*C
*b^2*c*d^3 - 8*(2*C*a*b + B*b^2)*d^4)*f^4*x^2 + 2*(35*C*b^2*c^2*d^2 - 40*(2
*C*a*b + B*b^2)*c*d^3 + 48*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*f^4*x - 3*(35*C*b
^2*c^3*d - 40*(2*C*a*b + B*b^2)*c^2*d^2 + 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^
3 - 64*(B*a^2 + 2*A*a*b)*d^4)*f^4 + 5*(14*C*b^2*d^4*f^2*x - (19*C*b^2*c*d^3
 - 24*(2*C*a*b + B*b^2)*d^4)*f^2)*e^2 - (56*C*b^2*d^4*f^3*x^2 - 4*(17*C*b^2
*c*d^3 - 20*(2*C*a*b + B*b^2)*d^4)*f^3*x + (95*C*b^2*c^2*d^2 - 112*(2*C*a*b
 + B*b^2)*c*d^3 + 144*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*f^3)*e)*sqrt(d*x + c)*
sqrt(f*x + e))/(d^5*f^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [A]
time = 1.33, size = 951, normalized size = 1.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/192*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*
x + c)*C*b^2/(d^5*f) - (25*C*b^2*c*d^19*f^6 - 16*C*a*b*d^20*f^6 - 8*B*b^2*d
^20*f^6 + 7*C*b^2*d^20*f^5*e)/(d^24*f^7)) + (163*C*b^2*c^2*d^19*f^6 - 208*C
*a*b*c*d^20*f^6 - 104*B*b^2*c*d^20*f^6 + 48*C*a^2*d^21*f^6 + 96*B*a*b*d^21*
f^6 + 48*A*b^2*d^21*f^6 + 90*C*b^2*c*d^20*f^5*e - 80*C*a*b*d^21*f^5*e - 40*
B*b^2*d^21*f^5*e + 35*C*b^2*d^21*f^4*e^2)/(d^24*f^7)) - 3*(93*C*b^2*c^3*d^1
9*f^6 - 176*C*a*b*c^2*d^20*f^6 - 88*B*b^2*c^2*d^20*f^6 + 80*C*a^2*c*d^21*f^
6 + 160*B*a*b*c*d^21*f^6 + 80*A*b^2*c*d^21*f^6 - 64*B*a^2*d^22*f^6 - 128*A*
a*b*d^22*f^6 + 73*C*b^2*c^2*d^20*f^5*e - 128*C*a*b*c*d^21*f^5*e - 64*B*b^2*
c*d^21*f^5*e + 48*C*a^2*d^22*f^5*e + 96*B*a*b*d^22*f^5*e + 48*A*b^2*d^22*f^
5*e + 55*C*b^2*c*d^21*f^4*e^2 - 80*C*a*b*d^22*f^4*e^2 - 40*B*b^2*d^22*f^4*e
^2 + 35*C*b^2*d^22*f^3*e^3)/(d^24*f^7))*sqrt(d*x + c) - 3*(35*C*b^2*c^4*f^4
 - 80*C*a*b*c^3*d*f^4 - 40*B*b^2*c^3*d*f^4 + 48*C*a^2*c^2*d^2*f^4 + 96*B*a*
b*c^2*d^2*f^4 + 48*A*b^2*c^2*d^2*f^4 - 64*B*a^2*c*d^3*f^4 - 128*A*a*b*c*d^3
```

```
*f^4 + 128*A*a^2*d^4*f^4 + 20*C*b^2*c^3*d*f^3*e - 48*C*a*b*c^2*d^2*f^3*e -
24*B*b^2*c^2*d^2*f^3*e + 32*C*a^2*c*d^3*f^3*e + 64*B*a*b*c*d^3*f^3*e + 32*A
*b^2*c*d^3*f^3*e - 64*B*a^2*d^4*f^3*e - 128*A*a*b*d^4*f^3*e + 18*C*b^2*c^2*
d^2*f^2*e^2 - 48*C*a*b*c*d^3*f^2*e^2 - 24*B*b^2*c*d^3*f^2*e^2 + 48*C*a^2*d^
4*f^2*e^2 + 96*B*a*b*d^4*f^2*e^2 + 48*A*b^2*d^4*f^2*e^2 + 20*C*b^2*c*d^3*f*
e^3 - 80*C*a*b*d^4*f*e^3 - 40*B*b^2*d^4*f*e^3 + 35*C*b^2*d^4*e^4)*log(abs(-
sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*
d^4*f^4))*d/abs(d)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.55 $\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=371

$$\frac{C(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{3bdf}-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(8a^2Cd^2f^2-6abdf(4Bdf-3C(de+cf))-b^2(C(15d^2e\right.}{}$$

```
[Out] 1/8*(2*a*d*f*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e))
)-b*(C*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3)+2*d*f*(4*A*d*f*(c*
f+d*e)-B*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(7/2)+1/3*C*(b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)
^(1/2)/b/d/f-1/24*(8*a^2*C*d^2*f^2-6*a*b*d*f*(4*B*d*f-3*C*(c*f+d*e))-b^2*(C
*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)+6*d*f*(4*A*d*f-3*B*(c*f+d*e)))+2*b*d*f*
(2*a*C*d*f-b*(6*B*d*f-5*C*(c*f+d*e)))*x)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^3/
f^3
```

Rubi [A]
time = 0.33, antiderivative size = 369, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1629, 152, 65, 223, 212}

$$-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))-\left(b^2\left(6df(4Adf-3B(cf+de))+C\left(15c^2f^2+14cdef+15d^2e^2\right)\right)\right)\right)}{24bd^3f^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)\left(2adf\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)-b\left(2df\left(4Adf(cf+de)-B\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+C\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)\right)}{8d^{7/2}f^{7/2}}+\frac{C(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{3bdf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - (Sqrt[c + d*x]*Sqrt
[e + f*x]*(8*a^2*C*d^2*f^2 - 6*a*b*d*f*(4*B*d*f - 3*C*(d*e + c*f)) - b^2*(C
*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))
) - 2*b*d*f*(6*b*B*d*f - 2*a*C*d*f - 5*b*C*(d*e + c*f))*x))/(24*b*d^3*f^3)
+ ((2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*
e + c*f))) - b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) +
 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*ArcT
anh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*d^(7/2)*f^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = \frac{C(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{3bdf} + \frac{\int \frac{(a+bx)\left(-\frac{1}{2}b(4bcCe+aCde+acCf-6Abdf)+\frac{1}{2}b(6b\ldots}{\sqrt{c+dx}\,\sqrt{e+f\ldots}}}{3b^2df}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(8a^2Cd^2f^2-6abdf(\ldots}{\ldots}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(8a^2Cd^2f^2-6abdf(\ldots}{\ldots}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(8a^2Cd^2f^2-6abdf(\ldots}{\ldots}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(8a^2Cd^2f^2-6abdf(\ldots}{\ldots}$$

Mathematica [**A**]
time = 1.06, size = 314, normalized size = 0.85

$$\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(6adf(4Bdf+C(-3de-3cf+2dfx))+b(6df(4Adf+B(-3de-3cf+2dfx))+C(15c^2f^2+2cdf(7e-5fx)+d^2(15e^2-10efx+8f^2x^2))))}{24d^3f^3} - \frac{(-2adf(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))+b(C(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3)+2df(4Adf(de+cf)-B(3d^2e^2+2cdef+3c^2f^2))))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{e+fx}}{\sqrt{f}\,\sqrt{c+dx}}\right)}{8d^{7/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(6*a*d*f*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f*
x)) + b*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c*f + 2*d*f*x)) + C*(15*c^2*f^2 + 2
*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)))))/(24*d^3*f^3)
 - ((-2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(
d*e + c*f))) + b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3)
 + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*Ar
cTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(8*d^(7/2)*f^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(345) = 690$.
time = 0.11, size = 1199, normalized size = 3.23

method	result

default	$\frac{(48B\sqrt{df}\ \sqrt{(dx+c)(fx+e)}\ a\,d^{2}f^{2}+30C\sqrt{df}\ \sqrt{(dx+c)(fx+e)}\ b\,c^{2}f^{2}+30C\sqrt{df}\ \sqrt{(dx+c)(fx}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/48*(48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^2*f^2+30*C*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)*b*c^2*f^2+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b
*d^2*e^2-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*a*c*d^2*f^3-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*e*f^2+18*B*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d*f^3+18*B*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*
e^2*f+18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*a*c^2*d*f^3+18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*e^2*f+48*A*(d*f)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)*b*d^2*f^2-24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e*f^2-24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*f^3+28*C*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*b*c*d*e*f+16*C*b*d^2*f^2*x^2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*
e)/(d*f)^(1/2))*a*d^3*f^3-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*f^3-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e^3-36*B*(d*f)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*b*d^2*e*f-9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d*e*f^2-9*C*ln(1/2*(2*d*f*x+2*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*e^2*f-36*
B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d*f^2+24*B*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2)*b*d^2*f^2*x+12*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*e*f^2+24*C*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)*a*d^2*f^2*x+12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^2*e*f^2-20*C*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)*b*d^2*e*f*x-36*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d*f^2-36
*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^2*e*f-20*C*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)*b*c*d*f^2*x)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/f^3/d^3/(d*f)^(1/2)
/((d*x+c)*(f*x+e))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 2.08, size = 729, normalized size = 1.96

$$\left[\frac{3\,{\left(5\,C b d^{3} e^{3} + {\left(5\,C b c^{3} - 16\,A a d^{3} - 6\,{\left(C a + B b\right)} c^{2} d + 8\,{\left(B a + A b\right)} c d^{2}\right)} f^{3} + {\left(3\,C b c^{2} d - 4\,{\left(C a + B b\right)} c d^{2} + 8\,{\left(B a + A b\right)} d^{3}\right)} f^{2} e + 3\,{\left(C b c d^{2} - 2\,{\left(C a + B b\right)} d^{3}\right)} f e^{2}\right)} \sqrt{d f} \log\left(8\,d^{2} f^{2} x^{2} + 8\,c d f^{2} x + c^{2} f^{2} + d^{2} e^{2} - 4\,{\left(2\,d f x + c f + d e\right)} \sqrt{d f} \sqrt{d x + c} \sqrt{f x + e} + 2\,{\left(4\,d^{2} f x + 3\,c d f\right)} e\right) + 4\,{\left(8\,C b d^{3} f^{3} x^{2} + 15\,C b d^{3} f e^{2} - 2\,{\left(5\,C b c d^{2} - 6\,{\left(C a + B b\right)} d^{3}\right)} f^{3} x + 3\,{\left(5\,C b c^{2} d - 6\,{\left(C a + B b\right)} c d^{2} + 8\,{\left(B a + A b\right)} d^{3}\right)} f^{3} - 2\,{\left(5\,C b d^{3} f^{2} x - {\left(7\,C b c d^{2} - 9\,{\left(C a + B b\right)} d^{3}\right)} f^{2}\right)} e\right)} \sqrt{d x + c} \sqrt{f x + e}}{96\,d^{4} f^{4}}, \frac{3\,{\left(5\,C b d^{3} e^{3} + {\left(5\,C b c^{3} - 16\,A a d^{3} - 6\,{\left(C a + B b\right)} c^{2} d + 8\,{\left(B a + A b\right)} c d^{2}\right)} f^{3} + {\left(3\,C b c^{2} d - 4\,{\left(C a + B b\right)} c d^{2} + 8\,{\left(B a + A b\right)} d^{3}\right)} f^{2} e + 3\,{\left(C b c d^{2} - 2\,{\left(C a + B b\right)} d^{3}\right)} f e^{2}\right)} \sqrt{-d f} \arctan\left(\frac{{\left(2\,d f x + c f + d e\right)} \sqrt{-d f} \sqrt{d x + c} \sqrt{f x + e}}{2\,{\left(d^{2} f^{2} x^{2} + c d f^{2} x + {\left(d^{2} f x + c d f\right)} e\right)}}\right) + 2\,{\left(8\,C b d^{3} f^{3} x^{2} + 15\,C b d^{3} f e^{2} - 2\,{\left(5\,C b c d^{2} - 6\,{\left(C a + B b\right)} d^{3}\right)} f^{3} x + 3\,{\left(5\,C b c^{2} d - 6\,{\left(C a + B b\right)} c d^{2} + 8\,{\left(B a + A b\right)} d^{3}\right)} f^{3} - 2\,{\left(5\,C b d^{3} f^{2} x - {\left(7\,C b c d^{2} - 9\,{\left(C a + B b\right)} d^{3}\right)} f^{2}\right)} e\right)} \sqrt{d x + c} \sqrt{f x + e}}{48\,d^{4} f^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/96*(3*(5*C*b*d^3*e^3 + (5*C*b*c^3 - 16*A*a*d^3 - 6*(C*a + B*b)*c^2*d + 8
*(B*a + A*b)*c*d^2)*f^3 + (3*C*b*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b
)*d^3)*f^2*e + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*f*e^2)*sqrt(d*f)*log(8*d^2
*f^2*x^2 + 8*c*d*f^2*x + c^2*f^2 + d^2*e^2 - 4*(2*d*f*x + c*f + d*e)*sqrt(d
*f)*sqrt(d*x + c)*sqrt(f*x + e) + 2*(4*d^2*f*x + 3*c*d*f)*e) + 4*(8*C*b*d^3
*f^3*x^2 + 15*C*b*d^3*f*e^2 - 2*(5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3*x + 3
*(5*C*b*c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b*d^3
*f^2*x - (7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*f^2)*e)*sqrt(d*x + c)*sqrt(f*x +
 e))/(d^4*f^4), 1/48*(3*(5*C*b*d^3*e^3 + (5*C*b*c^3 - 16*A*a*d^3 - 6*(C*a +
 B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3 + (3*C*b*c^2*d - 4*(C*a + B*b)*c*d^2
 + 8*(B*a + A*b)*d^3)*f^2*e + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*f*e^2)*sqrt
(-d*f)*arctan(1/2*(2*d*f*x + c*f + d*e)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x +
 e)/(d^2*f^2*x^2 + c*d*f^2*x + (d^2*f*x + c*d*f)*e)) + 2*(8*C*b*d^3*f^3*x^2
 + 15*C*b*d^3*f*e^2 - 2*(5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3*x + 3*(5*C*b*
c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b*d^3*f^2*x -
 (7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*f^2)*e)*sqrt(d*x + c)*sqrt(f*x + e))/(d^
4*f^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)\left(A + Bx + Cx^2\right)}{\sqrt{c + dx}\,\sqrt{e + fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [A]
time = 1.07, size = 447, normalized size = 1.20

$$\frac{\left(\sqrt{(dx+c)df-cdf+d^2e}\,\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)Cb}{d^4f}-\frac{13\,Cbcd^{11}f^4-6\,Cad^{12}f^4-6\,Bbd^{12}f^4+5\,Cbd^{12}f^3e}{d^{15}f^5}\right)+\frac{3\,(11\,Cbc^2d^{11}f^4-10\,Cacd^{12}f^4-10\,Bbcd^{12}f^4+8\,Bad^{13}f^4+8\,Abd^{13}f^4+8\,Cbcd^{12}f^3e-6\,Cad^{13}f^3e-6\,Bbd^{13}f^3e+5\,Cbd^{13}f^2e^2)}{d^{15}f^5}\right)+\frac{3\,(5\,Cbc^3f^3-6\,Cac^2df^3-6\,Bbc^2df^3+8\,Bacd^2f^3+8\,Abcd^2f^3-16\,Aad^3f^3+3\,Cbc^2df^2e-4\,Cacd^2f^2e-4\,Bbcd^2f^2e+8\,Bad^3f^2e+8\,Abd^3f^2e+3\,Cbcd^2fe^2-6\,Cad^3fe^2-6\,Bbd^3fe^2+5\,Cbd^3e^3)\log\left(\left|-\sqrt{df}\,\sqrt{dx+c}+\sqrt{(dx+c)df-cdf+d^2e}\right|\right)}{\sqrt{df}\,d^3f^3}\right)d}{24\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/24*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)*C*b/(d^4*f) - (13*C*b*c*d^11*f^4 - 6*C*a*d^12*f^4 - 6*B*b*d^12*f^4 +
 5*C*b*d^12*f^3*e)/(d^15*f^5)) + 3*(11*C*b*c^2*d^11*f^4 - 10*C*a*c*d^12*f^4
 - 10*B*b*c*d^12*f^4 + 8*B*a*d^13*f^4 + 8*A*b*d^13*f^4 + 8*C*b*c*d^12*f^3*e
 - 6*C*a*d^13*f^3*e - 6*B*b*d^13*f^3*e + 5*C*b*d^13*f^2*e^2)/(d^15*f^5)) +
3*(5*C*b*c^3*f^3 - 6*C*a*c^2*d*f^3 - 6*B*b*c^2*d*f^3 + 8*B*a*c*d^2*f^3 + 8*
A*b*c*d^2*f^3 - 16*A*a*d^3*f^3 + 3*C*b*c^2*d*f^2*e - 4*C*a*c*d^2*f^2*e - 4*
B*b*c*d^2*f^2*e + 8*B*a*d^3*f^2*e + 8*A*b*d^3*f^2*e + 3*C*b*c*d^2*f*e^2 - 6
*C*a*d^3*f*e^2 - 6*B*b*d^3*f*e^2 + 5*C*b*d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d
*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^3*f^3))*d/abs(
d)
```

Mupad [B]
time = 105.19, size = 2500, normalized size = 6.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((((c + d*x)^(1/2) - c^(1/2))*(2*A*b*c*f + 2*A*b*d*e))/(f^3*((e + f*x)^(1/2
) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1/2))^3*(2*A*b*c*f + 2*A*b*d*e))/(d*
f^2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*A*b*c^(1/2)*e^(1/2)*((c + d*x)^(1/2
) - c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/((((c + d*x)^(1/2) - c^
(1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) -
c^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/
2))*((3*C*a*d^3*e^2)/2 + (3*C*a*c^2*d*f^2)/2 + C*a*c*d^2*e*f))/(f^6*((e + f
*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*a*c^2*f^2)/2
+ (11*C*a*d^2*e^2)/2 + 25*C*a*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3)
 + (((c + d*x)^(1/2) - c^(1/2))^7*((3*C*a*c^2*f^2)/2 + (3*C*a*d^2*e^2)/2 +
C*a*c*d*e*f))/(d^2*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) -
 c^(1/2))^5*((11*C*a*c^2*f^2)/2 + (11*C*a*d^2*e^2)/2 + 25*C*a*c*d*e*f))/(d*
f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^
(1/2))^4*(32*C*a*c*f + 32*C*a*d*e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4))/((
(c + d*x)^(1/2) - c^(1/2))^8/((e + f*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d
*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*
```

```
((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2
*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e^(1/2))^4)) - ((((
c + d*x)^(1/2) - c^(1/2))^3*((85*C*b*d^4*e^3)/12 + (85*C*b*c^3*d*f^3)/12 +
(17*C*b*c*d^3*e^2*f)/4 + (17*C*b*c^2*d^2*e*f^2)/4))/(f^8*((e + f*x)^(1/2) -
 e^(1/2))^3) - (((c + d*x)^(1/2) - c^(1/2))*((5*C*b*d^5*e^3)/4 + (5*C*b*c^3
*d^2*f^3)/4 + (3*C*b*c*d^4*e^2*f)/4 + (3*C*b*c^2*d^3*e*f^2)/4))/(f^9*((e +
f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^5*((33*C*b*c^3*f^3)/2
 + (33*C*b*d^3*e^3)/2 + (327*C*b*c*d^2*e^2*f)/2 + (327*C*b*c^2*d*e*f^2)/2))
/(f^7*((e + f*x)^(1/2) - e^(1/2))^5) - (((c + d*x)^(1/2) - c^(1/2))^11*((5*
C*b*c^3*f^3)/4 + (5*C*b*d^3*e^3)/4 + (3*C*b*c*d^2*e^2*f)/4 + (3*C*b*c^2*d*e
*f^2)/4))/(d^3*f^4*((e + f*x)^(1/2) - e^(1/2))^11) + (((c + d*x)^(1/2) - c^
(1/2))^9*((85*C*b*c^3*f^3)/12 + (85*C*b*d^3*e^3)/12 + (17*C*b*c*d^2*e^2*f)/
4 + (17*C*b*c^2*d*e*f^2)/4))/(d^2*f^5*((e + f*x)^(1/2) - e^(1/2))^9) - (((c
 + d*x)^(1/2) - c^(1/2))^7*((33*C*b*c^3*f^3)/2 + (33*C*b*d^3*e^3)/2 + (327*
C*b*c*d^2*e^2*f)/2 + (327*C*b*c^2*d*e*f^2)/2))/(d*f^6*((e + f*x)^(1/2) - e^
(1/2))^7) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6*(128*C*b*c^2*f^2
 + 128*C*b*d^2*e^2 + (896*C*b*c*d*e*f)/3))/(f^6*((e + f*x)^(1/2) - e^(1/2))
^6) + (64*C*b*c^(3/2)*e^(3/2)*((c + d*x)^(1/2) - c^(1/2))^8)/(f^4*((e + f*x
)^(1/2) - e^(1/2))^8) + (64*C*b*c^(3/2)*d^2*e^(3/2)*((c + d*x)^(1/2) - c^(1
/2))^4)/(f^6*((e + f*x)^(1/2) - e^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^1
2/((e + f*x)^(1/2) - e^(1/2))^12 + d^6/f^6 - (6*d*((c + d*x)^(1/2) - c^(1/2
))^10)/(f*((e + f*x)^(1/2) - e^(1/2))^10) - (6*d^5*((c + d*x)^(1/2) - c^(1/
2))^2)/(f^5*((e + f*x)^(1/2) - e^(1/2))^2) + (15*d^4*((c + d*x)^(1/2) - c^(
1/2))^4)/(f^4*((e + f*x)^(1/2) - e^(1/2))^4) - (20*d^3*((c + d*x)^(1/2) - c
^(1/2))^6)/(f^3*((e + f*x)^(1/2) - e^(1/2))^6) + (15*d^2*((c + d*x)^(1/2) -
 c^(1/2))^8)/(f^2*((e + f*x)^(1/2) - e^(1/2))^8)) - ((((c + d*x)^(1/2) - c^
(1/2))*((3*B*b*d^3*e^2)/2 + (3*B*b*c^2*d*f^2)/2 + B*b*c*d^2*e*f))/(f^6*((e
+ f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*B*b*c^2*f^2)
/2 + (11*B*b*d^2*e^2)/2 + 25*B*b*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))
^3) + (((c + d*x)^(1/2) - c^(1/2))^7*((3*B*b*c^2*f^2)/2 + (3*B*b*d^2*e^2)/2
 + B*b*c*d*e*f))/(d^2*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2
) - c^(1/2))^5*((11*B*b*c^2*f^2)/2 + (11*B*b*d^2*e^2)/2 + 25*B*b*c*d*e*f))/
(d*f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) -
 c^(1/2))^4*(32*B*b*c*f + 32*B*b*d*e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4))
/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (
4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/2))^6) - (4*d
^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(1/2))^2) + (6*
d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e^(1/2))^4)) + (
(((c + d*x)^(1/2) - c^(1/2))*(2*B*a*c*f + 2*B*a*d*e))/(f^3*((e + f*x)^(1/2)
 - e^(1/2))) + (((c + d*x)^(1/2) - c^(1/2))^3*(2*B*a*c*f + 2*B*a*d*e))/(d*f
^2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*B*a*c^(1/2)*e^(1/2)*((c + d*x)^(1/2)
 - c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(
1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c
^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - (4*A*a*atan((d*((e + f*x)^(
1/2) - e^(1/2)))/((-d*f)^(1/2)*((c + d*x)^(1/2) - c^(1/2)))))/(-d*f)^(1/2)
```

```
+ (B*b*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2
) - e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*f))/(2*d^(5/2)*f^(5/2)) + (
C*a*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) -
 e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*f))/(2*d^(5/2)*f^(5/2)) - (2*A
*b*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) -
e^(1/2))))*(c*f + d*e))/(d^(3/2)*f^(3/2)) - (2*...
```

3.56 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=164

$$-\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\,\sqrt{e+fx}}{4d^2f^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}+\frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2}{}$$

```
[Out] 1/4*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e)))*arctanh
(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(5/2)/f^(5/2)+1/2*C*(d*x+c)
^(3/2)*(f*x+e)^(1/2)/d^2/f-1/4*(-4*B*d*f+5*C*c*f+3*C*d*e)*(d*x+c)^(1/2)*(f*
x+e)^(1/2)/d^2/f^2
```

Rubi [A]
time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {965, 81, 65, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))}{4d^{5/2}f^{5/2}}-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(-4Bdf+5cCf+3Cde)}{4d^2f^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/4*((3*C*d*e + 5*c*C*f - 4*B*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/(d^2*f^2)
+ (C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d^2*f) + ((C*(3*d^2*e^2 + 2*c*d*e*f
+ 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d
*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*f^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\int \frac{\frac{1}{2}(-3cCde-c^2Cf+4Ad^2f)-\frac{1}{2}d(3Cde+5cCf-4Bdf)x}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx}{2d^2f}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\,\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C}{}$$

$$(C$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\,\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{}{}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\,\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C}{}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\,\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C}{}$$

Mathematica [A]
time = 0.40, size = 141, normalized size = 0.86

$$\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(4Bdf+C(-3de-3cf+2dfx))}{4d^2f^2}+\frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{e+fx}}{\sqrt{f}\,\sqrt{c+dx}}\right)}{4d^{5/2}f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f*x)))/(4*d
^2*f^2) + ((C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e
 + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(4*d^(5
/2)*f^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(138) = 276.
time = 0.10, size = 425, normalized size = 2.59

method	result
default	$\frac{\left(8A\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)d^2f^2-4B\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)cd\,f^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*d^2*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*c*d*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^2*e*f+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*f^2+2*C*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d*e*f+3*C*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
d^2*e^2+4*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d*f*x+8*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*d*f-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*f-6*C*(d*f
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d*e)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(d*f)^(1/2
)/f^2/d^2/((d*x+c)*(f*x+e))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-%e*d>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 1.11, size = 391, normalized size = 2.38

$$\left[\frac{\left(3\,Cd^2e^2+\left(3\,Cc^2-4\,Bcd+8\,Ad^2\right)f^2+2\,\left(Ccd-2\,Bd^2\right)fe\right)\sqrt{df}\,\log\left(8\,d^2f^2x^2+8\,cdf^2x+c^2f^2+d^2e^2+4\,\left(2\,dfx+cf+de\right)\sqrt{df}\,\sqrt{dx+c}\,\sqrt{fx+e}+2\,\left(4\,d^2fx+3\,cdf\right)e\right)+4\,\left(2\,Cd^2f^2x-3\,Cd^2fe-\left(3\,Ccd-4\,Bd^2\right)f^2\right)\sqrt{dx+c}\,\sqrt{fx+e}}{16\,d^3f^3},-\frac{\left(3\,Cd^2e^2+\left(3\,Cc^2-4\,Bcd+8\,Ad^2\right)f^2+2\,\left(Ccd-2\,Bd^2\right)fe\right)\sqrt{-df}\,\arctan\left(\frac{\left(2\,dfx+cf+de\right)\sqrt{-df}\,\sqrt{dx+c}\,\sqrt{fx+e}}{2\,\left(d^2f^2x^2+cdf^2x+\left(d^2fx+cdf\right)e\right)}\right)-2\,\left(2\,Cd^2f^2x-3\,Cd^2fe-\left(3\,Ccd-4\,Bd^2\right)f^2\right)\sqrt{dx+c}\,\sqrt{fx+e}}{8\,d^3f^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*d^2*e^2 + (3*C*c^2 - 4*B*c*d + 8*A*d^2)*f^2 + 2*(C*c*d - 2*B*d^
2)*f*e)*sqrt(d*f)*log(8*d^2*f^2*x^2 + 8*c*d*f^2*x + c^2*f^2 + d^2*e^2 + 4*(
2*d*f*x + c*f + d*e)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 2*(4*d^2*f*x +
 3*c*d*f)*e) + 4*(2*C*d^2*f^2*x - 3*C*d^2*f*e - (3*C*c*d - 4*B*d^2)*f^2)*sq
rt(d*x + c)*sqrt(f*x + e))/(d^3*f^3), -1/8*((3*C*d^2*e^2 + (3*C*c^2 - 4*B*c
*d + 8*A*d^2)*f^2 + 2*(C*c*d - 2*B*d^2)*f*e)*sqrt(-d*f)*arctan(1/2*(2*d*f*x
 + c*f + d*e)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*f^2
*x + (d^2*f*x + c*d*f)*e)) - 2*(2*C*d^2*f^2*x - 3*C*d^2*f*e - (3*C*c*d - 4*
B*d^2)*f^2)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [A]
time = 0.89, size = 194, normalized size = 1.18

$$\frac{\left(\sqrt{(dx+c)df-cdf+d^2e}\,\sqrt{dx+c}\left(\frac{2\,(dx+c)C}{d^3f}-\frac{5\,Ccd^5f^2-4\,Bd^6f^2+3\,Cd^6fe}{d^8f^3}\right)-\frac{\left(3\,Cc^2f^2-4\,Bcdf^2+8\,Ad^2f^2+2\,Ccdfe-4\,Bd^2fe+3\,Cd^2e^2\right)\log\left(\left|-\sqrt{df}\,\sqrt{dx+c}+\sqrt{(dx+c)df-cdf+d^2e}\right|\right)}{\sqrt{df}\,d^2f^2}\right)d}{4\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*C/(d^3*
f) - (5*C*c*d^5*f^2 - 4*B*d^6*f^2 + 3*C*d^6*f*e)/(d^8*f^3)) - (3*C*c^2*f^2
```

```
- 4*B*c*d*f^2 + 8*A*d^2*f^2 + 2*C*c*d*f*e - 4*B*d^2*f*e + 3*C*d^2*e^2)*log(
abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(
d*f)*d^2*f^2))*d/abs(d)
```

Mupad [B]
time = 25.89, size = 833, normalized size = 5.08

$$\frac{\frac{(2Bcf+2Bde)\left(\sqrt{c+dx}-\sqrt{c}\right)}{f^3\left(\sqrt{e+fx}-\sqrt{e}\right)}+\frac{(2Bcf+2Bde)\left(\sqrt{c+dx}-\sqrt{c}\right)^3}{d\,f^2\left(\sqrt{e+fx}-\sqrt{e}\right)^3}-\frac{8B\sqrt{c}\,\sqrt{e}\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{f^2\left(\sqrt{e+fx}-\sqrt{e}\right)^2}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{e+fx}-\sqrt{e}\right)^4}+\frac{d^2}{f^2}-\frac{2d\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{f\left(\sqrt{e+fx}-\sqrt{e}\right)^2}}-\frac{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)\left(\frac{3Cc^2df^2}{2}+Ccd^2ef+\frac{3Cd^3e^2}{2}\right)}{f^6\left(\sqrt{e+fx}-\sqrt{e}\right)}-\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^3\left(\frac{11Cc^2f^2}{2}+25Ccdef+\frac{11Cd^2e^2}{2}\right)}{f^5\left(\sqrt{e+fx}-\sqrt{e}\right)^3}+\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^7\left(\frac{3Cc^2f^2}{2}+Ccdef+\frac{3Cd^2e^2}{2}\right)}{d^2f^3\left(\sqrt{e+fx}-\sqrt{e}\right)^7}-\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^5\left(\frac{11Cc^2f^2}{2}+25Ccdef+\frac{11Cd^2e^2}{2}\right)}{d\,f^4\left(\sqrt{e+fx}-\sqrt{e}\right)^5}+\frac{\sqrt{c}\,\sqrt{e}\,(32Ccf+32Cde)\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{f^4\left(\sqrt{e+fx}-\sqrt{e}\right)^4}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^8}{\left(\sqrt{e+fx}-\sqrt{e}\right)^8}+\frac{d^4}{f^4}-\frac{4d\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{f\left(\sqrt{e+fx}-\sqrt{e}\right)^6}-\frac{4d^3\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{f^3\left(\sqrt{e+fx}-\sqrt{e}\right)^2}+\frac{6d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{f^2\left(\sqrt{e+fx}-\sqrt{e}\right)^4}}-\frac{4A\,\mathrm{atan}\left(\frac{d\left(\sqrt{e+fx}-\sqrt{e}\right)}{\sqrt{-df}\left(\sqrt{c+dx}-\sqrt{c}\right)}\right)}{\sqrt{-df}}-\frac{2B\,\mathrm{atanh}\left(\frac{\sqrt{f}\left(\sqrt{c+dx}-\sqrt{c}\right)}{\sqrt{d}\left(\sqrt{e+fx}-\sqrt{e}\right)}\right)(cf+de)}{d^{3/2}f^{3/2}}+\frac{C\,\mathrm{atanh}\left(\frac{\sqrt{f}\left(\sqrt{c+dx}-\sqrt{c}\right)}{\sqrt{d}\left(\sqrt{e+fx}-\sqrt{e}\right)}\right)\left(3c^2f^2+2cdef+3d^2e^2\right)}{2d^{5/2}f^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2)))/(f^3*((e + f*x)^(1/2) -
e^(1/2))) + ((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(d*f^2*((e
+ f*x)^(1/2) - e^(1/2))^3) - (8*B*c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2
))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))^4/(
(e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/2))^2
)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/2))*((3*C*
d^3*e^2)/2 + (3*C*c^2*d*f^2)/2 + C*c*d^2*e*f))/(f^6*((e + f*x)^(1/2) - e^(1
/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*c^2*f^2)/2 + (11*C*d^2*e^2)/2
 + 25*C*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3) + (((c + d*x)^(1/2) -
 c^(1/2))^7*((3*C*c^2*f^2)/2 + (3*C*d^2*e^2)/2 + C*c*d*e*f))/(d^2*f^3*((e +
 f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) - c^(1/2))^5*((11*C*c^2*f^2)/
2 + (11*C*d^2*e^2)/2 + 25*C*c*d*e*f))/(d*f^4*((e + f*x)^(1/2) - e^(1/2))^5)
 + (c^(1/2)*e^(1/2)*(32*C*c*f + 32*C*d*e)*((c + d*x)^(1/2) - c^(1/2))^4)/(f
^4*((e + f*x)^(1/2) - e^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f*x
)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e
 + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((
e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*(
(e + f*x)^(1/2) - e^(1/2))^4)) - (4*A*atan((d*((e + f*x)^(1/2) - e^(1/2)))/
((-d*f)^(1/2)*((c + d*x)^(1/2) - c^(1/2)))))/(-d*f)^(1/2) - (2*B*atanh((f^(
1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c
*f + d*e))/(d^(3/2)*f^(3/2)) + (C*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)
))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*
f))/(2*d^(5/2)*f^(5/2))
```

3.57 $\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=188

$$\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}-\frac{(2aCdf+b(Cde+cCf-2Bdf))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2d^{3/2}f^{3/2}}-\frac{2(Ab^2-a(bB-aC)) \ldots}{b^2\sqrt{bc \ldots}}$$

```
[Out] -(2*a*C*d*f+b*(-2*B*d*f+C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/(f*x+e)^(1/2))/b^2/d^(3/2)/f^(3/2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e
)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^2/(-a*d+b*c)^(1/2)/
(-a*f+b*e)^(1/2)+C*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f
```

Rubi [A]
time = 0.22, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1629, 163, 65, 223, 212, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2aCdf+b(-2Bdf+cCf+Cde))}{b^2d^{3/2}f^{3/2}}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f -
2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^2*d^
(3/2)*f^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^2*Sqrt[b*c - a*d]*Sqrt[b*e -
a*f])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = \frac{C\sqrt{c+dx}\,\sqrt{e+fx}}{bdf} + \frac{\int \frac{\frac{1}{2}b(2Abdf-aC(de+cf))-\frac{1}{2}b(2aCdf+b(Cde+cCf-2Bdf))}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}}}{b^2df}$$

$$= \frac{C\sqrt{c+dx}\,\sqrt{e+fx}}{bdf} + \left(A-\frac{a(bB-aC)}{b^2}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e}}$$

$$= \frac{C\sqrt{c+dx}\,\sqrt{e+fx}}{bdf} + \left(2\left(A-\frac{a(bB-aC)}{b^2}\right)\right)\text{Subst}\left(\int \frac{}{-bc+ac}\right.$$

$$= \frac{C\sqrt{c+dx}\,\sqrt{e+fx}}{bdf} - \frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\tanh^{-1}\left(\frac{\sqrt{be-af}\,\sqrt{c+d}}{\sqrt{bc-ad}\,\sqrt{e+f}}\right.}{\sqrt{bc-ad}\,\sqrt{be-af}}$$

$$= \frac{C\sqrt{c+dx}\,\sqrt{e+fx}}{bdf} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\tanh^{-1}\left(\right.}{b^2d^{3/2}f^{3/2}}$$

Mathematica [A]
time = 0.65, size = 183, normalized size = 0.97

$$\frac{\frac{bC\sqrt{c+dx}\,\sqrt{e+fx}}{df} + \frac{2(Ab^2+a(-bB+aC))\tan^{-1}\left(\frac{\sqrt{bc-ad}\,\sqrt{e+fx}}{\sqrt{-be+af}\,\sqrt{c+dx}}\right)}{\sqrt{bc-ad}\,\sqrt{-be+af}} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{e+fx}}{\sqrt{f}\,\sqrt{c+dx}}\right)}{d^{3/2}f^{3/2}}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((b*C*Sqrt[c + d*x]*Sqrt[e + f*x])/(d*f) + (2*(A*b^2 + a*(-(b*B) + a*C))*Ar
cTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/(
Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2*B*
d*f))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(3/2)*f^
(3/2)))/b^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(160) = 320.
time = 0.12, size = 746, normalized size = 3.97

method	result

default	$-\frac{\left(2A\sqrt{df}\,\ln\left(\frac{-2adfx+bcfx+bdex+2\sqrt{(dx+c)(fx+e)}\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\,b-acf-ade+2bce}{bx+a}\right)b^2df-2B\sqrt{df}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2*(2*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1
/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b
*x+a))*b^2*d*f-2*B*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f
*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*
b*c*e)/(b*x+a))*a*b*d*f-2*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
b^2*d*f+2*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^
(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/
(b*x+a))*a^2*d*f+2*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(
2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d*f
+C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*f+C*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d*e-2*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2)*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/((d*x+c)*(f*x+e))^(1/2)/b^3/(d*f)^(1/2)/((a^2*d*f-a*b*c*f-a*b*d*e+
b^2*c*e)/b^2)^(1/2)/d/f
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(a + bx)\sqrt{c + dx}\,\sqrt{e + fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.58 $\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=254

$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)}+\frac{2C\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)}{b^2\sqrt{d}\,\sqrt{f}}+\frac{(2a^3Cdf-3a^2bC(de+cf)-b^3(}{}$$

```
[Out] (2*a^3*C*d*f-3*a^2*b*C*(c*f+d*e)-b^3*(-A*c*f-A*d*e+2*B*c*e)+a*b^2*(-2*A*d*f
+B*c*f+B*d*e+4*C*c*e))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1
/2)/(f*x+e)^(1/2))/b^2/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(3/2)+2*C*arctanh(f^(1/2
)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^2/d^(1/2)/f^(1/2)-(A*b^2-a*(B*b-C*
a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)
```

Rubi [A]
time = 0.42, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1627, 163, 65, 223, 212, 95, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{be-af}}{\sqrt{e+fx}\,\sqrt{bc-ad}}\right)(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce))}{b^2(bc-ad)^{3/2}(be-af)^{3/2}}-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(Ab^2-a(bB-aC))}{b(a+bx)(bc-ad)(be-af)}+\frac{2C\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)}{b^2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -(((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e
 - a*f)*(a + b*x))) + (2*C*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
+ f*x])])/(b^2*Sqrt[d]*Sqrt[f]) + ((2*a^3*C*d*f - 3*a^2*b*C*(d*e + c*f) - b
^3*(2*B*c*e - A*d*e - A*c*f) + a*b^2*(4*c*C*e + B*d*e + B*c*f - 2*A*d*f))*A
rcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b
^2*(b*c - a*d)^(3/2)*(b*e - a*f)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int -\frac{a^2C(de+cf)+b^2(2Bce-Ade-A}{(a+bx)}}{(b}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{C\int \frac{1}{\sqrt{c+dx}\,\sqrt{e+fx}}}{b^2}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{(2C)\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}}}\right.}{b}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{(2a^3Cdf-3a^2bC(de+c}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{2C\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+d}}{\sqrt{d}\,\sqrt{e+f}}\right.}{b^2\sqrt{d}\,\sqrt{f}}$$

Mathematica [A]
time = 1.48, size = 249, normalized size = 0.98

$$-\frac{\frac{b(Ab^2+a(-bB+aC))\sqrt{c+dx}\,\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)} + \frac{(-2a^3Cdf+3a^2bC(de+cf)-ab^2(4cCe+Bde+Bcf-2Adf)+b^3(2Bce-A(de+cf)))\tan^{-1}\left(\frac{\sqrt{bc-ad}\,\sqrt{e+fx}}{\sqrt{-be+af}\,\sqrt{c+dx}}\right)}{(bc-ad)^{3/2}(-be+af)^{3/2}} - \frac{2C\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{e+fx}}{\sqrt{f}\,\sqrt{c+dx}}\right)}{\sqrt{d}\,\sqrt{f}}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -(((b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*
(b*e - a*f)*(a + b*x)) + ((-2*a^3*C*d*f + 3*a^2*b*C*(d*e + c*f) - a*b^2*(4*
c*C*e + B*d*e + B*c*f - 2*A*d*f) + b^3*(2*B*c*e - A*(d*e + c*f)))*ArcTan[(S
qrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/((b*c -
a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) - (2*C*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sq
rt[f]*Sqrt[c + d*x])])/(Sqrt[d]*Sqrt[f]))/b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. 2(226) = 452.
time = 0.10, size = 2973, normalized size = 11.70

method	result	size

default	Expression too large to display	2973

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*(f*x+e)^(1/2)*(d*x+c)^(1/2)*(2*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*
x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a
*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*f*x*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d
*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f*x*(d*f)^(1/2)-B*ln((-2*a*d*f*x+
b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*e*x*(d*f)^(1/2)-2*C*ln
((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*f*x*(d*f)
^(1/2)+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*
b^2*c*f*x*(d*f)^(1/2)+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e)
)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e
)/(b*x+a))*a^2*b^2*d*e*x*(d*f)^(1/2)-4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*(
(d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*
f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*x*(d*f)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d*f*x*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*f*x*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*e*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)+2*C*a^2*b^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d
*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-
a*d*e+2*b*c*e)/(b*x+a))*a^4*d*f*(d*f)^(1/2)-2*B*a*b^3*(d*f)^(1/2)*((d*x+c)*
(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln(1/2*(2*
d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*e*x
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*A*ln((-2*a*d*f*x+b*c*f*x+b
*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*f*(d*f)^(1/2)-A*ln((-2*a*d*f*
x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f*(d*f)^(1/2)-A*ln((
-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*e*(d*f)^(1/
2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b
*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*
f*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((
```

```
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))
*a^2*b^2*d*e*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x
+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*
c*e)/(b*x+a))*a*b^3*c*e*(d*f)^(1/2)+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f
-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*c*f*(d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d
*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*d*e*x*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+
b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*e*x*(d*f)^(1/2)+3*C*ln((
-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*e*(d*f)^(1/
2)-4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*
c*e*(d*f)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*a^3*b*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*a^2*b^2*c*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b
^2*d*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*e*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*(
(d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*
f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*f*x*(d*f)^(1/2)+2*A*b^4*(d*f)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2))/((d*x+c)*(f
*x+e))^(1/2)/(a*d-b*c)/(a*f-b*e)/(b*x+a)/(d*f)^(1/2)/((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)/b^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(a + bx)^2 \sqrt{c + dx} \sqrt{e + fx}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. $2(232) = 464$.
time = 2.06, size = 1356, normalized size = 5.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] (3*sqrt(d*f)*C*a^2*b*c*d^2*f - sqrt(d*f)*B*a*b^2*c*d^2*f - sqrt(d*f)*A*b^3*
c*d^2*f - 2*sqrt(d*f)*C*a^3*d^3*f + 2*sqrt(d*f)*A*a*b^2*d^3*f - 4*sqrt(d*f)
*C*a*b^2*c*d^2*e + 2*sqrt(d*f)*B*b^3*c*d^2*e + 3*sqrt(d*f)*C*a^2*b*d^3*e -
sqrt(d*f)*B*a*b^2*d^3*e - sqrt(d*f)*A*b^3*d^3*e)*arctan(-1/2*(b*c*d*f - 2*a
*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d
))/((a*b^3*c*f*abs(d) - a^2*b^2*d*f*abs(d) - b^4*c*abs(d)*e + a*b^3*d*abs(d
)*e)*sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d) + 2*(sq
rt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b
^3*c^2*d^3*f^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*f*e + 2*sqrt(d*f)*B*a*b^2*c*d^4*
f*e - 2*sqrt(d*f)*A*b^3*c*d^4*f*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^2*c*d^2*f - s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*
A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
 c*d*f + d^2*e))^2*C*a^3*d^3*f - 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b*d^3*f + 2*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^2*d^3*f + sqrt
```

```
(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2
 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*B*a*b^2*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*d^3*e)/((b*c^2*d^2*f^2 - 2*b*c*
d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*
f - c*d*f + d^2*e))^4*b)*(a*b^3*c*f*abs(d) - a^2*b^2*d*f*abs(d) - b^4*c*abs
(d)*e + a*b^3*d*abs(d)*e)) - sqrt(d*f)*C*log((sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2)/(b^2*f*abs(d))
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.59 $\int \frac{A+Bx+Cx^2}{(a+bx)^3 \sqrt{c+dx} \sqrt{e+fx}} \, dx$

Optimal. Leaf size=424

$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2}+\frac{(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Adf)-b^3(4Bce-3A(\ldots}{4b(bc-ad)^2(be-\ldots}$$

```
[Out] -1/4*(b^2*(3*A*d^2*e^2-2*c*d*e*(-A*f+2*B*e)+c^2*(3*A*f^2-4*B*e*f+8*C*e^2))+
a*b*(d^2*e*(-8*A*f+B*e)-c^2*f*(-B*f+8*C*e)-2*c*d*(4*A*f^2-7*B*e*f+4*C*e^2))
+a^2*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e))))*arcta
nh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(-a*d+b*c
)^(5/2)/(-a*f+b*e)^(5/2)-1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2
)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2+1/4*(2*a^3*C*d*f+a*b^2*(-6*A*d*f+B*c*f+
B*d*e+8*C*c*e)-b^3*(4*B*c*e-3*A*(c*f+d*e))+a^2*b*(2*B*d*f-5*C*(c*f+d*e)))*(
d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)
```

Rubi [A]
time = 0.63, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {1627, 156, 12, 95, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(a^2(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))+ab(-2cd(4Af^2-7Bef+4Ce^2)+d^2e(Be-8Af)+c^2(-f)(8Ce-Bf))+b^2(c^2(3Af^2-4Bef+8Ce^2)-2cde(2Be-Af)+3Ad^2e^2)\right)}{4(bc-ad)^{5/2}(be-af)^{5/2}}+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(2Bdf-5C(cf+de))+ab^2(-6Adf+Bcf+Bde+8cCe)-b^3(4Bce-3A(cf+de))\right)}{4b(a+bx)(bc-ad)^2(be-af)^2}-\frac{\sqrt{c+dx}\sqrt{e+fx}(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/2*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^2) + ((2*a^3*C*d*f + a*b^2*(8*c*C*e + B*d*e + B*c*f -
6*A*d*f) - b^3*(4*B*c*e - 3*A*(d*e + c*f)) + a^2*b*(2*B*d*f - 5*C*(d*e + c*
f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x
)) - ((b^2*(3*A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) + c^2*(8*C*e^2 - 4*B*e*f +
3*A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f) - c^2*f*(8*C*e - B*f) - 2*c*d*(4*C*e^2
 - 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f
*(2*A*d*f - B*(d*e + c*f))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[
b*c - a*d]*Sqrt[e + f*x])])/(4*(b*c - a*d)^(5/2)*(b*e - a*f)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int -\frac{a^2C(de+cf)-ab(4cCe+Bde+\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(2a^3Cdf+ab^2(8cCe+\cdots}{\cdots}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(2a^3Cdf+ab^2(8cCe+\cdots}{\cdots}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(2a^3Cdf+ab^2(8cCe+\cdots}{\cdots}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(2a^3Cdf+ab^2(8cCe+\cdots}{\cdots}$$

Mathematica [A]
time = 3.54, size = 420, normalized size = 0.99

$$\frac{1}{4}\left(-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(4b^3Bcex-ab^2(8cCex+B(-2ce+dex+cfx))+a^2b(5Cdex+Bd(e-2fx)+c(-6Ce+Bf+5Cfx))+a^3(-4Bdf+C(3de+3cf-2dfx))+Ab(8a^2df+b^2(2ce-3dex-3cfx)+ab(-5de-5cf+6dfx)))}{(bc-ad)^2(be-af)^2(a+bx)^2}+\frac{(b^2(3Ad^2e^2+2cde(-2Be+Af)+c^2(8Ce^2-4Bef+3Af^2))+ab(d^2e(Be-8Af)+c^2f(-8Ce+Bf)-2cd(4Ce^2-7Bef+4Af^2))+a^2(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf))))\tan^{-1}\left(\frac{\sqrt{bc-ad}\,\sqrt{e+fx}}{\sqrt{-be+af}\,\sqrt{c+dx}}\right)}{(bc-ad)^{5/2}(-be+af)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-((Sqrt[c + d*x]*Sqrt[e + f*x]*(4*b^3*B*c*e*x - a*b^2*(8*c*C*e*x + B*(-2*c
*e + d*e*x + c*f*x)) + a^2*b*(5*C*d*e*x + B*d*(e - 2*f*x) + c*(-6*C*e + B*f
 + 5*C*f*x)) + a^3*(-4*B*d*f + C*(3*d*e + 3*c*f - 2*d*f*x)) + A*b*(8*a^2*d*
f + b^2*(2*c*e - 3*d*e*x - 3*c*f*x) + a*b*(-5*d*e - 5*c*f + 6*d*f*x))))/((b
*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^2)) + ((b^2*(3*A*d^2*e^2 + 2*c*d*e*(-2*
B*e + A*f) + c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f)
+ c^2*f*(-8*C*e + B*f) - 2*c*d*(4*C*e^2 - 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d
^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))))*ArcTan[
(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/((b*c
- a*d)^(5/2)*(-(b*e) + a*f)^(5/2)))/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7118 vs. $2(398) = 796$.
time = 0.11, size = 7119, normalized size = 16.79

method	result	size

default	Expression too large to display	7119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1988 vs. 2(413) = 826.
time = 207.91, size = 4038, normalized size = 9.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/16*(((8*A*a^2*b^2*d^2 + (3*C*a^2*b^2 + B*a*b^3 + 3*A*b^4)*c^2 - 4*(B*a^
2*b^2 + 2*A*a*b^3)*c*d)*f^2*x^2 + 2*(8*A*a^3*b*d^2 + (3*C*a^3*b + B*a^2*b^2
 + 3*A*a*b^3)*c^2 - 4*(B*a^3*b + 2*A*a^2*b^2)*c*d)*f^2*x + (8*A*a^4*d^2 + (
3*C*a^4 + B*a^3*b + 3*A*a^2*b^2)*c^2 - 4*(B*a^4 + 2*A*a^3*b)*c*d)*f^2 + (8*
C*a^2*b^2*c^2 - 4*(2*C*a^3*b + B*a^2*b^2)*c*d + (3*C*a^4 + B*a^3*b + 3*A*a^
2*b^2)*d^2 + (8*C*b^4*c^2 - 4*(2*C*a*b^3 + B*b^4)*c*d + (3*C*a^2*b^2 + B*a*
b^3 + 3*A*b^4)*d^2)*x^2 + 2*(8*C*a*b^3*c^2 - 4*(2*C*a^2*b^2 + B*a*b^3)*c*d
+ (3*C*a^3*b + B*a^2*b^2 + 3*A*a*b^3)*d^2)*x)*e^2 - 2*((2*(2*C*a*b^3 + B*b^
4)*c^2 - (C*a^2*b^2 + 7*B*a*b^3 + A*b^4)*c*d + 2*(B*a^2*b^2 + 2*A*a*b^3)*d^
2)*f*x^2 + 2*(2*(2*C*a^2*b^2 + B*a*b^3)*c^2 - (C*a^3*b + 7*B*a^2*b^2 + A*a*
b^3)*c*d + 2*(B*a^3*b + 2*A*a^2*b^2)*d^2)*f*x + (2*(2*C*a^3*b + B*a^2*b^2)*
c^2 - (C*a^4 + 7*B*a^3*b + A*a^2*b^2)*c*d + 2*(B*a^4 + 2*A*a^3*b)*d^2)*f)*e
```

)*sqrt(-(a*b*c - a^2*d)*f + (b^2*c - a*b*d)*e)*log((a^2*c^2*f^2 + (b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*f^2*x^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*f^2*x + 4*(a*c*f - (b*c - 2*a*d)*f*x - (b*d*x + 2*b*c - a*d)*e)*sqrt(-(a*b*c - a^2*d)*f + (b^2*c - a*b*d)*e)*sqrt(d*x + c)*sqrt(f*x + e) + (b^2*d^2*x^2 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x)*e^2 + 2*((3*b^2*c*d - 4*a*b*d^2)*f*x^2 + 2*(2*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*f*x - (4*a*b*c^2 - 3*a^2*c*d)*f)*e)/(b^2*x^2 + 2*a*b*x + a^2)) + 4*(((5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*c^2 - (7*C*a^4*b + B*a^3*b^2 - 9*A*a^2*b^3)*c*d + 2*(C*a^5 + B*a^4*b - 3*A*a^3*b^2)*d^2)*f^2*x + ((3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*c^2 - (3*C*a^5 + 5*B*a^4*b - 13*A*a^3*b^2)*c*d + 4*(B*a^5 - 2*A*a^4*b)*d^2)*f^2 + (2*(3*C*a^2*b^3 - B*a*b^4 - A*b^5)*c^2 - (9*C*a^3*b^2 - B*a^2*b^3 - 7*A*a*b^4)*c*d + (3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*d^2 + (4*(2*C*a*b^4 - B*b^5)*c^2 - (13*C*a^2*b^3 - 5*B*a*b^4 - 3*A*b^5)*c*d + (5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*d^2)*x)*e^2 - (((13*C*a^2*b^3 - 5*B*a*b^4 - 3*A*b^5)*c^2 - 4*(5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*c*d + (7*C*a^4*b + B*a^3*b^2 - 9*A*a^2*b^3)*d^2)*f*x + ((9*C*a^3*b^2 - B*a^2*b^3 - 7*A*a*b^4)*c^2 - 4*(3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*c*d + (3*C*a^5 + 5*B*a^4*b - 13*A*a^3*b^2)*d^2)*f)*e)*sqrt(d*x + c)*sqrt(f*x + e))/((a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*f^3*x^2 + 2*(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*f^3*x + (a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3)*f^3 - (a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^2 + 2*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x)*e^3 + 3*((a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*f*x^2 + 2*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*f*x + (a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*f)*e^2 - 3*((a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*f^2*x^2 + 2*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*f^2*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*f^2)*e), 1/8*(((8*A*a^2*b^2*d^2 + (3*C*a^2*b^2 + B*a*b^3 + 3*A*b^4)*c^2 - 4*(B*a^2*b^2 + 2*A*a*b^3)*c*d)*f^2*x^2 + 2*(8*A*a^3*b*d^2 + (3*C*a^3*b + B*a^2*b^2 + 3*A*a*b^3)*c^2 - 4*(B*a^3*b + 2*A*a^2*b^2)*c*d)*f^2*x + (8*A*a^4*d^2 + (3*C*a^4 + B*a^3*b + 3*A*a^2*b^2)*c^2 - 4*(B*a^4 + 2*A*a^3*b)*c*d)*f^2 + (8*C*a^2*b^2*c^2 - 4*(2*C*a^3*b + B*a^2*b^2)*c*d + (3*C*a^4 + B*a^3*b + 3*A*a^2*b^2)*d^2 + (8*C*b^4*c^2 - 4*(2*C*a*b^3 + B*b^4)*c*d + (3*C*a^2*b^2 + B*a*b^3 + 3*A*b^4)*d^2)*x^2 + 2*(8*C*a*b^3*c^2 - 4*(2*C*a^2*b^2 + B*a*b^3)*c*d + (3*C*a^3*b + B*a^2*b^2 + 3*A*a*b^3)*d^2)*x)*e^2 - 2*((2*(2*C*a*b^3 + B*b^4)*c^2 - (C*a^2*b^2 + 7*B*a*b^3 + A*b^4)*c*d + 2*(B*a^2*b^2 + 2*A*a*b^3)*d^2)*f*x^2 + 2*(2*(2*C*a^2*b^2 + B*a*b^3)*c^2 - (C*a^3*b + 7*B*a^2*b^2 + A*a*b^3)*c*d + 2*(B*a^3*b + 2*A*a^2*b^2)*d^2)*f*x + (2*(2*C*a^3*b + B*a^2*b^2)*c^2 - (C*a^4 + 7*B*a^3*b + A*a^2*b^2)*c*d + 2*(B*a^4 + 2*A*a^3*b)*d^2)*f)*e)*sqrt((a*b*c - a^2*d)*f - (b^2*c - a*b*d)*e)*arctan(-1/2*(a*c*f - (b*c - 2*a*d)*f*x - (b*d*x + 2*b*c - a*d)*e)*sqrt((a*b*c - a^2*d)*f - (b^2*c - a*b*d)*e)*sqrt(d*x + c)*sqrt(f*x + e)/((a*b*c*d - a^2*d^2)*f^2*x^2 + (a*b*c^2 - a^2*c*d)*f^2*x - (b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)*e^2 - ((b^2*c*d - a*

```
b*d^2)*f*x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f*x - (a*b*c^2 - a^2*c*d)*f)
*e)) - 2*(((5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*c^2 - (7*C*a^4*b + B*a^3*b
^2 - 9*A*a^2*b^3)*c*d + 2*(C*a^5 + B*a^4*b - 3*A*a^3*b^2)*d^2)*f^2*x + ((3*
C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*c^2 - (3*C*a^5 + 5*B*a^4*b - 13*A*a^3*b^
2)*c*d + 4*(B*a^5 - 2*A*a^4*b)*d^2)*f^2 + (2*(3*C*a^2*b^3 - B*a*b^4 - A*b^5
)*c^2 - (9*C*a^3*b^2 - B*a^2*b^3 - 7*A*a*b^4)*c*d + (3*C*a^4*b + B*a^3*b^2
- 5*A*a^2*b^3)*d^2 + (4*(2*C*a*b^4 - B*b^5)*c^2 - (13*C*a^2*b^3 - 5*B*a*b^4
 - 3*A*b^5)*c*d + (5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*d^2)*x)*e^2 - (((13
*C*a^2*b^3 - 5*B*a*b^4 - 3*A*b^5)*c^2 - 4*(5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*
b^4)*c*d + (7*C*a^4*b + B*a^3*b^2 - 9*A*a^2*b^3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8019 vs. 2(413) = 826.
time = 50.50, size = 8019, normalized size = 18.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
[Out] -1/4*(3*sqrt(d*f)*C*a^2*c^2*d^2*f^2 + sqrt(d*f)*B*a*b*c^2*d^2*f^2 + 3*sqrt(
d*f)*A*b^2*c^2*d^2*f^2 - 4*sqrt(d*f)*B*a^2*c*d^3*f^2 - 8*sqrt(d*f)*A*a*b*c*
d^3*f^2 + 8*sqrt(d*f)*A*a^2*d^4*f^2 - 8*sqrt(d*f)*C*a*b*c^2*d^2*f*e - 4*sqr
t(d*f)*B*b^2*c^2*d^2*f*e + 2*sqrt(d*f)*C*a^2*c*d^3*f*e + 14*sqrt(d*f)*B*a*b
*c*d^3*f*e + 2*sqrt(d*f)*A*b^2*c*d^3*f*e - 4*sqrt(d*f)*B*a^2*d^4*f*e - 8*sq
rt(d*f)*A*a*b*d^4*f*e + 8*sqrt(d*f)*C*b^2*c^2*d^2*e^2 - 8*sqrt(d*f)*C*a*b*c
*d^3*e^2 - 4*sqrt(d*f)*B*b^2*c*d^3*e^2 + 3*sqrt(d*f)*C*a^2*d^4*e^2 + sqrt(d
*f)*B*a*b*d^4*e^2 + 3*sqrt(d*f)*A*b^2*d^4*e^2)*arctan(-1/2*(b*c*d*f - 2*a*d
^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^
2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d))
/((a^2*b^2*c^2*f^2*abs(d) - 2*a^3*b*c*d*f^2*abs(d) + a^4*d^2*f^2*abs(d) - 2
*a*b^3*c^2*f*abs(d)*e + 4*a^2*b^2*c*d*f*abs(d)*e - 2*a^3*b*d^2*f*abs(d)*e +
 b^4*c^2*abs(d)*e^2 - 2*a*b^3*c*d*abs(d)*e^2 + a^2*b^2*d^2*abs(d)*e^2)*sqrt
(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d) - 1/2*(5*sqrt(d*
f)*C*a^2*b^3*c^5*d^5*f^5 - sqrt(d*f)*B*a*b^4*c^5*d^5*f^5 - 3*sqrt(d*f)*A*b^
```

5*c^5*d^5*f^5 - 2*sqrt(d*f)*C*a^3*b^2*c^4*d^6*f^5 - 2*sqrt(d*f)*B*a^2*b^3*c
^4*d^6*f^5 + 6*sqrt(d*f)*A*a*b^4*c^4*d^6*f^5 - 8*sqrt(d*f)*C*a*b^4*c^5*d^5*
f^4*e + 4*sqrt(d*f)*B*b^5*c^5*d^5*f^4*e - 15*sqrt(d*f)*C*a^2*b^3*c^4*d^6*f^
4*e + 3*sqrt(d*f)*B*a*b^4*c^4*d^6*f^4*e + 9*sqrt(d*f)*A*b^5*c^4*d^6*f^4*e +
 8*sqrt(d*f)*C*a^3*b^2*c^3*d^7*f^4*e + 8*sqrt(d*f)*B*a^2*b^3*c^3*d^7*f^4*e
- 24*sqrt(d*f)*A*a*b^4*c^3*d^7*f^4*e - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^4*d^4*f^4 + 3*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b
^4*c^4*d^4*f^4 + 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*A*b^5*c^4*d^4*f^4 + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^2*c^3*d^5*f^4 + 4*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^
2*b^3*c^3*d^5*f^4 - 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*
d*f - c*d*f + d^2*e))^2*A*a*b^4*c^3*d^5*f^4 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c^2*d^6*f^4 - 16*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*
B*a^3*b^2*c^2*d^6*f^4 + 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*c^2*d^6*f^4 + 32*sqrt(d*f)*C*a*b^4*c^
4*d^6*f^3*e^2 - 16*sqrt(d*f)*B*b^5*c^4*d^6*f^3*e^2 + 10*sqrt(d*f)*C*a^2*b^3
*c^3*d^7*f^3*e^2 - 2*sqrt(d*f)*B*a*b^4*c^3*d^7*f^3*e^2 - 6*sqrt(d*f)*A*b^5*
c^3*d^7*f^3*e^2 - 12*sqrt(d*f)*C*a^3*b^2*c^2*d^8*f^3*e^2 - 12*sqrt(d*f)*B*a
^2*b^3*c^2*d^8*f^3*e^2 + 36*sqrt(d*f)*A*a*b^4*c^2*d^8*f^3*e^2 + 24*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^4
*c^4*d^4*f^3*e - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*B*b^5*c^4*d^4*f^3*e - 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^3*d^5*f^3*e + 20
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^
2*B*a*b^4*c^3*d^5*f^3*e + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
+ c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^3*d^5*f^3*e - 32*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^2*c^2*d^6*f^
3*e - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d
^2*e))^2*B*a^2*b^3*c^2*d^6*f^3*e + 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^4*c^2*d^6*f^3*e + 16*sqrt(d*f)
*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*
c*d^7*f^3*e + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^2*B*a^3*b^2*c*d^7*f^3*e - 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*c*d^7*f^3*e + 15*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C
*a^2*b^3*c^3*d^3*f^3 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c
)*d*f - c*d*f + d^2*e))^4*B*a*b^4*c^3*d^3*f^3 - 9*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*b^5*c^3*d^3*f^3 - 46*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*
C*a^3*b^2*c^2*d^4*f^3 + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^4*B*a^2*b^3*c^2*d^4*f^3 + 42*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*c^2*d^4*f^3

```
+ 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^4*C*a^4*b*c*d^5*f^3 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
+ c)*d*f - c*d*f + d^2*e))^4*B*a^3*b^2*c*d^5*f^3 - 72*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^2*b^3*c*d^5*f^3
- 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^4*C*a^5*d^6*f^3 - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c
)*d*f - c*d*f + d^2*e))^4*B*a^4*b*d^6*f^3 + 48*...
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^3*(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.60 $\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=826

$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}+\frac{(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf)-b^3(6Bce-5A}{12b(bc-ad)^2(be}$$

```
[Out] 1/8*(b^3*(5*A*d^3*e^3-3*c*d^2*e^2*(-A*f+2*B*e)+c^2*d*e*(3*A*f^2-4*B*e*f+8*C
*e^2)+c^3*f*(5*A*f^2-6*B*e*f+8*C*e^2))+a*b^2*(d^3*e^2*(-18*A*f+B*e)-c^3*f^2
*(-B*f+4*C*e)-c*d^2*e*(12*A*f^2-23*B*e*f+4*C*e^2)-c^2*d*f*(18*A*f^2-23*B*e*
f+40*C*e^2))-2*a^3*d*f*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*
(c*f+d*e)))+a^2*b*(C*(c^3*f^3+23*c^2*d*e*f^2+23*c*d^2*e^2*f+d^3*e^3)+4*d*f*
(6*A*d*f*(c*f+d*e)-B*(c^2*f^2+10*c*d*e*f+d^2*e^2))))*arctanh((-a*f+b*e)^(1/
2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(-a*d+b*c)^(7/2)/(-a*f+b*e
)^(7/2)-1/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-
a*f+b*e)/(b*x+a)^3+1/12*(2*a^3*C*d*f+a*b^2*(-10*A*d*f+B*c*f+B*d*e+12*C*c*e)
-b^3*(6*B*c*e-5*A*(c*f+d*e))+a^2*b*(4*B*d*f-7*C*(c*f+d*e)))*(d*x+c)^(1/2)*(
f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^2+1/24*(4*a^4*C*d^2*f^2+8*
a^3*b*d*f*(B*d*f-2*C*(c*f+d*e))-b^4*(15*A*d^2*e^2-2*c*d*e*(-7*A*f+9*B*e)+3*
c^2*(5*A*f^2-6*B*e*f+8*C*e^2))-a*b^3*(d^2*e*(-44*A*f+3*B*e)-3*c^2*f*(-B*f+4
*C*e)-2*c*d*(22*A*f^2-29*B*e*f+6*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-34*c*d*e*f+3
*d^2*e^2)+2*d*f*(22*A*d*f-5*B*(c*f+d*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-
a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)
```

Rubi [A]
time = 1.60, antiderivative size = 826, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {1627, 156, 12, 95, 214}

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/3*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^3) + ((2*a^3*C*d*f + a*b^2*(12*c*C*e + B*d*e + B*c*f -
 10*A*d*f) - b^3*(6*B*c*e - 5*A*(d*e + c*f)) + a^2*b*(4*B*d*f - 7*C*(d*e +
c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(12*b*(b*c - a*d)^2*(b*e - a*f)^2*(a +
b*x)^2) + ((4*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f - 2*C*(d*e + c*f)) - b^4*(
15*A*d^2*e^2 - 2*c*d*e*(9*B*e - 7*A*f) + 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2
)) - a*b^3*(d^2*e*(3*B*e - 44*A*f) - 3*c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2
 - 29*B*e*f + 22*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 34*c*d*e*f + 3*c^2*f^2)
+ 2*d*f*(22*A*d*f - 5*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b*(
```

```
b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)) + ((b^3*(5*A*d^3*e^3 - 3*c*d^2*e^2*(2
*B*e - A*f) + c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*
e*f + 5*A*f^2)) + a*b^2*(d^3*e^2*(B*e - 18*A*f) - c^3*f^2*(4*C*e - B*f) - c
*d^2*e*(4*C*e^2 - 23*B*e*f + 12*A*f^2) - c^2*d*f*(40*C*e^2 - 23*B*e*f + 18*
A*f^2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f
 - B*(d*e + c*f))) + a^2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f^2 +
c^3*f^3) + 4*d*f*(6*A*d*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2*f^2))
))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])]
)/(8*(b*c - a*d)^(7/2)*(b*e - a*f)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
```

```
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} - \frac{\int -\frac{a^2C(de+cf)-ab(6cCe+Bde+}{}}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf+ab^2(12cCe-}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf+ab^2(12cCe-}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf+ab^2(12cCe-}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf+ab^2(12cCe-}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf+ab^2(12cCe-}{}$$

Mathematica [A]
time = 10.29, size = 1036, normalized size = 1.25

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/24*(Sqrt[c + d*x]*Sqrt[e + f*x]*(6*b^5*c*e*x*(4*c*C*e*x + B*(2*c*e - 3*d
*e*x - 3*c*f*x)) + 6*a^5*d*f*(-4*B*d*f + C*(3*d*e + 3*c*f - 2*d*f*x)) + a*b
^4*(-12*c*C*e*x*(-2*c*e + d*e*x + c*f*x) + B*(3*d^2*e^2*x^2 + 2*c*d*e*x*(-2
5*e + 29*f*x) + c^2*(4*e^2 - 50*e*f*x + 3*f^2*x^2))) + a^4*b*(12*B*d*f*(c*f
 + d*(e - 2*f*x)) - C*(3*c^2*f^2 + 2*c*d*f*(29*e - 25*f*x) + d^2*(3*e^2 - 5
0*e*f*x + 4*f^2*x^2))) + a^2*b^3*(d^2*e*x*(8*B*e + 3*C*e*x - 10*B*f*x) + c^
2*(8*B*f*(-2*e + f*x) + C*(8*e^2 + 14*e*f*x + 3*f^2*x^2)) - 2*c*d*(C*e*x*(-
7*e + 17*f*x) + B*(8*e^2 - 62*e*f*x + 5*f^2*x^2))) + a^3*b^2*(c^2*f*(10*C*e
```

```
 - 3*B*f - 8*C*f*x) + 2*c*d*(B*f*(17*e - 7*f*x) + C*(5*e^2 - 62*e*f*x + 8*f
^2*x^2)) - d^2*(8*C*e*x*(e - 2*f*x) + B*(3*e^2 + 14*e*f*x + 8*f^2*x^2))) +
A*b*(72*a^4*d^2*f^2 + 18*a^3*b*d*f*(-5*d*e - 5*c*f + 6*d*f*x) + b^4*(15*d^2
*e^2*x^2 + 2*c*d*e*x*(-5*e + 7*f*x) + c^2*(8*e^2 - 10*e*f*x + 15*f^2*x^2))
- 2*a*b^3*(c^2*f*(13*e - 20*f*x) + 2*d^2*e*x*(-10*e + 11*f*x) + c*d*(13*e^2
 - 34*e*f*x + 22*f^2*x^2)) + a^2*b^2*(33*c^2*f^2 + 2*c*d*f*(43*e - 59*f*x)
+ d^2*(33*e^2 - 118*e*f*x + 44*f^2*x^2)))))/((b*c - a*d)^3*(b*e - a*f)^3*(a
 + b*x)^3) + ((a*b^2*(d^3*e^2*(B*e - 18*A*f) + c^3*f^2*(-4*C*e + B*f) + c^2
*d*f*(-40*C*e^2 + 23*B*e*f - 18*A*f^2) + c*d^2*e*(-4*C*e^2 + 23*B*e*f - 12*
A*f^2)) + b^3*(5*A*d^3*e^3 + 3*c*d^2*e^2*(-2*B*e + A*f) + c^2*d*e*(8*C*e^2
- 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) - 2*a^3*d*f*(C*
(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) + a^
2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f^2 + c^3*f^3) + 4*d*f*(6*A*d
*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2*f^2))))*ArcTan[(Sqrt[b*c - a
*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/(8*(b*c - a*d)^(7/2
)*(-(b*e) + a*f)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 18801 vs. $2(794) = 1588$.
time = 0.18, size = 18802, normalized size = 22.76

method	result	size
default	Expression too large to display	18802

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re deta
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**4/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 25778 vs. $2(818) = 1636$.
time = 92.84, size = 25778, normalized size = 31.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] -1/8*(sqrt(d*f)*C*a^2*b*c^3*d^2*f^3 + sqrt(d*f)*B*a*b^2*c^3*d^2*f^3 + 5*sqr
t(d*f)*A*b^3*c^3*d^2*f^3 - 6*sqrt(d*f)*C*a^3*c^2*d^3*f^3 - 4*sqrt(d*f)*B*a^
2*b*c^2*d^3*f^3 - 18*sqrt(d*f)*A*a*b^2*c^2*d^3*f^3 + 8*sqrt(d*f)*B*a^3*c*d^
4*f^3 + 24*sqrt(d*f)*A*a^2*b*c*d^4*f^3 - 16*sqrt(d*f)*A*a^3*d^5*f^3 - 4*sqr
t(d*f)*C*a*b^2*c^3*d^2*f^2*e - 6*sqrt(d*f)*B*b^3*c^3*d^2*f^2*e + 23*sqrt(d*
f)*C*a^2*b*c^2*d^3*f^2*e + 23*sqrt(d*f)*B*a*b^2*c^2*d^3*f^2*e + 3*sqrt(d*f)
*A*b^3*c^2*d^3*f^2*e - 4*sqrt(d*f)*C*a^3*c*d^4*f^2*e - 40*sqrt(d*f)*B*a^2*b
*c*d^4*f^2*e - 12*sqrt(d*f)*A*a*b^2*c*d^4*f^2*e + 8*sqrt(d*f)*B*a^3*d^5*f^2
*e + 24*sqrt(d*f)*A*a^2*b*d^5*f^2*e + 8*sqrt(d*f)*C*b^3*c^3*d^2*f*e^2 - 40*
sqrt(d*f)*C*a*b^2*c^2*d^3*f*e^2 - 4*sqrt(d*f)*B*b^3*c^2*d^3*f*e^2 + 23*sqrt
(d*f)*C*a^2*b*c*d^4*f*e^2 + 23*sqrt(d*f)*B*a*b^2*c*d^4*f*e^2 + 3*sqrt(d*f)*
A*b^3*c*d^4*f*e^2 - 6*sqrt(d*f)*C*a^3*d^5*f*e^2 - 4*sqrt(d*f)*B*a^2*b*d^5*f
*e^2 - 18*sqrt(d*f)*A*a*b^2*d^5*f*e^2 + 8*sqrt(d*f)*C*b^3*c^2*d^3*e^3 - 4*s
qrt(d*f)*C*a*b^2*c*d^4*e^3 - 6*sqrt(d*f)*B*b^3*c*d^4*e^3 + sqrt(d*f)*C*a^2*
b*d^5*e^3 + sqrt(d*f)*B*a*b^2*d^5*e^3 + 5*sqrt(d*f)*A*b^3*d^5*e^3)*arctan(-
1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*
e + a*b*d^2*f*e)*d))/((a^3*b^3*c^3*f^3*abs(d) - 3*a^4*b^2*c^2*d*f^3*abs(d)
+ 3*a^5*b*c*d^2*f^3*abs(d) - a^6*d^3*f^3*abs(d) - 3*a^2*b^4*c^3*f^2*abs(d)*
```

e + 9*a^3*b^3*c^2*d*f^2*abs(d)*e - 9*a^4*b^2*c*d^2*f^2*abs(d)*e + 3*a^5*b*d
^3*f^2*abs(d)*e + 3*a*b^5*c^3*f*abs(d)*e^2 - 9*a^2*b^4*c^2*d*f*abs(d)*e^2 +
 9*a^3*b^3*c*d^2*f*abs(d)*e^2 - 3*a^4*b^2*d^3*f*abs(d)*e^2 - b^6*c^3*abs(d)
*e^3 + 3*a*b^5*c^2*d*abs(d)*e^3 - 3*a^2*b^4*c*d^2*abs(d)*e^3 + a^3*b^3*d^3*
abs(d)*e^3)*sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d)
+ 1/12*(3*sqrt(d*f)*C*a^2*b^5*c^8*d^7*f^8 + 3*sqrt(d*f)*B*a*b^6*c^8*d^7*f^8
 + 15*sqrt(d*f)*A*b^7*c^8*d^7*f^8 + 16*sqrt(d*f)*C*a^3*b^4*c^7*d^8*f^8 - 10
*sqrt(d*f)*B*a^2*b^5*c^7*d^8*f^8 - 44*sqrt(d*f)*A*a*b^6*c^7*d^8*f^8 - 4*sqr
t(d*f)*C*a^4*b^3*c^6*d^9*f^8 - 8*sqrt(d*f)*B*a^3*b^4*c^6*d^9*f^8 + 44*sqrt(
d*f)*A*a^2*b^5*c^6*d^9*f^8 - 12*sqrt(d*f)*C*a*b^6*c^8*d^7*f^7*e - 18*sqrt(d
*f)*B*b^7*c^8*d^7*f^7*e - 52*sqrt(d*f)*C*a^2*b^5*c^7*d^8*f^7*e + 40*sqrt(d*
f)*B*a*b^6*c^7*d^8*f^7*e - 76*sqrt(d*f)*A*b^7*c^7*d^8*f^7*e - 80*sqrt(d*f)*
C*a^3*b^4*c^6*d^9*f^7*e + 50*sqrt(d*f)*B*a^2*b^5*c^6*d^9*f^7*e + 220*sqrt(d
*f)*A*a*b^6*c^6*d^9*f^7*e + 24*sqrt(d*f)*C*a^4*b^3*c^5*d^10*f^7*e + 48*sqrt
(d*f)*B*a^3*b^4*c^5*d^10*f^7*e - 264*sqrt(d*f)*A*a^2*b^5*c^5*d^10*f^7*e - 1
5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^2*C*a^2*b^5*c^7*d^6*f^7 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*
x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^6*c^7*d^6*f^7 - 75*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^7*c^7*d^6*f^7
 - 78*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^2*C*a^3*b^4*c^6*d^7*f^7 + 84*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^5*c^6*d^7*f^7 + 390*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^6*c^6*
d^7*f^7 + 216*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d
*f + d^2*e))^2*C*a^4*b^3*c^5*d^8*f^7 - 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3*b^4*c^5*d^8*f^7 - 720*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a
^2*b^5*c^5*d^8*f^7 - 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)
*d*f - c*d*f + d^2*e))^2*C*a^5*b^2*c^4*d^9*f^7 - 96*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^4*b^3*c^4*d^9*f^7
 + 480*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2
*e))^2*A*a^3*b^4*c^4*d^9*f^7 + 24*sqrt(d*f)*C*b^7*c^8*d^7*f^6*e^2 + 60*sqrt
(d*f)*C*a*b^6*c^7*d^8*f^6*e^2 + 90*sqrt(d*f)*B*b^7*c^7*d^8*f^6*e^2 + 252*sq
rt(d*f)*C*a^2*b^5*c^6*d^9*f^6*e^2 - 300*sqrt(d*f)*B*a*b^6*c^6*d^9*f^6*e^2 +
 156*sqrt(d*f)*A*b^7*c^6*d^9*f^6*e^2 + 144*sqrt(d*f)*C*a^3*b^4*c^5*d^10*f^6
*e^2 - 90*sqrt(d*f)*B*a^2*b^5*c^5*d^10*f^6*e^2 - 396*sqrt(d*f)*A*a*b^6*c^5*
d^10*f^6*e^2 - 60*sqrt(d*f)*C*a^4*b^3*c^4*d^11*f^6*e^2 - 120*sqrt(d*f)*B*a^
3*b^4*c^4*d^11*f^6*e^2 + 660*sqrt(d*f)*A*a^2*b^5*c^4*d^11*f^6*e^2 + 60*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a
*b^6*c^7*d^6*f^6*e + 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)
*d*f - c*d*f + d^2*e))^2*B*b^7*c^7*d^6*f^6*e + 171*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^5*c^6*d^7*f^6*e
 - 453*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^
2*e))^2*B*a*b^6*c^6*d^7*f^6*e + 135*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^7*c^6*d^7*f^6*e - 300*sqrt(d*f)*(s

```
qrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^4*c
^5*d^8*f^6*e + 360*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*B*a^2*b^5*c^5*d^8*f^6*e - 9...
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^4*(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.61 $\int \sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)\,dx$

Optimal. Leaf size=1182

$$\frac{2(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2((c^2C-7Ad^2)f+Bd(de-2cf))-b^3(C(16d^3e^3-\cdots}{315b^3d^3f\cdots}$$

```
[Out] 2/9*C*(b*x+a)^(3/2)*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-2/21*(2*a*C*d*f-b*(3*
B*d*f-2*C*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(3/2)*(b*x+a)^(1/2)/b/d^2/f^2-2
/105*(7*b*d*f*(-3*A*b*d*f+C*a*c*f+C*a*d*e+C*b*c*e)+(a*d*f-4*b*(c*f+d*e))*(2
*a*C*d*f-b*(3*B*d*f-2*C*(c*f+d*e))))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c)^(1
/2)/b^2/d^2/f^3+2/315*(8*a^3*C*d^3*f^3+3*a^2*b*d^2*f^2*(-4*B*d*f-C*c*f+C*d*
e)-3*a*b^2*d*f^2*((-7*A*d^2+C*c^2)*f+B*d*(-2*c*f+d*e))-b^3*(C*(-8*c^3*f^3-3
*c^2*d*e*f^2+16*d^3*e^3)+3*d*f*(7*A*d*f*(-c*f+2*d*e)-B*(-4*c^2*f^2-c*d*e*f+
8*d^2*e^2))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d^3/f^3-2/315*(
16*a^4*C*d^4*f^4-8*a^3*b*d^3*f^3*(3*B*d*f+C*c*f+C*d*e)+3*a^2*b^2*d^2*f^2*(d
*f*(14*A*d*f+5*B*c*f+5*B*d*e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2))-a*b^3*d*f*(C*(
8*c^3*f^3-6*c^2*d*e*f^2-6*c*d^2*e^2*f+8*d^3*e^3)+3*d*f*(14*A*d*f*(c*f+d*e)-
B*(5*c^2*f^2-6*c*d*e*f+5*d^2*e^2)))+b^4*(2*C*(8*c^4*f^4-4*c^3*d*e*f^3-3*c^2
*d^2*e^2*f^2-4*c*d^3*e^3*f+8*d^4*e^4)+3*d*f*(14*A*d*f*(c^2*f^2-c*d*e*f+d^2*
e^2)-B*(8*c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+8*d^3*e^3))))*EllipticE(d^(1/
2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*
c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(7/2)/f^4/(d*x+c)
^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/315*(-a*f+b*e)*(-c*f+d*e)*(8*a^3*C*d^
3*f^3+3*a^2*b*d^2*f^2*(-4*B*d*f-C*c*f+C*d*e)-3*a*b^2*d*f^2*((-7*A*d^2+C*c^2
)*f+B*d*(-2*c*f+d*e))-b^3*(C*(-8*c^3*f^3-3*c^2*d*e*f^2+16*d^3*e^3)+3*d*f*(7
*A*d*f*(-c*f+2*d*e)-B*(-4*c^2*f^2-c*d*e*f+8*d^2*e^2))))*EllipticF(d^(1/2)*(
b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(
1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/d^(7/2)/
f^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 2.79, antiderivative size = 1154, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (2*((8*a^3*C*d*f)/b - 3*a*b*(B*d*e - 2*B*c*f + (c^2*C*f)/d - 7*A*d*f) + 3*a
^2*(C*d*e - c*C*f - 4*B*d*f) + b^2*((3*c^2*C*e)/d - 42*A*d*e - (16*C*d*e^3)
```

```
/f^2 + 21*A*c*f + (8*c^3*C*f)/d^2 - B*(3*c*e - (24*d*e^2)/f + (12*c^2*f)/d)
))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(315*b^2*d*f) - (2*(7*b*d*f*(
b*c*C*e + a*C*d*e + a*c*C*f - 3*A*b*d*f) - (a*d*f - 4*b*(d*e + c*f))*(3*b*B
*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x
)^(3/2))/(105*b^2*d^2*f^3) + (2*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))
*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(21*b*d^2*f^2) + (2*C*(a +
b*x)^(3/2)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(9*b*d*f) - (2*Sqrt[-(b*c) + a*
d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) + 3*a^2*b^
2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^
2*f^2)) - a*b^3*d*f*(C*(8*d^3*e^3 - 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + 8*c^3*f
^3) + 3*d*f*(14*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 - 6*c*d*e*f + 5*c^2*f^2)))
 + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3
+ 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e*f + c^2*f^2) - B*(8*d^3*e^3
 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c -
a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) +
a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(315*b^4*d^(7/2)*f^4*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e
- c*f)*(8*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*f - 4*B*d*f) - 3*a*b
^2*d*f^2*((c^2*C - 7*A*d^2)*f + B*d*(d*e - 2*c*f)) - b^3*(C*(16*d^3*e^3 - 3
*c^2*d*e*f^2 - 8*c^3*f^3) + 3*d*f*(7*A*d*f*(2*d*e - c*f) - B*(8*d^2*e^2 - c
*d*e*f - 4*c^2*f^2))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(
b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], (
(b*c - a*d)*f)/(d*(b*e - a*f))])/(315*b^4*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[e
+ f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&& !LtQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
```

```
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx = \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} + \frac{2\int \sqrt{a+bx}\ldots}{\ldots}$$

$$= \frac{2(3bBdf - 2aCdf - 2bC(de+cf))\sqrt{a+bx}\,(c+d\ldots}{21bd^2f^2}$$

$$= -\frac{2(7bdf(bcCe + aCde + acCf - 3Abdf) - (adf - \ldots}{\ldots}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab\ldots}{\ldots}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab\ldots}{\ldots}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab\ldots}{\ldots}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab\ldots}{\ldots}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.44, size = 1362, normalized size = 1.15

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (2*(-(b^2*Sqrt[-a + (b*c)/d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c
*C*f + 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2
*C*(d^2*e^2 - c*d*e*f + c^2*f^2)) + a*b^3*d*f*(C*(-8*d^3*e^3 + 6*c*d^2*e^2*
f + 6*c^2*d*e*f^2 - 8*c^3*f^3) - 3*d*f*(14*A*d*f*(d*e + c*f) + B*(-5*d^2*e^
2 + 6*c*d*e*f - 5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*
d^2*e^2*f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e
*f + c^2*f^2) + B*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 8*c^3*f^3))
))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x)*(c + d*x)*(e
 + f*x)*(8*a^3*C*d^3*f^3 - 3*a^2*b*d^2*f^2*(c*C*f + 4*B*d*f + C*d*(e + 2*f*
x)) + a*b^2*d*f*(3*d*f*(7*A*d*f + B*(2*d*e + 2*c*f + 3*d*f*x)) + C*(-3*c^2*
f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 5*f^2*x^2))) + b^3*(C*(8*
c^3*f^3 - 3*c^2*d*f^2*(e + 2*f*x) + c*d^2*f*(-3*e^2 + 2*e*f*x + 5*f^2*x^2)
+ d^3*(8*e^3 - 6*e^2*f*x + 5*e*f^2*x^2 + 35*f^3*x^3)) + 3*d*f*(7*A*d*f*(c*f
 + d*(e + 3*f*x)) + B*(-4*c^2*f^2 + c*d*f*(2*e + 3*f*x) + d^2*(-4*e^2 + 3*e
*f*x + 15*f^2*x^2))))) - I*(b*c - a*d)*f*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))
/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*((16*a^4*C*d^4*f^4 - 8*a^
3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5
*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)) + a*b^3*d*f*(C*(-8*
d^3*e^3 + 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 8*c^3*f^3) - 3*d*f*(14*A*d*f*(d*e
 + c*f) + B*(-5*d^2*e^2 + 6*c*d*e*f - 5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 -
4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*
A*d*f*(d^2*e^2 - c*d*e*f + c^2*f^2) + B*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2
*d*e*f^2 - 8*c^3*f^3))))*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - b*(d*e - c*f)*(8*a^3*C*d^3*f^3 - 3*
a^2*b*d^2*f^2*(C*d*e - c*C*f + 4*B*d*f) - 3*a*b^2*d^2*f*(C*d*e^2 + f*(-2*B*
d*e + B*c*f - 7*A*d*f)) + b^3*(C*(8*d^3*e^3 + 3*c*d^2*e^2*f - 16*c^3*f^3) -
 3*d*f*(-7*A*d*f*(d*e - 2*c*f) + B*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2))))*Ell
ipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f
- a*d*f)])))/(315*b^5*Sqrt[-a + (b*c)/d]*d^4*f^4*Sqrt[a + b*x]*Sqrt[c + d*x
]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 14777 vs. $2(1112) = 2224$.
time = 0.11, size = 14778, normalized size = 12.50

method	result	size
elliptic	Expression too large to display	2077
default	Expression too large to display	14778

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 1910, normalized size = 1.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/945*(3*(35*C*b^5*d^5*f^5*x^3 + 8*C*b^5*d^5*f^2*e^3 + 5*(C*b^5*c*d^4 + (C*
a*b^4 + 9*B*b^5)*d^5)*f^5*x^2 - (6*C*b^5*c^2*d^3 - (2*C*a*b^4 + 9*B*b^5)*c*
d^4 + 3*(2*C*a^2*b^3 - 3*B*a*b^4 - 21*A*b^5)*d^5)*f^5*x + (8*C*b^5*c^3*d^2
- 3*(C*a*b^4 + 4*B*b^5)*c^2*d^3 - 3*(C*a^2*b^3 - 2*B*a*b^4 - 7*A*b^5)*c*d^4
 + (8*C*a^3*b^2 - 12*B*a^2*b^3 + 21*A*a*b^4)*d^5)*f^5 - 3*(2*C*b^5*d^5*f^3*
x + (C*b^5*c*d^4 + (C*a*b^4 + 4*B*b^5)*d^5)*f^3)*e^2 + (5*C*b^5*d^5*f^4*x^2
 + (2*C*b^5*c*d^4 + (2*C*a*b^4 + 9*B*b^5)*d^5)*f^4*x - (3*C*b^5*c^2*d^3 - 2
*(C*a*b^4 + 3*B*b^5)*c*d^4 + 3*(C*a^2*b^3 - 2*B*a*b^4 - 7*A*b^5)*d^5)*f^4)*
e)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) + (16*C*b^5*d^5*e^5 + (16*C*b^
5*c^5 - 8*(2*C*a*b^4 + 3*B*b^5)*c^4*d - (5*C*a^2*b^3 - 27*B*a*b^4 - 42*A*b^
5)*c^3*d^2 - (5*C*a^3*b^2 - 12*B*a^2*b^3 + 63*A*a*b^4)*c^2*d^3 - (16*C*a^4*
b - 27*B*a^3*b^2 + 63*A*a^2*b^3)*c*d^4 + 2*(8*C*a^5 - 12*B*a^4*b + 21*A*a^3
*b^2)*d^5)*f^5 - (16*C*b^5*c^4*d - (20*C*a*b^4 + 27*B*b^5)*c^3*d^2 - 3*(2*C
*a^2*b^3 - 14*B*a*b^4 - 21*A*b^5)*c^2*d^3 - 2*(10*C*a^3*b^2 - 21*B*a^2*b^3
+ 126*A*a*b^4)*c*d^4 + (16*C*a^4*b - 27*B*a^3*b^2 + 63*A*a^2*b^3)*d^5)*f^4*
e - (5*C*b^5*c^3*d^2 - 6*(C*a*b^4 + 2*B*b^5)*c^2*d^3 - 3*(2*C*a^2*b^3 - 14*
B*a*b^4 - 21*A*b^5)*c*d^4 + (5*C*a^3*b^2 - 12*B*a^2*b^3 + 63*A*a*b^4)*d^5)*
f^3*e^2 - (5*C*b^5*c^2*d^3 - (20*C*a*b^4 + 27*B*b^5)*c*d^4 + (5*C*a^2*b^3 -
 27*B*a*b^4 - 42*A*b^5)*d^5)*f^2*e^3 - 8*(2*C*b^5*c*d^4 + (2*C*a*b^4 + 3*B*
b^5)*d^5)*f*e^4)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^
2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*
(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^
3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*
d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))
 + 3*(16*C*b^5*d^5*f*e^4 + (16*C*b^5*c^4*d - 8*(C*a*b^4 + 3*B*b^5)*c^3*d^2
```

```
- 3*(2*C*a^2*b^3 - 5*B*a*b^4 - 14*A*b^5)*c^2*d^3 - (8*C*a^3*b^2 - 15*B*a^2*
b^3 + 42*A*a*b^4)*c*d^4 + 2*(8*C*a^4*b - 12*B*a^3*b^2 + 21*A*a^2*b^3)*d^5)*
f^5 - (8*C*b^5*c^3*d^2 - 3*(2*C*a*b^4 + 5*B*b^5)*c^2*d^3 - 6*(C*a^2*b^3 - 3
*B*a*b^4 - 7*A*b^5)*c*d^4 + (8*C*a^3*b^2 - 15*B*a^2*b^3 + 42*A*a*b^4)*d^5)*
f^4*e - 3*(2*C*b^5*c^2*d^3 - (2*C*a*b^4 + 5*B*b^5)*c*d^4 + (2*C*a^2*b^3 - 5
*B*a*b^4 - 14*A*b^5)*d^5)*f^3*e^2 - 8*(C*b^5*c*d^4 + (C*a*b^4 + 3*B*b^5)*d^
5)*f^2*e^3)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c
*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^
3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^
3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^
2)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d
 + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*
e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*
c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)
/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^5*d^5
*f^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] int((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2), x)
```

3.62 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=774

$$-\frac{2(5bdf(3aC(de+cf)+b(cCe-7Adf))-(2bde-bcf+4adf)(6aCdf-b(7Bdf-4C(de+cf))))\sqrt{a+}}{105b^3d^2f^2}$$

```
[Out] 2/7*C*(d*x+c)^(3/2)*(f*x+e)^(3/2)*(b*x+a)^(1/2)/b/d/f-2/35*(6*a*C*d*f-b*(7*
B*d*f-4*C*(c*f+d*e)))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d/f^2-2
/105*(5*b*d*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*e))-(4*a*d*f-b*c*f+2*b*d*e)*
(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^
(1/2)/b^3/d^2/f^2-2/105*(3*b*d*f*(5*b*c*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*
e))-(3*a*c*f+a*d*e+b*c*e)*(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e))))+2*(1/2*b*d
*e-(a*d+b*c)*f)*(5*b*d*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*e))-(4*a*d*f-b*c*
f+2*b*d*e)*(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e)))))*EllipticE(d^(1/2)*(b*x+a
)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*
(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(5/2)/f^3/(d*x+c)^(1/2)/(b
*(f*x+e)/(-a*f+b*e))^(1/2)-2/105*(-a*f+b*e)*(-c*f+d*e)*(24*a^2*C*d^2*f^2+a*
b*d*f*(-28*B*d*f-5*C*c*f+13*C*d*e)-b^2*(7*d*f*(-5*A*d*f-B*c*f+2*B*d*e)-C*(-
4*c^2*f^2-c*d*e*f+8*d^2*e^2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1
/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c
))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/d^(5/2)/f^3/(d*x+c)^(1/2)/(f*x+e)
^(1/2)
```

Rubi [A]
time = 1.42, antiderivative size = 769, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (-2*(((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C*d*f - 4*b*C*(d*e + c*f
)))/(b*d*f) + 5*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f)))*Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x])/(105*b^2*d*f) + (2*(7*b*B*d*f - 6*a*C*d*f - 4*b*
C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(35*b^2*d*f^2)
+ (2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(7*b*d*f) - (2*Sqrt[-
(b*c) + a*d]*(3*b*d*f*((b*c*e + a*d*e + 3*a*c*f)*(7*b*B*d*f - 6*a*C*d*f - 4
```

```
*b*C*(d*e + c*f)) + 5*b*c*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))) + 2*
((b*d*e)/2 - (b*c + a*d)*f)*((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C
*d*f - 4*b*C*(d*e + c*f)) + 5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f
))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(10
5*b^4*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[
-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(24*a^2*C*d^2*f^2 + a*b*d*f*(13*C*d*e
 - 5*c*C*f - 28*B*d*f) - b^2*(7*d*f*(2*B*d*e - B*c*f - 5*A*d*f) - C*(8*d^2*
e^2 - c*d*e*f - 4*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f
*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*
d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(105*b^4*d^(5/2)*f^3*Sqrt[c + d*x]*S
qrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
```

```
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \frac{2C\sqrt{a+bx}\,(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} + \frac{2\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(\cdots}{\cdots}}{\cdots}$$

$$= \frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}\,\sqrt{c+dx}\,(e+fx)\cdots}{35b^2df^2}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5\cdots}{10\cdots}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5\cdots}{10\cdots}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5\cdots}{10\cdots}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5\cdots}{10\cdots}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5\cdots}{10\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.17, size = 917, normalized size = 1.18

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f +
2*C*(d*e + c*f)) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-9*
d^2*e^2 + 8*c*d*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*
c^2*d*e*f^2 - 8*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 - c*d*
e*f + c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x
)*(c + d*x)*(e + f*x)*(-24*a^2*C*d^2*f^2 + a*b*d*f*(28*B*d*f + C*(5*d*e + 5
*c*f + 18*d*f*x)) + b^2*(-7*d*f*(B*c*f + 5*A*d*f + B*d*(e + 3*f*x)) + C*(4*
c^2*f^2 - c*d*f*(2*e + 3*f*x) + d^2*(4*e^2 - 3*e*f*x - 15*f^2*x^2)))) + I*(
b*c - a*d)*f*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f + 2*C*(d*e + c*f)
) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-9*d^2*e^2 + 8*c*d
*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 8
*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 - c*d*e*f + c^2*f^2))
))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*
(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e -
 a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f)*(24*a^2*C*d^2*f^2
+ a*b*d*f*(-5*C*d*e + 13*c*C*f - 28*B*d*f) + b^2*(7*d*f*(B*d*e - 2*B*c*f +
5*A*d*f) - C*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c
 + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSi
nh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(1
05*b^5*Sqrt[-a + (b*c)/d]*d^3*f^3*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9542 vs. $2(710) = 1420$.

time = 0.11, size = 9543, normalized size = 12.33

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cx^2\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+}}{7b}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 1391, normalized size = 1.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/315*(3*(15*C*b^4*d^4*f^4*x^2 - 4*C*b^4*d^4*f^2*e^2 + 3*(C*b^4*c*d^3 - (6*
C*a*b^3 - 7*B*b^4)*d^4)*f^4*x - (4*C*b^4*c^2*d^2 + (5*C*a*b^3 - 7*B*b^4)*c*
d^3 - (24*C*a^2*b^2 - 28*B*a*b^3 + 35*A*b^4)*d^4)*f^4 + (3*C*b^4*d^4*f^3*x
+ (2*C*b^4*c*d^3 - (5*C*a*b^3 - 7*B*b^4)*d^4)*f^3)*e)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(f*x + e) - (8*C*b^4*d^4*e^4 + (8*C*b^4*c^4 + (5*C*a*b^3 - 14*B*
b^4)*c^3*d + (10*C*a^2*b^2 - 14*B*a*b^3 + 35*A*b^4)*c^2*d^2 + (40*C*a^3*b -
 49*B*a^2*b^2 + 70*A*a*b^3)*c*d^3 - 2*(24*C*a^4 - 28*B*a^3*b + 35*A*a^2*b^2
)*d^4)*f^4 - (9*C*b^4*c^3*d + 7*(C*a*b^3 - 3*B*b^4)*c^2*d^2 + 14*(3*C*a^2*b
^2 - 4*B*a*b^3 + 10*A*b^4)*c*d^3 - (40*C*a^3*b - 49*B*a^2*b^2 + 70*A*a*b^3)
*d^4)*f^3*e - (4*C*b^4*c^2*d^2 + 7*(C*a*b^3 - 3*B*b^4)*c*d^3 - (10*C*a^2*b^
2 - 14*B*a*b^3 + 35*A*b^4)*d^4)*f^2*e^2 - (9*C*b^4*c*d^3 - (5*C*a*b^3 - 14*
B*b^4)*d^4)*f*e^3)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*
c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/2
7*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*
f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^
2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f
)) - 3*(8*C*b^4*d^4*f*e^3 + (8*C*b^4*c^3*d + (9*C*a*b^3 - 14*B*b^4)*c^2*d^2
 + (16*C*a^2*b^2 - 21*B*a*b^3 + 35*A*b^4)*c*d^3 - 2*(24*C*a^3*b - 28*B*a^2*
b^2 + 35*A*a*b^3)*d^4)*f^4 - (5*C*b^4*c^2*d^2 + 2*(4*C*a*b^3 - 7*B*b^4)*c*d
^3 - (16*C*a^2*b^2 - 21*B*a*b^3 + 35*A*b^4)*d^4)*f^3*e - (5*C*b^4*c*d^3 - (
9*C*a*b^3 - 14*B*b^4)*d^4)*f^2*e^2)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^
2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d
^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
+ 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3
*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*
```

```
e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2
*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c
*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d
)*f)/(b*d*f))))/(b^5*d^4*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/sqrt(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(1/2),x)
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(1/2), x)
```

3.63 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=706

$$\frac{2(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf)))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+}}{15b^3df(be-af)}$$

```
[Out] -2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/
(b*x+a)^(1/2)+2/5*(6*a^2*C*d*f+b^2*(5*A*d*f+C*c*e)-a*b*(5*B*d*f+C*c*f+C*d*e
))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)+2/
15*(24*a^2*C*d*f^2-a*b*f*(20*B*d*f+C*c*f+7*C*d*e)+b^2*(5*d*f*(3*A*f+B*e)-C*
e*(-c*f+2*d*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d/f/(-a*f+b*
e)+2/15*(48*a^2*C*d^2*f^2-8*a*b*d*f*(5*B*d*f+C*c*f+C*d*e)+b^2*(5*d*f*(6*A*d
*f+B*c*f+B*d*e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2)))*EllipticE(d^(1/2)*(b*x+a)^(
1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*
(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(3/2)/f^2/(d*x+c)^(1/2)/(b*(f
*x+e)/(-a*f+b*e))^(1/2)-2/15*(-c*f+d*e)*(24*a^2*C*d*f^2-a*b*f*(20*B*d*f+C*c
*f+7*C*d*e)+b^2*(5*d*f*(3*A*f+B*e)-C*e*(-c*f+2*d*e)))*EllipticF(d^(1/2)*(b*
x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/
2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/d^(3/2)/f^
2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 1.24, antiderivative size = 706, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 159, 164, 115, 114, 122, 121}

$$\frac{2\sqrt{e+fx}\,\sqrt{ad-bc}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2(5df(6Adf+Bcf+Bde)-2C(c^2f^2-cdef+d^2e^2)))\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{15b^4d^{3/2}f^2\sqrt{c+dx}\,\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{2\sqrt{ad-bc}\,(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sqrt{\frac{b(e+fx)}{be-af}}\,(24a^2Cdf^2-abf(20Bdf+cCf+7Cde)+b^2(5df(3Af+Be)-Ce(2de-cf)))\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{15b^4d^{3/2}f^2\sqrt{c+dx}\,\sqrt{e+fx}}+\frac{2\sqrt{a+bx}\,\sqrt{c+dx}\,(e+fx)^{3/2}(6a^2Cdf-ab(5Bdf+cCf+Cde)+b^2(5Adf+cCe))}{5b^2f(bc-ad)(be-af)}+\frac{2\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,(24a^2Cdf^2-abf(20Bdf+cCf+7Cde)+b^2(5df(3Af+Be)-Ce(2de-cf)))}{15b^3df(be-af)}-\frac{2(c+dx)^{3/2}(e+fx)^{3/2}(Ab^2-a(bB-aC))}{b\sqrt{a+bx}\,(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),x]
```

```
[Out] (2*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(B*e +
 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(1
5*b^3*d*f*(b*e - a*f)) + (2*(6*a^2*C*d*f + b^2*(c*C*e + 5*A*d*f) - a*b*(C*d
*e + c*C*f + 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(5*b^2*
(b*c - a*d)*f*(b*e - a*f)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e
+ f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]) + (2*Sqrt[-(b*c) +
a*d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(
B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*Sqrt[(b*(c +
```

```
 d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/
Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^4*d^(3/2)*f^2*
Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e
 - c*f)*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(
B*e + 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*
(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c
) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^4*d^(3/2)*f^2*Sqrt[c + d
*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
```

```
1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int \frac{\sqrt{c+dx}}{}}{}$$

$$= \frac{2(6a^2Cdf+b^2(cCe+5Adf)-ab(Cde+cCf+5Bdf))\,\sqrt{a+}}{5b^2(bc-ad)f(be-af)}$$

$$= \frac{2(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3A}{15b^3df(be-a}$$

$$= \frac{2(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3A}{15b^3df(be-a}$$

$$= \frac{2(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3A}{15b^3df(be-a}$$

$$= \frac{2(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3A}{15b^3df(be-a}$$

$$= \frac{2(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3A}{15b^3df(be-a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 25.76, size = 633, normalized size = 0.90

$$-\frac{2\left(-b^2\sqrt{-a+\frac{bc}{d}}\,(48a^2Cd^2f^2-8abdf(Cde+cCf+5Bdf)+b^2(5df(Bde+Bcf+6Adf)-2C(d^2e^2-cdef+c^2f^2)))\,(c+dx)(e+fx)+b^2\sqrt{-a+\frac{bc}{d}}\,df(c+dx)(e+fx)\,(15(Ab^2+a(-bB+aC))\,df-(-9aCdf+b(Cde+cCf+5Bdf))(a+bx)-3bCdfx(a+bx))-i(bc-ad)f(48a^2Cd^2f^2-8abdf(Cde+cCf+5Bdf)+b^2(5df(Bde+Bcf+6Adf)-2C(d^2e^2-cdef+c^2f^2)))\,(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\,\sqrt{\frac{b(e+fx)}{f(a+bx)}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)\middle|\frac{bde-adf}{bcf-adf}\right)-ibf(de-cf)\,(24a^2Cd^2f-abd(Cde+7cCf+20Bdf)+b^2(-2c^2Cf+15Ad^2f+cd(Ce+5Bf)))\,(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\,\sqrt{\frac{b(e+fx)}{f(a+bx)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)\middle|\frac{bde-adf}{bcf-adf}\right)\right)}{15b^5\sqrt{-a+\frac{bc}{d}}\,d^2f^2\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),x
]
```

```
[Out] (-2*(-(b^2*Sqrt[-a + (b*c)/d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f
+ 5*B*d*f) + b^2*(5*d*f*(B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f
+ c^2*f^2)))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e
 + f*x)*(15*(A*b^2 + a*(-(b*B) + a*C))*d*f - (-9*a*C*d*f + b*(C*d*e + c*C*f
 + 5*B*d*f))*(a + b*x) - 3*b*C*d*f*x*(a + b*x)) - I*(b*c - a*d)*f*(48*a^2*C
*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(B*d*e + B*c*f
+ 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c
 + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSi
nh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*
b*f*(d*e - c*f)*(24*a^2*C*d^2*f - a*b*d*(C*d*e + 7*c*C*f + 20*B*d*f) + b^2*
(-2*c^2*C*f + 15*A*d^2*f + c*d*(C*e + 5*B*f)))*(a + b*x)^(3/2)*Sqrt[(b*(c +
 d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh
[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*
b^5*Sqrt[-a + (b*c)/d]*d^2*f^2*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5786 vs. $2(646) = 1292$.
time = 0.12, size = 5787, normalized size = 8.20

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(-\frac{2\left(bdf\,x^{2}+bcfx+bdex+bce\right)\left(b^{2}A-abB+C\,a^{2}\right)}{b^{4}\sqrt{\left(x+\frac{a}{b}\right)\left(bdf\,x^{2}+bcfx+bdex+bce\right)}}+\frac{2Cx\sqrt{bdf\,x^{3}+adf\,x^{2}}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 1458, normalized size = 2.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="fricas")
```

```
[Out] 2/45*(3*(3*C*b^4*d^3*f^3*x^2 + (C*b^4*c*d^2 - (6*C*a*b^3 - 5*B*b^4)*d^3)*f^
3*x + (C*a*b^3*c*d^2 - (24*C*a^2*b^2 - 20*B*a*b^3 + 15*A*b^4)*d^3)*f^3 + (C
*b^4*d^3*f^2*x + C*a*b^3*d^3*f^2)*e)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x +
 e) + ((2*C*b^4*c^3 + (7*C*a*b^3 - 5*B*b^4)*c^2*d + (32*C*a^2*b^2 - 25*B*a*
b^3 + 15*A*b^4)*c*d^2 - 2*(24*C*a^3*b - 20*B*a^2*b^2 + 15*A*a*b^3)*d^3)*f^3
*x + (2*C*a*b^3*c^3 + (7*C*a^2*b^2 - 5*B*a*b^3)*c^2*d + (32*C*a^3*b - 25*B*
a^2*b^2 + 15*A*a*b^3)*c*d^2 - 2*(24*C*a^4 - 20*B*a^3*b + 15*A*a^2*b^2)*d^3)
*f^3 + 2*(C*b^4*d^3*x + C*a*b^3*d^3)*e^3 - ((3*C*b^4*c*d^2 - (7*C*a*b^3 - 5
*B*b^4)*d^3)*f*x + (3*C*a*b^3*c*d^2 - (7*C*a^2*b^2 - 5*B*a*b^3)*d^3)*f)*e^2
 - ((3*C*b^4*c^2*d + 4*(7*C*a*b^3 - 5*B*b^4)*c*d^2 - (32*C*a^2*b^2 - 25*B*a
*b^3 + 15*A*b^4)*d^3)*f^2*x + (3*C*a*b^3*c^2*d + 4*(7*C*a^2*b^2 - 5*B*a*b^3
)*c*d^2 - (32*C*a^3*b - 25*B*a^2*b^2 + 15*A*a*b^3)*d^3)*f^2)*e)*sqrt(b*d*f)
*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 -
 (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3
 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c
*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1
/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*((2*C*b^4*c^2*d + (8*C*
a*b^3 - 5*B*b^4)*c*d^2 - 2*(24*C*a^2*b^2 - 20*B*a*b^3 + 15*A*b^4)*d^3)*f^3*
x + (2*C*a*b^3*c^2*d + (8*C*a^2*b^2 - 5*B*a*b^3)*c*d^2 - 2*(24*C*a^3*b - 20
*B*a^2*b^2 + 15*A*a*b^3)*d^3)*f^3 + 2*(C*b^4*d^3*f*x + C*a*b^3*d^3*f)*e^2 -
 ((2*C*b^4*c*d^2 - (8*C*a*b^3 - 5*B*b^4)*d^3)*f^2*x + (2*C*a*b^3*c*d^2 - (8
*C*a^2*b^2 - 5*B*a*b^3)*d^3)*f^2)*e)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d
^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*
d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^
3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2
*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^
2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*
c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*
d)*f)/(b*d*f))))/(b^6*d^3*f^3*x + a*b^5*d^3*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(3/2), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(3/2),x)
```

```
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(3/2), x)
```

3.64 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=687

$$\frac{2(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{3b^3(bc-ad)(be-af)}-\frac{2(bB-2aC}{b^2(be}$$

```
[Out] -2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(3/2)-2*(B*b-2*C*a)*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/(-a*f+b*e)/(b
*x+a)^(1/2)+2/3*(8*a^2*C*d*f+b^2*(A*d*f+3*B*c*f+C*c*e)-a*b*(4*B*d*f+7*C*c*f
+C*d*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)/(-a*f+b*e
)+2/3*(16*a^3*C*d^2*f^2-8*a^2*b*d*f*(B*d*f+2*C*(c*f+d*e))-b^3*(c^2*C*e*f+A*
d^2*e*f+c*d*(A*f^2+6*B*e*f+C*e^2))+a*b^2*(d*f*(2*A*d*f+7*B*c*f+7*B*d*e)+C*(
c^2*f^2+16*c*d*e*f+d^2*e^2)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/
2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^
(1/2)/b^4/f/(-a*f+b*e)/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a
*f+b*e))^(1/2)+2/3*(-c*f+d*e)*(8*a^2*C*d*f+b^2*(A*d*f+3*B*c*f+C*c*e)-a*b*(4
*B*d*f+7*C*c*f+C*d*e))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a
*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*
f+b*e))^(1/2)/b^4/f/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 1.26, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1628, 155, 159, 164, 115, 114, 122, 121}

$$\frac{2(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe))\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^4\sqrt{d}\,f\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{ad-bc}}+\frac{2\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe))}{3b^3(bc-ad)(be-af)}+\frac{2\sqrt{e+fx}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,(16a^3Cd^2f^2-8a^2bdf(Bdf+2C(cf+de))+ab^2(df(2Adf+7Bcf+7Bde)+C(c^2f^2+16cdef+d^2e^2))-b^3(cd(Af^2+6Bef+Ce^2)+Ad^2ef+c^2Cef))\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^4\sqrt{d}\,f\sqrt{c+dx}\,\sqrt{ad-bc}\,(be-af)\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\,(Ab^2-a(bB-aC))}{3b(a+bx)^{3/2}(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\,(e+fx)^{3/2}(bB-2aC)}{b^2\sqrt{a+bx}\,(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),x]
```

```
[Out] (2*(8*a^2*C*d*f + b^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*
B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b^3*(b*c - a*d)*(b*e
- a*f)) - (2*(b*B - 2*a*C)*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b^2*(b*e - a*f)*
Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2)
)/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(3/2)) + (2*(16*a^3*C*d^2*f^2 - 8*
a^2*b*d*f*(B*d*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e
^2 + 6*B*e*f + A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*
e^2 + 16*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]
*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
```

```
*f)/(d*(b*e - a*f))]])/(3*b^4*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[
c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(8*a^2*C*d*f + b
^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*B*d*f))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
3*b^4*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
```

```
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{2\int \sqrt{c+dx}}{}$$

$$= -\frac{2(bB-2aC)\sqrt{c+dx}\,(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}} - \frac{2(Ab^2-a(bB-aC))\,(c}{3b(bc-ad)(be-}$$

$$= \frac{2(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf}{3b^3(bc-ad)(be-af)}$$

$$= \frac{2(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf}{3b^3(bc-ad)(be-af)}$$

$$= \frac{2(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf}{3b^3(bc-ad)(be-af)}$$

$$= \frac{2(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf}{3b^3(bc-ad)(be-af)}$$

$$= \frac{2(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf}{3b^3(bc-ad)(be-af)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.81, size = 815, normalized size = 1.19

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a
*C))*(b*c - a*d)*(b*e - a*f) + (-8*a^3*C*d*f + b^3*(3*B*c*e + A*d*e + A*c*f
) - 2*a*b^2*(3*c*C*e + 2*B*d*e + 2*B*c*f + A*d*f) + a^2*b*(5*B*d*f + 7*C*(d
*e + c*f)))*(a + b*x) - C*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (a + b*x)*
(b^2*Sqrt[-a + (b*c)/d]*(16*a^3*C*d^2*f^2 - 8*a^2*b*d*f*(B*d*f + 2*C*(d*e +
 c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e^2 + 6*B*e*f + A*f^2)) + a*b^
2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*e^2 + 16*c*d*e*f + c^2*f^2)))
*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(16*a^3*C*d^2*f^2 - 8*a^2*b*d*f*(B*d
*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e^2 + 6*B*e*f +
 A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*e^2 + 16*c*d*e
*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f)*(8*a^
2*C*d*f + b^2*(c*C*e + 3*B*d*e + A*d*f) - a*b*(7*C*d*e + c*C*f + 4*B*d*f))*
(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a
+ b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*
d*f)/(b*c*f - a*d*f)])))/(3*b^5*Sqrt[-a + (b*c)/d]*d*(b*c - a*d)*f*(b*e - a
*f)*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15768 vs. $2(627) = 1254$.

time = 0.12, size = 15769, normalized size = 22.95

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(-\frac{2\left(b^2A-abB+Ca^2\right)\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+ade}}{3b^5\left(x+\frac{a}{b}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 2584, normalized size = 3.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="fricas")
```

```
[Out] 2/9*(3*((C*a*b^5*c*d^2 - C*a^2*b^4*d^3)*f^3*x^2 + ((9*C*a^2*b^4 - 4*B*a*b^5
 + A*b^6)*c*d^2 - (10*C*a^3*b^3 - 5*B*a^2*b^4 + 2*A*a*b^5)*d^3)*f^3*x + ((7
*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 - (8*C*a^4*b^2 - 4*B*a^3*b^3 + A*a^2*b^4)*d
^3)*f^3 - ((C*b^6*c*d^2 - C*a*b^5*d^3)*f^2*x^2 + ((8*C*a*b^5 - 3*B*b^6)*c*d
^2 - (9*C*a^2*b^4 - 4*B*a*b^5 + A*b^6)*d^3)*f^2*x + ((6*C*a^2*b^4 - 2*B*a*b
^5 - A*b^6)*c*d^2 - (7*C*a^3*b^3 - 3*B*a^2*b^4)*d^3)*f^2)*e)*sqrt(b*x + a)*
sqrt(d*x + c)*sqrt(f*x + e) - ((C*a*b^5*c^3 + (6*C*a^2*b^4 - 2*B*a*b^5 - A*
b^6)*c^2*d - (24*C*a^3*b^3 - 11*B*a^2*b^4 + 2*A*a*b^5)*c*d^2 + 2*(8*C*a^4*b
^2 - 4*B*a^3*b^3 + A*a^2*b^4)*d^3)*f^3*x^2 + 2*(C*a^2*b^4*c^3 + (6*C*a^3*b^
3 - 2*B*a^2*b^4 - A*a*b^5)*c^2*d - (24*C*a^4*b^2 - 11*B*a^3*b^3 + 2*A*a^2*b
^4)*c*d^2 + 2*(8*C*a^5*b - 4*B*a^4*b^2 + A*a^3*b^3)*d^3)*f^3*x + (C*a^3*b^3
*c^3 + (6*C*a^4*b^2 - 2*B*a^3*b^3 - A*a^2*b^4)*c^2*d - (24*C*a^5*b - 11*B*a
^4*b^2 + 2*A*a^3*b^3)*c*d^2 + 2*(8*C*a^6 - 4*B*a^5*b + A*a^4*b^2)*d^3)*f^3
- (C*a^2*b^4*c*d^2 - C*a^3*b^3*d^3 + (C*b^6*c*d^2 - C*a*b^5*d^3)*x^2 + 2*(C
*a*b^5*c*d^2 - C*a^2*b^4*d^3)*x)*e^3 + ((4*C*b^6*c^2*d - (11*C*a*b^5 - 3*B*
b^6)*c*d^2 + (6*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*d^3)*f*x^2 + 2*(4*C*a*b^5*c^
2*d - (11*C*a^2*b^4 - 3*B*a*b^5)*c*d^2 + (6*C*a^3*b^3 - 2*B*a^2*b^4 - A*a*b
^5)*d^3)*f*x + (4*C*a^2*b^4*c^2*d - (11*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 + (6
*C*a^4*b^2 - 2*B*a^3*b^3 - A*a^2*b^4)*d^3)*f)*e^2 - ((C*b^6*c^3 + (11*C*a*b
^5 - 3*B*b^6)*c^2*d - 2*(19*C*a^2*b^4 - 8*B*a*b^5 + 2*A*b^6)*c*d^2 + (24*C*
a^3*b^3 - 11*B*a^2*b^4 + 2*A*a*b^5)*d^3)*f^2*x^2 + 2*(C*a*b^5*c^3 + (11*C*a
^2*b^4 - 3*B*a*b^5)*c^2*d - 2*(19*C*a^3*b^3 - 8*B*a^2*b^4 + 2*A*a*b^5)*c*d^
2 + (24*C*a^4*b^2 - 11*B*a^3*b^3 + 2*A*a^2*b^4)*d^3)*f^2*x + (C*a^2*b^4*c^3
 + (11*C*a^3*b^3 - 3*B*a^2*b^4)*c^2*d - 2*(19*C*a^4*b^2 - 8*B*a^3*b^3 + 2*A
*a^2*b^4)*c*d^2 + (24*C*a^5*b - 11*B*a^4*b^2 + 2*A*a^3*b^3)*d^3)*f^2)*e)*sq
```

```
rt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d
^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2
*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d -
4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^
3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*((C*a*b^5*c^2*
d - (16*C*a^2*b^4 - 7*B*a*b^5 + A*b^6)*c*d^2 + 2*(8*C*a^3*b^3 - 4*B*a^2*b^4
 + A*a*b^5)*d^3)*f^3*x^2 + 2*(C*a^2*b^4*c^2*d - (16*C*a^3*b^3 - 7*B*a^2*b^4
 + A*a*b^5)*c*d^2 + 2*(8*C*a^4*b^2 - 4*B*a^3*b^3 + A*a^2*b^4)*d^3)*f^3*x +
(C*a^3*b^3*c^2*d - (16*C*a^4*b^2 - 7*B*a^3*b^3 + A*a^2*b^4)*c*d^2 + 2*(8*C*
a^5*b - 4*B*a^4*b^2 + A*a^3*b^3)*d^3)*f^3 - ((C*b^6*c*d^2 - C*a*b^5*d^3)*f*
x^2 + 2*(C*a*b^5*c*d^2 - C*a^2*b^4*d^3)*f*x + (C*a^2*b^4*c*d^2 - C*a^3*b^3*
d^3)*f)*e^2 - ((C*b^6*c^2*d - 2*(8*C*a*b^5 - 3*B*b^6)*c*d^2 + (16*C*a^2*b^4
 - 7*B*a*b^5 + A*b^6)*d^3)*f^2*x^2 + 2*(C*a*b^5*c^2*d - 2*(8*C*a^2*b^4 - 3*
B*a*b^5)*c*d^2 + (16*C*a^3*b^3 - 7*B*a^2*b^4 + A*a*b^5)*d^3)*f^2*x + (C*a^2
*b^4*c^2*d - 2*(8*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 + (16*C*a^4*b^2 - 7*B*a^3*
b^3 + A*a^2*b^4)*d^3)*f^2)*e)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2
+ (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2
), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^
3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2
 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 +
(b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2),
 -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*
d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 +
 a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(
b*d*f))))/((a*b^8*c*d^2 - a^2*b^7*d^3)*f^3*x^2 + 2*(a^2*b^7*c*d^2 - a^3*b^6
*d^3)*f^3*x + (a^3*b^6*c*d^2 - a^4*b^5*d^3)*f^3 - ((b^9*c*d^2 - a*b^8*d^3)*
f^2*x^2 + 2*(a*b^8*c*d^2 - a^2*b^7*d^3)*f^2*x + (a^2*b^7*c*d^2 - a^3*b^6*d^
3)*f^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(5/2), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(5/2),x)
```

```
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(5/2), x)
```

3.65 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=964

$$\frac{2(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af)) + ab^2(15c^2Cf + d^2(3}{15b^3(bc-ad)^2(be-af)\sqrt{a+bx}}$$

```
[Out] -2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(5/2)+2/15*(6*a^3*C*d*f+a*b^2*(-4*A*d*f+3*B*c*f+3*B*d*e+10*C*c*e)
-b^3*(5*B*c*e-2*A*(c*f+d*e))-a^2*b*(B*d*f+8*C*(c*f+d*e)))*(f*x+e)^(3/2)*(d*
x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/15*(24*a^3*C*d^2*f-a
^2*b*d*(4*B*d*f+41*C*c*f+23*C*d*e)-b^3*(15*c^2*C*e-2*A*d^2*e+c*d*(A*f+5*B*e
))+a*b^2*(15*c^2*C*f+d^2*(-A*f+3*B*e)+c*(6*B*d*f+40*C*d*e)))*(d*x+c)^(1/2)*
(f*x+e)^(1/2)/b^3/(-a*d+b*c)^2/(-a*f+b*e)/(b*x+a)^(1/2)+2/15*(48*a^4*C*d^2*
f^2-8*a^3*b*d*f*(B*d*f+11*C*(c*f+d*e))-b^4*(2*A*d^2*e^2-c*d*e*(2*A*f+5*B*e)
-c^2*(-2*A*f^2+5*B*e*f+30*C*e^2))-a*b^3*(d^2*e*(-2*A*f+3*B*e)+c^2*f*(3*B*f+
70*C*e)+2*c*d*(-A*f^2+11*B*e*f+35*C*e^2))+a^2*b^2*(2*C*(19*c^2*f^2+81*c*d*e
*f+19*d^2*e^2)-d*f*(2*A*d*f-13*B*(c*f+d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/
2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-
a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2
)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/15*(-c*f+d*e)*(24*a^3*C*d^2*f-a^2*b*d*(4*B
*d*f+41*C*c*f+23*C*d*e)-b^3*(15*c^2*C*e-2*A*d^2*e+c*d*(A*f+5*B*e))+a*b^2*(1
5*c^2*C*f+d^2*(-A*f+3*B*e)+c*(6*B*d*f+40*C*d*e)))*EllipticF(d^(1/2)*(b*x+a)
^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+
b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/(a*d-b*c)^(3/2)/(-a*f+b*e)/d^(
1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]

time = 2.02, antiderivative size = 964, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 155, 164, 115, 114, 122, 121}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),x]
```

```
[Out] (2*(24*a^3*C*d^2*f - a^2*b*d*(23*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*
C*e - 2*A*d^2*e + c*d*(5*B*e + A*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f
) + c*(40*C*d*e + 6*B*d*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b^3*(b*c - a*
```

```
d)^2*(b*e - a*f)*Sqrt[a + b*x]) + (2*(6*a^3*C*d*f + a*b^2*(10*c*C*e + 3*B*d
*e + 3*B*c*f - 4*A*d*f) - b^3*(5*B*c*e - 2*A*(d*e + c*f)) - a^2*b*(B*d*f +
8*C*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(15*b^2*(b*c - a*d)*(b*e -
 a*f)^2*(a + b*x)^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e +
f*x)^(3/2))/(5*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(5/2)) + (2*Sqrt[d]*(48*
a^4*C*d^2*f^2 - 8*a^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) - b^4*(2*A*d^2*e^2 -
 c*d*e*(5*B*e + 2*A*f) - c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*b^3*(d^2*e
*(3*B*e - 2*A*f) + c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*
f^2)) + a^2*b^2*(2*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) - d*f*(2*A*d*f
- 13*B*(d*e + c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellipti
cE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(
b*e - a*f))])/(15*b^4*(-(b*c) + a*d)^(3/2)*(b*e - a*f)^2*Sqrt[c + d*x]*Sqrt
[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(24*a^3*C*d^2*f - a^2*b*d*(23
*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*C*e - 2*A*d^2*e + c*d*(5*B*e + A
*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f) + c*(40*C*d*e + 6*B*d*f)))*Sqr
t[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcS
in[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a
*f))])/(15*b^4*Sqrt[d]*(-(b*c) + a*d)^(3/2)*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[
e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \sqrt{c+dx}}{}$$

$$= \frac{2(6a^3Cdf + ab^2(10cCe+3Bde+3Bcf-4Adf) - b^3(5Bce -}{15b^2(bc-ad)(be}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde+41cCf+4Bdf) - b^3(15c^2Ce - 2}{}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde+41cCf+4Bdf) - b^3(15c^2Ce - 2}{}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde+41cCf+4Bdf) - b^3(15c^2Ce - 2}{}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde+41cCf+4Bdf) - b^3(15c^2Ce - 2}{}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde+41cCf+4Bdf) - b^3(15c^2Ce - 2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.14, size = 1444, normalized size = 1.50

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),x
]
```

```
[Out] Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5*
b^3*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e + A*b^3*d*e - 6*a*b^2*B
*d*e + 11*a^2*b*C*d*e + A*b^3*c*f - 6*a*b^2*B*c*f + 11*a^2*b*c*C*f - 2*a*A*
b^2*d*f + 7*a^2*b*B*d*f - 12*a^3*C*d*f))/(15*b^3*(b*c - a*d)*(b*e - a*f)*(a
 + b*x)^2) - (2*(15*b^4*c^2*C*e^2 + 5*b^4*B*c*d*e^2 - 40*a*b^3*c*C*d*e^2 -
2*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 23*a^2*b^2*C*d^2*e^2 + 5*b^4*B*c^2*e*
f - 40*a*b^3*c^2*C*e*f + 2*A*b^4*c*d*e*f - 22*a*b^3*B*c*d*e*f + 102*a^2*b^2
*c*C*d*e*f + 2*a*A*b^3*d^2*e*f + 13*a^2*b^2*B*d^2*e*f - 58*a^3*b*C*d^2*e*f
- 2*A*b^4*c^2*f^2 - 3*a*b^3*B*c^2*f^2 + 23*a^2*b^2*c^2*C*f^2 + 2*a*A*b^3*c*
d*f^2 + 13*a^2*b^2*B*c*d*f^2 - 58*a^3*b*c*C*d*f^2 - 2*a^2*A*b^2*d^2*f^2 - 8
*a^3*b*B*d^2*f^2 + 33*a^4*C*d^2*f^2))/(15*b^3*(b*c - a*d)^2*(b*e - a*f)^2*(
a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(48*a^4*C*d^2*f^2 - 8*a
^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) + b^4*(-2*A*d^2*e^2 + c*d*e*(5*B*e + 2*
A*f) + c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*b^3*(d^2*e*(3*B*e - 2*A*f) +
 c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*f^2)) + a^2*b^2*(2
*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) + d*f*(-2*A*d*f + 13*B*(d*e + c*f
))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f + (b*e)/(a + b*x) - (a*f)/(
a + b*x)) + (I*(-(b*c) + a*d)*f*(-48*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f + 1
1*C*(d*e + c*f)) - b^4*(-2*A*d^2*e^2 + c*d*e*(5*B*e + 2*A*f) + c^2*(30*C*e^
2 + 5*B*e*f - 2*A*f^2)) + a*b^3*(d^2*e*(3*B*e - 2*A*f) + c^2*f*(70*C*e + 3*
B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*f^2)) - a^2*b^2*(2*C*(19*d^2*e^2 + 81
*c*d*e*f + 19*c^2*f^2) + d*f*(-2*A*d*f + 13*B*(d*e + c*f))))*Sqrt[1 - a/(a
+ b*x) + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(f*(a + b*x))]*E
llipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*
f - a*d*f)])/Sqrt[a + b*x] - (I*b*(-(b*c) + a*d)*(d*e - c*f)*(-24*a^3*C*d*f
^2 + a^2*b*f*(41*C*d*e + 23*c*C*f + 4*B*d*f) + b^3*(15*c*C*e^2 + A*d*e*f +
c*f*(5*B*e - 2*A*f)) - a*b^2*(5*C*e*(3*d*e + 8*c*f) + f*(6*B*d*e + 3*B*c*f
- A*d*f)))*Sqrt[1 - a/(a + b*x) + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x)
 + (b*e)/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x
]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[a + b*x]))/(15*b^5*Sqrt[-a + (b*
c)/d]*(b*c - a*d)^2*(b*e - a*f)^2*Sqrt[c + ((a + b*x)*(d - (a*d)/(a + b*x))
)/b]*Sqrt[e + ((a + b*x)*(f - (a*f)/(a + b*x)))/b])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 34619 vs. $2(902) = 1804$.
time = 0.15, size = 34620, normalized size = 35.91

method	result	size
elliptic	Expression too large to display	2292
default	Expression too large to display	34620

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 4721, normalized size = 4.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="fricas")
```

```
[Out] -2/45*(3*(((23*C*a^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*c^2*d - (58*C*a^3*b^5 - 13*
B*a^2*b^6 - 2*A*a*b^7)*c*d^2 + (33*C*a^4*b^4 - 8*B*a^3*b^5 - 2*A*a^2*b^6)*d
^3)*f^3*x^2 + (5*(7*C*a^3*b^5 - A*a*b^7)*c^2*d - (93*C*a^4*b^4 - 13*B*a^3*b
^5 - 7*A*a^2*b^6)*c*d^2 + 3*(18*C*a^5*b^3 - 3*B*a^4*b^4 - 2*A*a^3*b^5)*d^3)
*f^3*x + (15*C*a^4*b^4*c^2*d - (41*C*a^5*b^3 - 6*B*a^4*b^4 + A*a^3*b^5)*c*d
^2 + (24*C*a^6*b^2 - 4*B*a^5*b^3 - A*a^4*b^4)*d^3)*f^3 + ((15*C*b^8*c^2*d -
 5*(8*C*a*b^7 - B*b^8)*c*d^2 + (23*C*a^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*d^3)*f*
x^2 + (5*(4*C*a*b^7 + B*b^8)*c^2*d - (59*C*a^2*b^6 + B*a*b^7 - A*b^8)*c*d^2
 + 5*(7*C*a^3*b^5 - A*a*b^7)*d^3)*f*x + (15*C*a^4*b^4*d^3 + (8*C*a^2*b^6 +
2*B*a*b^7 + 3*A*b^8)*c^2*d - 5*(5*C*a^3*b^5 + A*a*b^7)*c*d^2)*f)*e^2 - ((5*
(8*C*a*b^7 - B*b^8)*c^2*d - 2*(51*C*a^2*b^6 - 11*B*a*b^7 + A*b^8)*c*d^2 + (
58*C*a^3*b^5 - 13*B*a^2*b^6 - 2*A*a*b^7)*d^3)*f^2*x^2 + ((59*C*a^2*b^6 + B*
a*b^7 - A*b^8)*c^2*d - 20*(8*C*a^3*b^5 - B*a^2*b^6)*c*d^2 + (93*C*a^4*b^4 -
 13*B*a^3*b^5 - 7*A*a^2*b^6)*d^3)*f^2*x + (5*(5*C*a^3*b^5 + A*a*b^7)*c^2*d
- 10*(7*C*a^4*b^4 - B*a^3*b^5 + A*a^2*b^6)*c*d^2 + (41*C*a^5*b^3 - 6*B*a^4*
b^4 + A*a^3*b^5)*d^3)*f^2)*e)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (
((7*C*a^2*b^6 + 3*B*a*b^7 + 2*A*b^8)*c^3 - (73*C*a^3*b^5 - 8*B*a^2*b^6 + 3*
A*a*b^7)*c^2*d + (112*C*a^4*b^4 - 17*B*a^3*b^5 - 3*A*a^2*b^6)*c*d^2 - 2*(24
*C*a^5*b^3 - 4*B*a^4*b^4 - A*a^3*b^5)*d^3)*f^3*x^3 + 3*((7*C*a^3*b^5 + 3*B*
a^2*b^6 + 2*A*a*b^7)*c^3 - (73*C*a^4*b^4 - 8*B*a^3*b^5 + 3*A*a^2*b^6)*c^2*d
 + (112*C*a^5*b^3 - 17*B*a^4*b^4 - 3*A*a^3*b^5)*c*d^2 - 2*(24*C*a^6*b^2 - 4
```

*B*a^5*b^3 - A*a^4*b^4)*d^3)*f^3*x^2 + 3*((7*C*a^4*b^4 + 3*B*a^3*b^5 + 2*A*
a^2*b^6)*c^3 - (73*C*a^5*b^3 - 8*B*a^4*b^4 + 3*A*a^3*b^5)*c^2*d + (112*C*a^
6*b^2 - 17*B*a^5*b^3 - 3*A*a^4*b^4)*c*d^2 - 2*(24*C*a^7*b - 4*B*a^6*b^2 - A
*a^5*b^3)*d^3)*f^3*x + ((7*C*a^5*b^3 + 3*B*a^4*b^4 + 2*A*a^3*b^5)*c^3 - (73
*C*a^6*b^2 - 8*B*a^5*b^3 + 3*A*a^4*b^4)*c^2*d + (112*C*a^7*b - 17*B*a^6*b^2
 - 3*A*a^5*b^3)*c*d^2 - 2*(24*C*a^8 - 4*B*a^7*b - A*a^6*b^2)*d^3)*f^3 + (15
*C*a^3*b^5*c^2*d - 5*(4*C*a^4*b^4 + B*a^3*b^5)*c*d^2 + (7*C*a^5*b^3 + 3*B*a
^4*b^4 + 2*A*a^3*b^5)*d^3 + (15*C*b^8*c^2*d - 5*(4*C*a*b^7 + B*b^8)*c*d^2 +
 (7*C*a^2*b^6 + 3*B*a*b^7 + 2*A*b^8)*d^3)*x^3 + 3*(15*C*a*b^7*c^2*d - 5*(4*
C*a^2*b^6 + B*a*b^7)*c*d^2 + (7*C*a^3*b^5 + 3*B*a^2*b^6 + 2*A*a*b^7)*d^3)*x
^2 + 3*(15*C*a^2*b^6*c^2*d - 5*(4*C*a^3*b^5 + B*a^2*b^6)*c*d^2 + (7*C*a^4*b
^4 + 3*B*a^3*b^5 + 2*A*a^2*b^6)*d^3)*x)*e^3 + ((15*C*b^8*c^3 - 10*(13*C*a*b
^7 - 2*B*b^8)*c^2*d + (182*C*a^2*b^6 - 22*B*a*b^7 - 3*A*b^8)*c*d^2 - (73*C*
a^3*b^5 - 8*B*a^2*b^6 + 3*A*a*b^7)*d^3)*f*x^3 + 3*(15*C*a*b^7*c^3 - 10*(13*
C*a^2*b^6 - 2*B*a*b^7)*c^2*d + (182*C*a^3*b^5 - 22*B*a^2*b^6 - 3*A*a*b^7)*c
*d^2 - (73*C*a^4*b^4 - 8*B*a^3*b^5 + 3*A*a^2*b^6)*d^3)*f*x^2 + 3*(15*C*a^2*
b^6*c^3 - 10*(13*C*a^3*b^5 - 2*B*a^2*b^6)*c^2*d + (182*C*a^4*b^4 - 22*B*a^3
*b^5 - 3*A*a^2*b^6)*c*d^2 - (73*C*a^5*b^3 - 8*B*a^4*b^4 + 3*A*a^3*b^5)*d^3)
*f*x + (15*C*a^3*b^5*c^3 - 10*(13*C*a^4*b^4 - 2*B*a^3*b^5)*c^2*d + (182*C*a
^5*b^3 - 22*B*a^4*b^4 - 3*A*a^3*b^5)*c*d^2 - (73*C*a^6*b^2 - 8*B*a^5*b^3 +
3*A*a^4*b^4)*d^3)*f)*e^2 - ((5*(4*C*a*b^7 + B*b^8)*c^3 - (182*C*a^2*b^6 - 2
2*B*a*b^7 - 3*A*b^8)*c^2*d + 2*(134*C*a^3*b^5 - 19*B*a^2*b^6 - 6*A*a*b^7)*c
*d^2 - (112*C*a^4*b^4 - 17*B*a^3*b^5 - 3*A*a^2*b^6)*d^3)*f^2*x^3 + 3*(5*(4*
C*a^2*b^6 + B*a*b^7)*c^3 - (182*C*a^3*b^5 - 22*B*a^2*b^6 - 3*A*a*b^7)*c^2*d
 + 2*(134*C*a^4*b^4 - 19*B*a^3*b^5 - 6*A*a^2*b^6)*c*d^2 - (112*C*a^5*b^3 -
17*B*a^4*b^4 - 3*A*a^3*b^5)*d^3)*f^2*x^2 + 3*(5*(4*C*a^3*b^5 + B*a^2*b^6)*c
^3 - (182*C*a^4*b^4 - 22*B*a^3*b^5 - 3*A*a^2*b^6)*c^2*d + 2*(134*C*a^5*b^3
- 19*B*a^4*b^4 - 6*A*a^3*b^5)*c*d^2 - (112*C*a^6*b^2 - 17*B*a^5*b^3 - 3*A*a
^4*b^4)*d^3)*f^2*x + (5*(4*C*a^4*b^4 + B*a^3*b^5)*c^3 - (182*C*a^5*b^3 - 22
*B*a^4*b^4 - 3*A*a^3*b^5)*c^2*d + 2*(134*C*a^6*b^2 - 19*B*a^5*b^3 - 6*A*a^4
*b^4)*c*d^2 - (112*C*a^7*b - 17*B*a^6*b^2 - 3*A*a^5*b^3)*d^3)*f^2)*e)*sqrt(
b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)
*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^
3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a
*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f
^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(((38*C*a^2*b^6 -
 3*B*a*b^7 - 2*A*b^8)*c^2*d - (88*C*a^3*b^5 - 13*B*a^2*b^6 - 2*A*a*b^7)*c*d
^2 + 2*(24*C*a^4*b^4 - 4*B*a^3*b^5 - A*a^2*b^6)*d^3)*f^3*x^3 + 3*((38*C*a^3
*b^5 - 3*B*a^2*b^6 - 2*A*a*b^7)*c^2*d - (88*C*a^4*b^4 - 13*B*a^3*b^5 - 2*A*
a^2*b^6)*c*d^2 + 2*(24*C*a^5*b^3 - 4*B*a^4*b^4 - A*a^3*b^5)*d^3)*f^3*x^2 +
3*((38*C*a^4*b^4 - 3*B*a^3*b^5 - 2*A*a^2*b^6)*c^2*d - (88*C*a^5*b^3 - 13*B*
a^4*b^4 - 2*A*a^3*b^5)*c*d^2 + 2*(24*C*a^6*b^2 - 4*B*a^5*b^3 - A*a^4*b^4)*d
^3)*f^3*x + ((38*C*a^5*b^3 - 3*B*a^4*b^4 - 2*A*a^3*b^5)*c^2*d - (88*C*a^6*b
^2 - 13*B*a^5*b^3 - 2*A*a^4*b^4)*c*d^2 + 2*(24*C*a^7*b - 4*B*a^6*b^2 - A*a^
5*b^3)*d^3)*f^3 + ((30*C*b^8*c^2*d - 5*(14*C*a*b^7 - B*b^8)*c*d^2 + (38*C*a

```
^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*d^3)*f*x^3 + 3*(3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(7/2),x)
```

```
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(7/2), x)
```

3.66 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=1716

$$-\frac{2(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3(d^2e(Be-3Af)+2c^2f(7Ce-Bf)+cd(28Ce^2-}{}$$

```
[Out] -2/7*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(7/2)+2/35*(6*a^3*C*d*f+a*b^2*(-8*A*d*f+3*B*c*f+3*B*d*e+14*C*c*e)
-b^3*(7*B*c*e-4*A*(c*f+d*e))+a^2*b*(B*d*f-10*C*(c*f+d*e)))*(f*x+e)^(3/2)*(d
*x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(5/2)-2/105*(24*a^4*C*d^2*f
^2-a^3*b*d*f*(-4*B*d*f+43*C*c*f+61*C*d*e)-3*a*b^3*(d^2*e*(-3*A*f+B*e)+2*c^2
*f*(-B*f+7*C*e)+c*d*(5*A*f^2-5*B*e*f+28*C*e^2))-b^4*(4*A*d^2*e^2-c*d*e*(-A*
f+7*B*e)-c^2*(8*A*f^2-14*B*e*f+35*C*e^2))-3*a^2*b^2*(d*f*(-A*d*f+2*B*c*f+3*
B*d*e)-C*(5*c^2*f^2+37*c*d*e*f+15*d^2*e^2)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^
3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/105*(48*a^5*C*d^3*f^3+8*a^4*b*d
^2*f^2*(B*d*f-16*C*(c*f+d*e))-b^5*(8*A*d^3*e^3-c*d^2*e^2*(5*A*f+14*B*e)+c^2
*d*e*(-5*A*f^2+14*B*e*f+35*C*e^2)+c^3*f*(8*A*f^2-14*B*e*f+35*C*e^2))-a*b^4*
(d^3*e^2*(-19*A*f+6*B*e)-6*c^3*f^2*(-B*f+7*C*e)-c^2*d*f*(238*C*e^2-19*f*(-A
*f+B*e))-c*d^2*e*(42*C*e^2-f*(20*A*f+19*B*e)))+a^3*b^2*d*f*(C*(103*c^2*f^2+
344*c*d*e*f+103*d^2*e^2)+d*f*(6*A*d*f-19*B*(c*f+d*e)))-3*a^2*b^3*(C*(5*c^3*
f^3+94*c^2*d*e*f^2+94*c*d^2*e^2*f+5*d^3*e^3)+d*f*(3*A*d*f*(c*f+d*e)-B*(3*c^
2*f^2+16*c*d*e*f+3*d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)^3
/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/105*(48*a^5*C*d^3*f^3+8*a^4*b*d^2*f^2*(B*d*f-
16*C*(c*f+d*e))-b^5*(8*A*d^3*e^3-c*d^2*e^2*(5*A*f+14*B*e)+c^2*d*e*(-5*A*f^2
+14*B*e*f+35*C*e^2)+c^3*f*(8*A*f^2-14*B*e*f+35*C*e^2))-a*b^4*(d^3*e^2*(-19*
A*f+6*B*e)-6*c^3*f^2*(-B*f+7*C*e)-c^2*d*f*(238*C*e^2-19*f*(-A*f+B*e))-c*d^2
*e*(42*C*e^2-f*(20*A*f+19*B*e)))+a^3*b^2*d*f*(C*(103*c^2*f^2+344*c*d*e*f+10
3*d^2*e^2)+d*f*(6*A*d*f-19*B*(c*f+d*e)))-3*a^2*b^3*(C*(5*c^3*f^3+94*c^2*d*e
*f^2+94*c*d^2*e^2*f+5*d^3*e^3)+d*f*(3*A*d*f*(c*f+d*e)-B*(3*c^2*f^2+16*c*d*e
*f+3*d^2*e^2))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c
)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)
/b^4/(a*d-b*c)^(5/2)/(-a*f+b*e)^3/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2
)+2/105*(-c*f+d*e)*(24*a^4*C*d^2*f^2-a^3*b*d*f*(-4*B*d*f+61*C*c*f+43*C*d*e)
+b^4*(8*A*d^2*e^2-c*d*e*(A*f+14*B*e)+c^2*(-4*A*f^2+7*B*e*f+35*C*e^2))+3*a*b
^3*(d^2*e*(-5*A*f+2*B*e)-c^2*f*(B*f+28*C*e)-c*d*(-3*A*f^2-5*B*e*f+14*C*e^2)
)-3*a^2*b^2*(d*f*(-A*d*f+3*B*c*f+2*B*d*e)-C*(15*c^2*f^2+37*c*d*e*f+5*d^2*e^
2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+
b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1
/2)/b^4/(a*d-b*c)^(5/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]

time = 4.55, antiderivative size = 1716, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1628, 155, 157, 164, 115, 114, 122, 121}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(61*C*d*e + 43*c*C*f - 4*B*d*f) - 3*a*b^3
*(d^2*e*(B*e - 3*A*f) + 2*c^2*f*(7*C*e - B*f) + c*d*(28*C*e^2 - 5*B*e*f + 5
*A*f^2)) - b^4*(4*A*d^2*e^2 - c*d*e*(7*B*e - A*f) - c^2*(35*C*e^2 - 14*B*e*
f + 8*A*f^2)) - 3*a^2*b^2*(d*f*(3*B*d*e + 2*B*c*f - A*d*f) - C*(15*d^2*e^2
+ 37*c*d*e*f + 5*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^3*(b*c - a*
d)^2*(b*e - a*f)^2*(a + b*x)^(3/2)) + (2*(48*a^5*C*d^3*f^3 + 8*a^4*b*d^2*f^
2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c*d^2*e^2*(14*B*e + 5*A*f
) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f*(35*C*e^2 - 14*B*e*f +
8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3*f^2*(7*C*e - B*f) - c^2
*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C*e^2 - f*(19*B*e + 20*A*
f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f + 103*c^2*f^2) + d*f*(6*A*
d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3 + 94*c*d^2*e^2*f + 94*c^
2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 16*c*d*e
*f + 3*c^2*f^2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^3*(b*c - a*d)^3*(b*e
 - a*f)^3*Sqrt[a + b*x]) + (2*(6*a^3*C*d*f + a*b^2*(14*c*C*e + 3*B*d*e + 3*
B*c*f - 8*A*d*f) - b^3*(7*B*c*e - 4*A*(d*e + c*f)) + a^2*b*(B*d*f - 10*C*(d
*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(35*b^2*(b*c - a*d)*(b*e - a*f)^
2*(a + b*x)^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(
3/2))/(7*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(7/2)) + (2*Sqrt[d]*(48*a^5*C*
d^3*f^3 + 8*a^4*b*d^2*f^2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c
*d^2*e^2*(14*B*e + 5*A*f) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f
*(35*C*e^2 - 14*B*e*f + 8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3
*f^2*(7*C*e - B*f) - c^2*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C
*e^2 - f*(19*B*e + 20*A*f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f +
103*c^2*f^2) + d*f*(6*A*d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3
+ 94*c*d^2*e^2*f + 94*c^2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) -
 B*(3*d^2*e^2 + 16*c*d*e*f + 3*c^2*f^2))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*
Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]],
 ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*(-(b*c) + a*d)^(5/2)*(b*e - a*f
)^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[d]*(d*e - c*f)
*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(43*C*d*e + 61*c*C*f - 4*B*d*f) + b^4*(8*A*d
^2*e^2 - c*d*e*(14*B*e + A*f) + c^2*(35*C*e^2 + 7*B*e*f - 4*A*f^2)) + 3*a*b
^3*(d^2*e*(2*B*e - 5*A*f) - c^2*f*(28*C*e + B*f) - c*d*(14*C*e^2 - 5*B*e*f
- 3*A*f^2)) - 3*a^2*b^2*(d*f*(2*B*d*e + 3*B*c*f - A*d*f) - C*(5*d^2*e^2 + 3
7*c*d*e*f + 15*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x)
```

```
)/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]]
, ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*(-(b*c) + a*d)^(5/2)*(b*e - a*
f)^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}} - \frac{2\int \frac{\sqrt{c+dx}}{}}{}$$

$$= \frac{2(6a^3Cdf+ab^2(14cCe+3Bde+3Bcf-8Adf)-b^3(7Bce-}{35b^2(bc-ad)(b}$$

$$= -\frac{2(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3(d^2e}{}$$

$$= -\frac{2(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3(d^2e}{}$$

$$= -\frac{2(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3(d^2e}{}$$

$$= -\frac{2(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3(d^2e}{}$$

$$= -\frac{2(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3(d^2e}{}$$

$$= -\frac{2(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3(d^2e}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 36.52, size = 15719, normalized size = 9.16

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),x
]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65230 vs. $2(1646) = 3292$.
time = 0.24, size = 65231, normalized size = 38.01

method	result	size
elliptic	Expression too large to display	3900
default	Expression too large to display	65231

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 9152, normalized size = 5.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="fricas")
```

```
[Out] 2/315*(3*(((15*C*a^2*b^8 + 6*B*a*b^9 + 8*A*b^10)*c^3*d - (103*C*a^3*b^7 + 9
*B*a^2*b^8 + 19*A*a*b^9)*c^2*d^2 + (128*C*a^4*b^6 + 19*B*a^3*b^7 + 9*A*a^2*
b^8)*c*d^3 - 2*(24*C*a^5*b^5 + 4*B*a^4*b^6 + 3*A*a^3*b^7)*d^4)*f^4*x^3 + (7
*(3*B*a^2*b^8 + 4*A*a*b^9)*c^3*d - (158*C*a^4*b^6 + 45*B*a^3*b^7 + 67*A*a^2
*b^8)*c^2*d^2 + (221*C*a^5*b^5 + 80*B*a^4*b^6 + 39*A*a^3*b^7)*c*d^3 - (87*C
```

*a^6*b^4 + 32*B*a^5*b^5 + 24*A*a^4*b^6)*d^4)*f^4*x^2 + (35*A*a^2*b^8*c^3*d
- (145*C*a^5*b^5 - 12*B*a^4*b^6 + 89*A*a^3*b^7)*c^2*d^2 + (199*C*a^6*b^4 +
25*B*a^5*b^5 + 66*A*a^4*b^6)*c*d^3 - (78*C*a^7*b^3 + 13*B*a^6*b^4 + 36*A*a^
5*b^5)*d^4)*f^4*x - ((45*C*a^6*b^4 - 3*B*a^5*b^5 - 4*A*a^4*b^6)*c^2*d^2 - (
61*C*a^7*b^3 + 9*B*a^6*b^4 - 9*A*a^5*b^5)*c*d^3 + (24*C*a^8*b^2 + 4*B*a^7*b
^3 + 3*A*a^6*b^4)*d^4)*f^4 + ((35*C*b^10*c^2*d^2 - 14*(3*C*a*b^9 + B*b^10)*
c*d^3 + (15*C*a^2*b^8 + 6*B*a*b^9 + 8*A*b^10)*d^4)*f*x^3 + (35*C*b^10*c^3*d
- 7*(2*C*a*b^9 - B*b^10)*c^2*d^2 + (3*C*a^2*b^8 - 52*B*a*b^9 - 4*A*b^10)*c
d^3 + 7(3*B*a^2*b^8 + 4*A*a*b^9)*d^4)*f*x^2 + (35*A*a^2*b^8*d^4 + 7*(4*C*
a*b^9 + 3*B*b^10)*c^3*d - (4*C*a^2*b^8 + 52*B*a*b^9 - 3*A*b^10)*c^2*d^2 + 7
*(B*a^2*b^8 - 2*A*a*b^9)*c*d^3)*f*x + (35*A*a^2*b^8*c*d^3 + (8*C*a^2*b^8 +
6*B*a*b^9 + 15*A*b^10)*c^3*d - 14*(B*a^2*b^8 + 3*A*a*b^9)*c^2*d^2)*f)*e^3 +
((35*C*b^10*c^3*d - 14*(17*C*a*b^9 - B*b^10)*c^2*d^2 + (282*C*a^2*b^8 + 19
*B*a*b^9 - 5*A*b^10)*c*d^3 - (103*C*a^3*b^7 + 9*B*a^2*b^8 + 19*A*a*b^9)*d^4
)*f^2*x^3 - (7*(2*C*a*b^9 - B*b^10)*c^3*d + (313*C*a^2*b^8 + 2*B*a*b^9 - 2*
A*b^10)*c^2*d^2 - 7*(59*C*a^3*b^7 + 16*B*a^2*b^8 - A*a*b^9)*c*d^3 + (158*C*
a^4*b^6 + 45*B*a^3*b^7 + 67*A*a^2*b^8)*d^4)*f^2*x^2 - ((4*C*a^2*b^8 + 52*B*
a*b^9 - 3*A*b^10)*c^3*d + 7*(44*C*a^3*b^7 - 23*B*a^2*b^8 + 2*A*a*b^9)*c^2*d
^2 - 7*(55*C*a^4*b^6 - 7*B*a^3*b^7 + 4*A*a^2*b^8)*c*d^3 + (145*C*a^5*b^5 -
12*B*a^4*b^6 + 89*A*a^3*b^7)*d^4)*f^2*x - (14*(B*a^2*b^8 + 3*A*a*b^9)*c^3*d
+ 7*(14*C*a^4*b^6 - 6*B*a^3*b^7 - 17*A*a^2*b^8)*c^2*d^2 - 7*(17*C*a^5*b^5
- B*a^4*b^6 - 15*A*a^3*b^7)*c*d^3 + (45*C*a^6*b^4 - 3*B*a^5*b^5 - 4*A*a^4*b
^6)*d^4)*f^2)*e^2 - ((14*(3*C*a*b^9 + B*b^10)*c^3*d - (282*C*a^2*b^8 + 19*B
*a*b^9 - 5*A*b^10)*c^2*d^2 + 4*(86*C*a^3*b^7 + 12*B*a^2*b^8 - 5*A*a*b^9)*c*
d^3 - (128*C*a^4*b^6 + 19*B*a^3*b^7 + 9*A*a^2*b^8)*d^4)*f^3*x^3 - ((3*C*a^2
*b^8 - 52*B*a*b^9 - 4*A*b^10)*c^3*d + 7*(59*C*a^3*b^7 + 16*B*a^2*b^8 - A*a*
b^9)*c^2*d^2 - (565*C*a^4*b^6 + 212*B*a^3*b^7 - 44*A*a^2*b^8)*c*d^3 + (221*
C*a^5*b^5 + 80*B*a^4*b^6 + 39*A*a^3*b^7)*d^4)*f^3*x^2 - (7*(B*a^2*b^8 - 2*A
*a*b^9)*c^3*d + 7*(55*C*a^4*b^6 - 7*B*a^3*b^7 + 4*A*a^2*b^8)*c^2*d^2 - (512
*C*a^5*b^5 + 55*B*a^4*b^6 + 8*A*a^3*b^7)*c*d^3 + (199*C*a^6*b^4 + 25*B*a^5*
b^5 + 66*A*a^4*b^6)*d^4)*f^3*x - (35*A*a^2*b^8*c^3*d + 7*(17*C*a^5*b^5 - B*
a^4*b^6 - 15*A*a^3*b^7)*c^2*d^2 - (156*C*a^6*b^4 + 26*B*a^5*b^5 - 103*A*a^4
*b^6)*c*d^3 + (61*C*a^7*b^3 + 9*B*a^6*b^4 - 9*A*a^5*b^5)*d^4)*f^3)*e)*sqrt(
b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) + (((15*C*a^2*b^8 + 6*B*a*b^9 + 8*A*b^
10)*c^4 + (47*C*a^3*b^7 - 12*B*a^2*b^8 - 23*A*a*b^9)*c^3*d - (158*C*a^4*b^6
+ 17*B*a^3*b^7 - 17*A*a^2*b^8)*c^2*d^2 + (152*C*a^5*b^5 + 23*B*a^4*b^6 + 1
2*A*a^3*b^7)*c*d^3 - 2*(24*C*a^6*b^4 + 4*B*a^5*b^5 + 3*A*a^4*b^6)*d^4)*f^4*
x^4 + 4*((15*C*a^3*b^7 + 6*B*a^2*b^8 + 8*A*a*b^9)*c^4 + (47*C*a^4*b^6 - 12*
B*a^3*b^7 - 23*A*a^2*b^8)*c^3*d - (158*C*a^5*b^5 + 17*B*a^4*b^6 - 17*A*a^3*
b^7)*c^2*d^2 + (152*C*a^6*b^4 + 23*B*a^5*b^5 + 12*A*a^4*b^6)*c*d^3 - 2*(24*
C*a^7*b^3 + 4*B*a^6*b^4 + 3*A*a^5*b^5)*d^4)*f^4*x^3 + 6*((15*C*a^4*b^6 + 6*
B*a^3*b^7 + 8*A*a^2*b^8)*c^4 + (47*C*a^5*b^5 - 12*B*a^4*b^6 - 23*A*a^3*b^7)
*c^3*d - (158*C*a^6*b^4 + 17*B*a^5*b^5 - 17*A*a^4*b^6)*c^2*d^2 + (152*C*a^7
*b^3 + 23*B*a^6*b^4 + 12*A*a^5*b^5)*c*d^3 - 2*(24*C*a^8*b^2 + 4*B*a^7*b^3 +
3*A*a^6*b^4)*d^4)*f^4*x^2 + 4*((15*C*a^5*b^5 + 6*B*a^4*b^6 + 8*A*a^3*b^7)*

```
c^4 + (47*C*a^6*b^4 - 12*B*a^5*b^5 - 23*A*a^4*b^6)*c^3*d - (158*C*a^7*b^3 +
 17*B*a^6*b^4 - 17*A*a^5*b^5)*c^2*d^2 + (152*C*a^8*b^2 + 23*B*a^7*b^3 + 12*
A*a^6*b^4)*c*d^3 - 2*(24*C*a^9*b + 4*B*a^8*b^2 + 3*A*a^7*b^3)*d^4)*f^4*x +
((15*C*a^6*b^4 + 6*B*a^5*b^5 + 8*A*a^4*b^6)*c^4 + (47*C*a^7*b^3 - 12*B*a^6*
b^4 - 23*A*a^5*b^5)*c^3*d - (158*C*a^8*b^2 + 17*B*a^7*b^3 - 17*A*a^6*b^4)*c
^2*d^2 + (152*C*a^9*b + 23*B*a^8*b^2 + 12*A*a^7*b^3)*c*d^3 - 2*(24*C*a^10 +
 4*B*a^9*b + 3*A*a^8*b^2)*d^4)*f^4 + (35*C*a^4*b^6*c^2*d^2 - 14*(3*C*a^5*b^
5 + B*a^4*b^6)*c*d^3 + (15*C*a^6*b^4 + 6*B*a^5*b^5 + 8*A*a^4*b^6)*d^4 + (35
*C*b^10*c^2*d^2 - 14*(3*C*a*b^9 + B*b^10)*c*d^3 + (15*C*a^2*b^8 + 6*B*a*b^9
 + 8*A*b^10)*d^4)*x^4 + 4*(35*C*a*b^9*c^2*d^2 - 14*(3*C*a^2*b^8 + B*a*b^9)*
c*d^3 + (15*C*a^3*b^7 + 6*B*a^2*b^8 + 8*A*a*b^9)*d^4)*x^3 + 6*(35*C*a^2*b^8
*c^2*d^2 - 14*(3*C*a^3*b^7 + B*a^2*b^8)*c*d^3 + (15*C*a^4*b^6 + 6*B*a^3*b^7
 + 8*A*a^2*b^8)*d^4)*x^2 + 4*(35*C*a^3*b^7*c^2*d^2 - 14*(3*C*a^4*b^6 + B*a^
3*b^7)*c*d^3 + (15*C*a^5*b^5 + 6*B*a^4*b^6 + 8*A*a^3*b^7)*d^4)*x)*e^4 - ((1
40*C*b^10*c^3*d - 7*(34*C*a*b^9 + 3*B*b^10)*c^2*d^2 + (177*C*a^2*b^8 - 23*B
*a*b^9 + 9*A*b^10)*c*d^3 - (47*C*a^3*b^7 - 12*B*a^2*b^8 - 23*A*a*b^9)*d^4)*
f*x^4 + 4*(140*C*a*b^9*c^3*d - 7*(34*C*a^2*b^8 ...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(9/2),x)
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(9/2), x)
```

3.67 $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1235

$$-\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf}{}$$

```
[Out] -2/63*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e))*(b*x+a)^(3/2)*(d*x+c)^(3/2)*
(f*x+e)^(1/2)/b/d^2/f^2+2/9*C*(b*x+a)^(5/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d
/f-2/315*(7*b*d*f*(-9*A*b*d*f+C*a*c*f+3*C*a*d*e+5*C*b*c*e)-(-3*a*d*f+4*b*c*
f+6*b*d*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e)))*(d*x+c)^(3/2)*(b*x+a)^
(1/2)*(f*x+e)^(1/2)/b/d^3/f^3-2/945*(5*b*d*f*(7*a*d*f*(-9*A*b*d*f+C*a*c*f+3
*C*a*d*e+5*C*b*c*e)-(a*c*f+3*a*d*e+3*b*c*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+
8*C*d*e)))+2*(1/2*a*d*f-b*(c*f+2*d*e))*(7*b*d*f*(-9*A*b*d*f+C*a*c*f+3*C*a*d
*e+5*C*b*c*e)-(-3*a*d*f+4*b*c*f+6*b*d*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C
*d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d^3/f^4+2/315*(8*a^4
*C*d^4*f^4+a^3*b*d^3*f^3*(-18*B*d*f-7*C*c*f+11*C*d*e)-3*a^2*b^2*d^2*f^2*(3*
d*f*(-7*A*d*f-3*B*c*f+4*B*d*e)-C*(-3*c^2*f^2-5*c*d*e*f+9*d^2*e^2))-a*b^3*d*
f*(2*C*(-16*c^3*f^3-18*c^2*d*e*f^2-33*c*d^2*e^2*f+92*d^3*e^3)+3*d*f*(7*A*d*
f*(-7*c*f+13*d*e)-B*(-19*c^2*f^2-29*c*d*e*f+72*d^2*e^2)))+b^4*(C*(-16*c^4*f
^4-16*c^3*d*e*f^3-21*c^2*d^2*e^2*f^2-40*c*d^3*e^3*f+128*d^4*e^4)+3*d*f*(7*A
*d*f*(-2*c^2*f^2-3*c*d*e*f+8*d^2*e^2)-B*(-8*c^3*f^3-9*c^2*d*e*f^2-16*c*d^2*
e^2*f+48*d^3*e^3))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d
+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(
f*x+e)^(1/2)/b^3/d^(7/2)/f^5/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/3
15*(-a*f+b*e)*(-c*f+d*e)*(4*a^3*C*d^3*f^3+3*a^2*b*d^2*f^2*(-3*B*d*f-C*c*f+3
*C*d*e)-3*a*b^2*d*f*(3*d*f*(-21*A*d*f+3*B*c*f+16*B*d*e)-5*C*(c^2*f^2+2*c*d*
e*f+8*d^2*e^2))-b^3*(C*(8*c^3*f^3+15*c^2*d*e*f^2+24*c*d^2*e^2*f+128*d^3*e^3
)+3*d*f*(7*A*d*f*(c*f+8*d*e)-4*B*(c^2*f^2+2*c*d*e*f+12*d^2*e^2))))*Elliptic
F(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*
(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b
^3/d^(7/2)/f^5/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 2.83, antiderivative size = 1235, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (-2*(5*b*d*f*(7*a*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (3*b*
c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f))) + 2*(
(a*d*f)/2 - b*(2*d*e + c*f))*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*
A*b*d*f) - (6*b*d*e + 4*b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f
- 9*B*d*f))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(945*b^2*d^3*f^4)
- (2*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (6*b*d*e + 4*
b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f)))*Sqrt[a + b*
x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(315*b*d^3*f^3) - (2*(4*a*C*d*f + b*(8*C*
d*e + 6*c*C*f - 9*B*d*f))*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(6
3*b*d^2*f^2) + (2*C*(a + b*x)^(5/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(9*b*d*f
) + (2*Sqrt[-(b*c) + a*d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e - 7*c*
C*f - 18*B*d*f) - 3*a^2*b^2*d^2*f^2*(3*d*f*(4*B*d*e - 3*B*c*f - 7*A*d*f) -
C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) - a*b^3*d*f*(2*C*(92*d^3*e^3 - 33*c*
d^2*e^2*f - 18*c^2*d*e*f^2 - 16*c^3*f^3) + 3*d*f*(7*A*d*f*(13*d*e - 7*c*f)
- B*(72*d^2*e^2 - 29*c*d*e*f - 19*c^2*f^2))) + b^4*(C*(128*d^4*e^4 - 40*c*d
^3*e^3*f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) + 3*d*f*(7*A*d
*f*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) - B*(48*d^3*e^3 - 16*c*d^2*e^2*f - 9
*c^2*d*e*f^2 - 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*
EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*
f)/(d*(b*e - a*f))])/(315*b^3*d^(7/2)*f^5*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/
(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^3*C*d^3*
f^3 + 3*a^2*b*d^2*f^2*(3*C*d*e - c*C*f - 3*B*d*f) - 3*a*b^2*d*f*(3*d*f*(16*
B*d*e + 3*B*c*f - 21*A*d*f) - 5*C*(8*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) - b^3*
(C*(128*d^3*e^3 + 24*c*d^2*e^2*f + 15*c^2*d*e*f^2 + 8*c^3*f^3) + 3*d*f*(7*A
*d*f*(8*d*e + c*f) - 4*B*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[
d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3
15*b^3*d^(7/2)*f^5*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
```

```
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} + \frac{2\int \frac{(a+bx)^{3/2}\sqrt{c+dx}\,(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/\cdots}}{63bd^2f^2}$$

$$= -\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf\cdots}{\cdots}$$

$$= -\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+\cdots}{\cdots}$$

$$= -\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+\cdots}{\cdots}$$

$$= -\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+\cdots}{\cdots}$$

$$= -\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+\cdots}{\cdots}$$

$$= -\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.88, size = 12483, normalized size = 10.11

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x
]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15735 vs. $2(1163) = 2326$.
time = 0.11, size = 15736, normalized size = 12.74

method	result	size
elliptic	Expression too large to display	2108
default	Expression too large to display	15736

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 1925, normalized size = 1.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/945*(3*(35*C*b^5*d^5*f^5*x^3 - 64*C*b^5*d^5*f^2*e^3 + 5*(C*b^5*c*d^4 + (1
0*C*a*b^4 + 9*B*b^5)*d^5)*f^5*x^2 - (6*C*b^5*c^2*d^3 - (11*C*a*b^4 + 9*B*b^
5)*c*d^4 - 3*(C*a^2*b^3 + 24*B*a*b^4 + 21*A*b^5)*d^5)*f^5*x + (8*C*b^5*c^3*
d^2 - 3*(5*C*a*b^4 + 4*B*b^5)*c^2*d^3 + 3*(C*a^2*b^3 + 9*B*a*b^4 + 7*A*b^5)
*c*d^4 - (4*C*a^3*b^2 - 9*B*a^2*b^3 - 126*A*a*b^4)*d^5)*f^5 + 12*(4*C*b^5*d
```

```
^5*f^3*x + (C*b^5*c*d^4 + (7*C*a*b^4 + 6*B*b^5)*d^5)*f^3)*e^2 - (40*C*b^5*d
^5*f^4*x^2 + (7*C*b^5*c*d^4 + (61*C*a*b^4 + 54*B*b^5)*d^5)*f^4*x - (9*C*b^5
*c^2*d^3 - (19*C*a*b^4 + 15*B*b^5)*c*d^4 - 3*(2*C*a^2*b^3 + 33*B*a*b^4 + 28
*A*b^5)*d^5)*f^4)*e)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (128*C*b^5
*d^5*e^5 - (16*C*b^5*c^5 - 8*(5*C*a*b^4 + 3*B*b^5)*c^4*d + (22*C*a^2*b^3 +
69*B*a*b^4 + 42*A*b^5)*c^3*d^2 + (7*C*a^3*b^2 - 51*B*a^2*b^3 - 168*A*a*b^4)
*c^2*d^3 + (11*C*a^4*b - 36*B*a^3*b^2 + 357*A*a^2*b^3)*c*d^4 - (8*C*a^5 - 1
8*B*a^4*b + 63*A*a^3*b^2)*d^5)*f^5 - (8*C*b^5*c^4*d - (22*C*a*b^4 + 15*B*b^
5)*c^3*d^2 + 3*(5*C*a^2*b^3 + 21*B*a*b^4 + 14*A*b^5)*c^2*d^3 + 7*(2*C*a^3*b
^2 - 21*B*a^2*b^3 - 54*A*a*b^4)*c*d^4 - (7*C*a^4*b - 27*B*a^3*b^2 + 168*A*a
^2*b^3)*d^5)*f^4*e - (10*C*b^5*c^3*d^2 - 15*(3*C*a*b^4 + 2*B*b^5)*c^2*d^3 +
 3*(37*C*a^2*b^3 + 91*B*a*b^4 + 49*A*b^5)*c*d^4 - (20*C*a^3*b^2 - 117*B*a^2
*b^3 - 357*A*a*b^4)*d^5)*f^3*e^2 - (25*C*b^5*c^2*d^3 - 2*(113*C*a*b^4 + 60*
B*b^5)*c*d^4 - (95*C*a^2*b^3 + 288*B*a*b^4 + 168*A*b^5)*d^5)*f^2*e^3 - 8*(1
3*C*b^5*c*d^4 + (31*C*a*b^4 + 18*B*b^5)*d^5)*f*e^4)*sqrt(b*d*f)*weierstrass
PInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d +
a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b
*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*
x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(128*C*b^5*d^5*f*e^4 - (16*C*b^5*c^
4*d - 8*(4*C*a*b^4 + 3*B*b^5)*c^3*d^2 + 3*(3*C*a^2*b^3 + 19*B*a*b^4 + 14*A*
b^5)*c^2*d^3 + (7*C*a^3*b^2 - 27*B*a^2*b^3 - 147*A*a*b^4)*c*d^4 - (8*C*a^4*
b - 18*B*a^3*b^2 + 63*A*a^2*b^3)*d^5)*f^5 - (16*C*b^5*c^3*d^2 - 9*(4*C*a*b^
4 + 3*B*b^5)*c^2*d^3 + 3*(5*C*a^2*b^3 + 29*B*a*b^4 + 21*A*b^5)*c*d^4 - (11*
C*a^3*b^2 - 36*B*a^2*b^3 - 273*A*a*b^4)*d^5)*f^4*e - 3*(7*C*b^5*c^2*d^3 - 2
*(11*C*a*b^4 + 8*B*b^5)*c*d^4 - (9*C*a^2*b^3 + 72*B*a*b^4 + 56*A*b^5)*d^5)*
f^3*e^2 - 8*(5*C*b^5*c*d^4 + (23*C*a*b^4 + 18*B*b^5)*d^5)*f^2*e^3)*sqrt(b*d
*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 -
(b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 -
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*
d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), we
ierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b
^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^
2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*
(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^4*d^5*f^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] int(((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2), x)
```

3.68 $\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=766

$$-\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6Cde+4cCf-7Bdf)))\sqrt{a}}{105b^2d^2f^3}$$

```
[Out] 2/7*C*(b*x+a)^(3/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f-2/35*(4*a*C*d*f+b*(-7
*B*d*f+4*C*c*f+6*C*d*e))*(d*x+c)^(3/2)*(b*x+a)^(1/2)*(f*x+e)^(1/2)/b/d^2/f^
2-2/105*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+3*C*a*d*e+3*C*b*c*e)+(a*d*f-2*b*(c*f+2
*d*e))*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*d*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2
)*(f*x+e)^(1/2)/b^2/d^2/f^3-2/105*(3*b*d*f*(5*a*d*f*(-7*A*b*d*f+C*a*c*f+3*C
*a*d*e+3*C*b*c*e)-(a*c*f+3*a*d*e+b*c*e)*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*
d*e)))+2*(1/2*b*c*f-d*(a*f+b*e))*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+3*C*a*d*e+3*C
*b*c*e)+(a*d*f-2*b*(c*f+2*d*e))*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*d*e))))*
EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))
^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/d^(5
/2)/f^4/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/105*(-a*f+b*e)*(-c*f+d
*e)*(4*a^2*C*d^2*f^2+a*b*d*f*(-7*B*d*f-2*C*c*f+8*C*d*e)-b^2*(7*d*f*(-10*A*d
*f+B*c*f+8*B*d*e)-4*C*(c^2*f^2+2*c*d*e*f+12*d^2*e^2)))*EllipticF(d^(1/2)*(b
*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1
/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/d^(5/2)/f
^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 1.28, antiderivative size = 766, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (-2*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7*A*b*d*f) + (a*d*f - 2*b*(
2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7*B*d*f)))*Sqrt[a + b*x]*
Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^2*d^2*f^3) - (2*(4*a*C*d*f + b*(6*C*d*e
 + 4*c*C*f - 7*B*d*f))*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(35*b*d
^2*f^2) + (2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(7*b*d*f) - (
2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(5*a*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7
```

```
*A*b*d*f) - (b*c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7
*B*d*f))) + 2*((b*c*f)/2 - d*(b*e + a*f))*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e +
 a*c*C*f - 7*A*b*d*f) + (a*d*f - 2*b*(2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e
 + 4*c*C*f - 7*B*d*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Elli
pticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(
d*(b*e - a*f))])/(105*b^3*d^(5/2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e
 - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^2*C*d^2*f^2
+ a*b*d*f*(8*C*d*e - 2*c*C*f - 7*B*d*f) - b^2*(7*d*f*(8*B*d*e + B*c*f - 10*
A*d*f) - 4*C*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^3*d^(5/2
)*f^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
```

```
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} + \frac{2\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(-}{}}{}$$

$$= -\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}\,(c+dx)^{3/2}\sqrt{e}}{35bd^2f^2}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+}{1}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+}{1}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+}{1}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+}{1}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+}{1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.93, size = 922, normalized size = 1.20

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (2*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e - 5*c*
C*f - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) + C*(16*d
^2*e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e^2*f + 9
*c^2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d^2*e^2 -
3*c*d*e*f - 2*c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*d*f*
(a + b*x)*(c + d*x)*(e + f*x)*(-4*a^2*C*d^2*f^2 + a*b*d*f*(7*B*d*f + C*(-5*
d*e + 2*c*f + 3*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e + c*f + 3*d*f*x))
 + C*(-4*c^2*f^2 + c*d*f*(-5*e + 3*f*x) + 3*d^2*(8*e^2 - 6*e*f*x + 5*f^2*x^
2)))) + I*(b*c - a*d)*f*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e - 5*c*C*f
 - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) + C*(16*d^2*
e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d^2*e^2 - 3*c
*d*e*f - 2*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sq
rt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt
[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f
)*(4*a^2*C*d^2*f^2 + a*b*d*f*(5*C*d*e + c*C*f - 7*B*d*f) - b^2*(7*d*f*(-4*B
*d*e - 2*B*c*f + 5*A*d*f) + C*(24*d^2*e^2 + 13*c*d*e*f + 8*c^2*f^2)))*(a +
b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x
))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/
(b*c*f - a*d*f)]))/(105*b^4*Sqrt[-a + (b*c)/d]*d^3*f^4*Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10267 vs. $2(702) = 1404$.
time = 0.11, size = 10268, normalized size = 13.40

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cx^2\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+}}{7f}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 1388, normalized size = 1.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/315*(3*(15*C*b^4*d^4*f^4*x^2 + 24*C*b^4*d^4*f^2*e^2 + 3*(C*b^4*c*d^3 + (C
*a*b^3 + 7*B*b^4)*d^4)*f^4*x - (4*C*b^4*c^2*d^2 - (2*C*a*b^3 + 7*B*b^4)*c*d
^3 + (4*C*a^2*b^2 - 7*B*a*b^3 - 35*A*b^4)*d^4)*f^4 - (18*C*b^4*d^4*f^3*x +
(5*C*b^4*c*d^3 + (5*C*a*b^3 + 28*B*b^4)*d^4)*f^3)*e)*sqrt(b*x + a)*sqrt(d*x
 + c)*sqrt(f*x + e) + (48*C*b^4*d^4*e^4 - (8*C*b^4*c^4 - (9*C*a*b^3 + 14*B*
b^4)*c^3*d - (4*C*a^2*b^2 - 21*B*a*b^3 - 35*A*b^4)*c^2*d^2 - (9*C*a^3*b - 2
1*B*a^2*b^2 + 140*A*a*b^3)*c*d^3 + (8*C*a^4 - 14*B*a^3*b + 35*A*a^2*b^2)*d^
4)*f^4 - (5*C*b^4*c^3*d - 7*(C*a*b^3 + 2*B*b^4)*c^2*d^2 - 7*(C*a^2*b^2 - 8*
B*a*b^3 - 10*A*b^4)*c*d^3 + (5*C*a^3*b - 14*B*a^2*b^2 + 70*A*a*b^3)*d^4)*f^
3*e - (10*C*b^4*c^2*d^2 - 7*(6*C*a*b^3 + 7*B*b^4)*c*d^3 + (10*C*a^2*b^2 - 4
9*B*a*b^3 - 70*A*b^4)*d^4)*f^2*e^2 - 8*(5*C*b^4*c*d^3 + (5*C*a*b^3 + 7*B*b^
4)*d^4)*f*e^3)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2
- a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2
*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3
- 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^
3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) +
 3*(48*C*b^4*d^4*f*e^3 - (8*C*b^4*c^3*d - (5*C*a*b^3 + 14*B*b^4)*c^2*d^2 -
(5*C*a^2*b^2 - 14*B*a*b^3 - 35*A*b^4)*c*d^3 + (8*C*a^3*b - 14*B*a^2*b^2 + 3
5*A*a*b^3)*d^4)*f^4 - (9*C*b^4*c^2*d^2 - (8*C*a*b^3 + 21*B*b^4)*c*d^3 + (9*
C*a^2*b^2 - 21*B*a*b^3 - 70*A*b^4)*d^4)*f^3*e - 8*(2*C*b^4*c*d^3 + (2*C*a*b
^3 + 7*B*b^4)*d^4)*f^2*e^2)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 +
(b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2),
 -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*
d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 +
 a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b
```

```
^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -
4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^
3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a
*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*
d*f))))/(b^4*d^4*f^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] int(((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2), x)
```

3.69 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{a+bx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=527

$$-\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15b^2df^2}+\frac{2C\sqrt{a+bx}\,(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{2\cdot}{}$$

```
[Out] 2/5*C*(d*x+c)^(3/2)*(b*x+a)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/15*(4*a*C*d*f+b*(-5
*B*d*f+2*C*c*f+4*C*d*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d/f^
2-2/15*(3*b*d*f*(-5*A*b*d*f+C*a*c*f+3*C*a*d*e+C*b*c*e)-(2*a*d*f-b*c*f+2*b*d
*e)*(4*a*C*d*f+b*(-5*B*d*f+2*C*c*f+4*C*d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/
2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d
*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/d^(3/2)/f^3/(d*x+c)^(1/2)/(b*(f*x
+e)/(-a*f+b*e))^(1/2)-2/15*(-c*f+d*e)*(4*a^2*C*d*f^2+a*b*f*(-5*B*d*f-C*c*f+
3*C*d*e)-b^2*(5*d*f*(-3*A*f+2*B*e)-C*e*(c*f+8*d*e)))*EllipticF(d^(1/2)*(b*x
+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2
)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/d^(3/2)/f^3
/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$-\frac{2\sqrt{ad-bc}\,(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sqrt{\frac{b(e+fx)}{be-af}}\,(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-(b^2(5df(2Be-3Af)-Ce(cf+8de))))\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{15b^3d^{3/2}f^3\sqrt{c+dx}\,\sqrt{e+fx}}-\frac{2\sqrt{e+fx}\,\sqrt{ad-bc}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,(3bdf(acCf+3aCde-5Abdf+bcCe)-(2adf-bcf+2bde)(4aCdf+b(-5Bdf+2cCf+4Cde)))E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{15b^3d^{3/2}f^3\sqrt{c+dx}\,\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{2\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,(4aCdf+b(-5Bdf+2cCf+4Cde))}{15b^2df^2}+\frac{2C\sqrt{a+bx}\,(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(4*a*C*d*f + b*(4*C*d*e + 2*c*C*f - 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x])/(15*b^2*d*f^2) + (2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[
e + f*x])/(5*b*d*f) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(b*c*C*e + 3*a*C*d*e +
 a*c*C*f - 5*A*b*d*f) - (2*b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(4*C*d*e
 + 2*c*C*f - 5*B*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellip
ticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(15*b^3*d^(3/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e -
 a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e - c*f)*(4*a^2*C*d*f^2 + a*b*f*(3*C*d*e
 - c*C*f - 5*B*d*f) - b^2*(5*d*f*(2*B*e - 3*A*f) - C*e*(8*d*e + c*f)))*Sqrt
[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSi
n[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*
f))])/(15*b^3*d^(3/2)*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&& !LtQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{a+bx}\,\sqrt{e+fx}}\,dx = \frac{2C\sqrt{a+bx}\,(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{2\int \frac{\sqrt{c+dx}\,(-\frac{1}{2}b(bcCe+3aCde+acCf-}{\sqrt{a+bx}}}{5}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15b^2df^2} +$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15b^2df^2} +$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15b^2df^2} +$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15b^2df^2} +$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15b^2df^2} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 26.85, size = 562, normalized size = 1.07

$$\frac{2\sqrt{a+bx}\left(\frac{b^2(8a^2Cd^2f^2+abdf(7Cde-3cCf-10Bdf)+b^2(5df(-2Bde+Bcf+3Adf)+C(8d^2e^2-3cdef-2c^2f^2)))(c+dx)(e+fx)}{a+bx}+b^2df(c+dx)(e+fx)(5bBdf-4aCdf+bC(-4de+cf+3dfx))+\frac{i(bc-ad)f(8a^2Cd^2f^2+abdf(7Cde-3cCf-10Bdf)+b^2(5df(-2Bde+Bcf+3Adf)+C(8d^2e^2-3cdef-2c^2f^2)))\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)|\frac{bde-adf}{bcf-adf}\right)}{\sqrt{-a+\frac{bc}{d}}}+ib\sqrt{-a+\frac{bc}{d}}df(de-cf)(5bBdf-4aCdf-2bC(2de+cf))\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)|\frac{bde-adf}{bcf-adf}\right)\right)}{15b^4d^2f^3\sqrt{c+dx}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (2*Sqrt[a + b*x]*((b^2*(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*B
*d*f) + b^2*(5*d*f*(-2*B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*f
```

```
- 2*c^2*f^2)))*(c + d*x)*(e + f*x))/(a + b*x) + b^2*d*f*(c + d*x)*(e + f*x)
*(5*b*B*d*f - 4*a*C*d*f + b*C*(-4*d*e + c*f + 3*d*f*x)) + (I*(b*c - a*d)*f*
(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*B*d*f) + b^2*(5*d*f*(-2*
B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2)))*Sqrt[a +
 b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*E
llipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*
f - a*d*f)])/Sqrt[-a + (b*c)/d] + I*b*Sqrt[-a + (b*c)/d]*d*f*(d*e - c*f)*(5
*b*B*d*f - 4*a*C*d*f - 2*b*C*(2*d*e + c*f))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x)
)/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt
[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*b^4*d
^2*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6048 vs. $2(473) = 946$.
time = 0.11, size = 6049, normalized size = 11.48

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cx\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ac}}{5fb}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 1036, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/45*(3*(3*C*b^3*d^3*f^3*x - 4*C*b^3*d^3*f^2*e + (C*b^3*c*d^2 - (4*C*a*b^2
- 5*B*b^3)*d^3)*f^3)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (8*C*b^3*d
^3*e^3 - (2*C*b^3*c^3 + (2*C*a*b^2 - 5*B*b^3)*c^2*d + (7*C*a^2*b - 10*B*a*b
^2 + 30*A*b^3)*c*d^2 - (8*C*a^3 - 10*B*a^2*b + 15*A*a*b^2)*d^3)*f^3 - (2*C*
b^3*c^2*d + 2*(C*a*b^2 - 5*B*b^3)*c*d^2 - (3*C*a^2*b - 5*B*a*b^2 + 15*A*b^3
)*d^3)*f^2*e - (7*C*b^3*c*d^2 - (3*C*a*b^2 - 10*B*b^3)*d^3)*f*e^2)*sqrt(b*d
*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^
2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c
^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^
2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3)
, 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(8*C*b^3*d^3*f*e^2 -
 (2*C*b^3*c^2*d + (3*C*a*b^2 - 5*B*b^3)*c*d^2 - (8*C*a^2*b - 10*B*a*b^2 + 1
5*A*b^3)*d^3)*f^3 - (3*C*b^3*c*d^2 - (7*C*a*b^2 - 10*B*b^3)*d^3)*f^2*e)*sqr
t(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f
^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*
c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b
^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3
), weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2
 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^
3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2
*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3),
 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^4*d^3*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{a+bx}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(1/2)),x
)
```

```
[Out] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(1/2)),
x)
```

3.70 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=540

$$\frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)}-\frac{2(Ab^2-a(bB-aC}{b(bc-ad)(b}$$

```
[Out] -2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/
(b*x+a)^(1/2)+2/3*(4*a^2*C*d*f+b^2*(3*A*d*f+C*c*e)-a*b*(3*B*d*f+C*c*f+C*d*e
))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)+2/
3*(8*a^2*C*d*f^2-a*b*f*(6*B*d*f+C*c*f+3*C*d*e)+b^2*(3*d*f*(A*f+B*e)-C*e*(-c
*f+2*d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d
/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1
/2)/b^3/f^2/(-a*f+b*e)/d^(1/2)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2
/3*(-c*f+d*e)*(-3*B*b*f+4*C*a*f+2*C*b*e)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a
*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)
/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/f^2/d^(1/2)/(d*x+c)^(1/
2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$,
Rules used = {1628, 159, 164, 115, 114, 122, 121}

$$\frac{2\sqrt{e+fx}\,\sqrt{ad-bc}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-Ce(2de-cf)))\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3\sqrt{d}\,f^2\sqrt{c+dx}\,(be-af)\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{2\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,(4a^2Cdf-ab(3Bdf+cCf+Cde)+b^2(3Adf+cCe))}{3b^2f(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\,(Ab^2-a(bB-aC))}{b\sqrt{a+bx}\,(bc-ad)(be-af)}+\frac{2\sqrt{ad-bc}\,(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sqrt{\frac{b(e+fx)}{be-af}}\,(4aCf-3bBf+2bCe)F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3\sqrt{d}\,f^2\sqrt{c+dx}\,\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x]),x]
```

```
[Out] (2*(4*a^2*C*d*f + b^2*(c*C*e + 3*A*d*f) - a*b*(C*d*e + c*C*f + 3*B*d*f))*Sq
rt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b^2*(b*c - a*d)*f*(b*e - a*f))
- (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*Sqrt[a + b*x]) + (2*Sqrt[-(b*c) + a*d]*(8*a^2*C*d*f^2 - a*b*f*(
3*C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) - C*e*(2*d*e - c*f)))*S
qrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sq
rt[d]*f^2*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*S
qrt[-(b*c) + a*d]*(d*e - c*f)*(2*b*C*e - 3*b*B*f + 4*a*C*f)*Sqrt[(b*(c + d*
x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*
Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^
3*Sqrt[d]*f^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{b^2(Bc+2Ad)}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\,\sqrt{c+d\cdots}}{3b^2(bc-ad)f(be-af)}$$

$$= \frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\,\sqrt{c+d\cdots}}{3b^2(bc-ad)f(be-af)}$$

$$= \frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\,\sqrt{c+d\cdots}}{3b^2(bc-ad)f(be-af)}$$

$$= \frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\,\sqrt{c+d\cdots}}{3b^2(bc-ad)f(be-af)}$$

$$= \frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\,\sqrt{c+d\cdots}}{3b^2(bc-ad)f(be-af)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.77, size = 551, normalized size = 1.02

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}\left(-8a^2Cdf^2+abf(3Cde+cCf+6Bdf)+b^2(-3df(Be+Af)+Ce(2de-cf))\right)(c+dx)(e+fx)+b^2\sqrt{-a+\frac{bc}{d}}\,df(c+dx)(e+fx)\left(3(Ab^2+a(-bB+aC))f-C(be-af)(a+bx)\right)-i(bc-ad)f\left(8a^2Cdf^2-abf(3Cde+cCf+6Bdf)+b^2(3df(Be+Af)+Ce(-2de+cf))\right)(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)\middle|\frac{bde-adf}{bcf-adf}\right)-ibf(de-cf)\left(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf)\right)(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)\middle|\frac{bde-adf}{bcf-adf}\right)\right)}{3b^4\sqrt{-a+\frac{bc}{d}}\,df^2(be-af)\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x])
,x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(-8*a^2*C*d*f^2 + a*b*f*(3*C*d*e + c*C*f + 6*B*
d*f) + b^2*(-3*d*f*(B*e + A*f) + C*e*(2*d*e - c*f)))*(c + d*x)*(e + f*x) +
```

```
b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*(3*(A*b^2 + a*(-(b*B) + a*C)
)*f - C*(b*e - a*f)*(a + b*x)) - I*(b*c - a*d)*f*(8*a^2*C*d*f^2 - a*b*f*(3*
C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) + C*e*(-2*d*e + c*f)))*(a
 + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a +
b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*
f)/(b*c*f - a*d*f)] - I*b*f*(d*e - c*f)*(4*a^2*C*d*f + b^2*(c*C*e + 3*A*d*f
) - a*b*(C*d*e + c*C*f + 3*B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a
 + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (
b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(3*b^4*Sqrt[-a +
 (b*c)/d]*d*f^2*(b*e - a*f)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5319 vs. $2(488) = 976$.
time = 0.12, size = 5320, normalized size = 9.85

method	result
elliptic	$\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2(bdf\,x^2+bcfx+bdex+bce)(b^2A-abB+C\,a^2)}{b^3(af-be)\sqrt{\left(x+\frac{a}{b}\right)(bdf\,x^2+bcfx+bdex+bce)}}+\frac{2C\sqrt{bdf\,x^3+aa}}{}\right.$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 1331, normalized size = 2.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/9*(3*(C*a*b^3*d^2*f^3*x + (4*C*a^2*b^2 - 3*B*a*b^3 + 3*A*b^4)*d^2*f^3 - (
C*b^4*d^2*f^2*x + C*a*b^3*d^2*f^2)*e)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x
+ e) - ((C*a*b^3*c^2 + (5*C*a^2*b^2 - 3*B*a*b^3 - 3*A*b^4)*c*d - (8*C*a^3*b
 - 6*B*a^2*b^2 + 3*A*a*b^3)*d^2)*f^3*x + (C*a^2*b^2*c^2 + (5*C*a^3*b - 3*B*
a^2*b^2 - 3*A*a*b^3)*c*d - (8*C*a^4 - 6*B*a^3*b + 3*A*a^2*b^2)*d^2)*f^3 + 2
*(C*b^4*d^2*x + C*a*b^3*d^2)*e^3 - ((2*C*b^4*c*d - (2*C*a*b^3 - 3*B*b^4)*d^
2)*f*x + (2*C*a*b^3*c*d - (2*C*a^2*b^2 - 3*B*a*b^3)*d^2)*f)*e^2 - ((C*b^4*c
^2 + 6*(C*a*b^3 - B*b^4)*c*d - (7*C*a^2*b^2 - 6*B*a*b^3 + 6*A*b^4)*d^2)*f^2
*x + (C*a*b^3*c^2 + 6*(C*a^2*b^2 - B*a*b^3)*c*d - (7*C*a^3*b - 6*B*a^2*b^2
+ 6*A*a*b^3)*d^2)*f^2)*e)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2
+ (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2
), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^
3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2
 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)
/(b*d*f)) - 3*((C*a*b^3*c*d - (8*C*a^2*b^2 - 6*B*a*b^3 + 3*A*b^4)*d^2)*f^3*
x + (C*a^2*b^2*c*d - (8*C*a^3*b - 6*B*a^2*b^2 + 3*A*a*b^3)*d^2)*f^3 + 2*(C*
b^4*d^2*f*x + C*a*b^3*d^2*f)*e^2 - ((C*b^4*c*d - 3*(C*a*b^3 - B*b^4)*d^2)*f
^2*x + (C*a*b^3*c*d - 3*(C*a^2*b^2 - B*a*b^3)*d^2)*f^2)*e)*sqrt(b*d*f)*weie
rstrassZeta(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d
 + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^
2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstras
sPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d +
 a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*
b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f
*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(a*b^5*d^2*f^4*x + a^2*b^4*d^2*f^4 -
 (b^6*d^2*f^3*x + a*b^5*d^2*f^3)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{3}{2}}\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(e + f*x)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(3/2)),x
)
```

```
[Out] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(3/2)),
x)
```

3.71 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=597

$$-\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\,\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}}-\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$

```
[Out] -2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(3/2)-2/3*(4*a^2*C*f+b^2*(-2*A*f+3*B*e)-a*b*(B*f+6*C*e))*(d*x+c)^
(1/2)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^(1/2)+2/3*(8*a^3*C*d*f^2-a^2*b
*f*(2*B*d*f+7*C*c*f+13*C*d*e)+a*b^2*(3*C*e*(4*c*f+d*e)+f*(-A*d*f+B*c*f+4*B*
d*e))-b^3*(A*d*e*f+c*(-2*A*f^2+3*B*e*f+3*C*e^2)))*EllipticE(d^(1/2)*(b*x+a)
^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c
)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/f/(-a*f+b*e)^2/(a*d-b*c)^(1/2)/(d*x+c
)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/3*(-c*f+d*e)*(4*a^2*C*d*f+b^2*(A*d*f
+3*C*c*e)-a*b*(B*d*f+3*C*(c*f+d*e)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b
*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(
b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/f/(-a*f+b*e)/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c
)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 155, 164, 115, 114, 122, 121}

$$\frac{2(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe))\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3\sqrt{d}\,f\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{ad-bc}\,(be-af)}-\frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,(4a^2Cf-ab(Bf+6Ce)+b^2(3Be-2Af))}{3b^2\sqrt{a+bx}\,(be-af)^2}+\frac{2\sqrt{d}\,\sqrt{e+fx}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,(8a^3Cdf^2-a^2bf(2Bdf+7cCf+13Cde)+ab^2(f(-Adf+Bcf+4Bde)+3Ce(4cf+de))-b^3(cf(3Be-2Af)+Adef+3cCe^2))\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3f\sqrt{c+dx}\,\sqrt{ad-bc}\,(be-af)^2\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\,(Ab^2-a(bB-aC))}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(4*a^2*C*f + b^2*(3*B*e - 2*A*f) - a*b*(6*C*e + B*f))*Sqrt[c + d*x]*Sqr
t[e + f*x])/(3*b^2*(b*e - a*f)^2*Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C)
)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(3/
2)) + (2*Sqrt[d]*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) -
b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f
) + f*(4*B*d*e + B*c*f - A*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e +
f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c -
a*d)*f)/(d*(b*e - a*f))]])/(3*b^3*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)^2*Sqrt[c
 + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(4*a^2*C*d*f + b^2
*(3*c*C*e + A*d*f) - a*b*(B*d*f + 3*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c
```

```
 - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a +
 b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sqrt[d
]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\,\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \cdots$$

$$= -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\,\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \cdots$$

$$= -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\,\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \cdots$$

$$= -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\,\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \cdots$$

$$= -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\,\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.50, size = 724, normalized size = 1.21

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x])
,x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*C
))*(b*c - a*d)*(b*e - a*f) + (-5*a^3*C*d*f + b^3*(3*B*c*e + A*d*e - 2*A*c*f
```

```
) - a*b^2*(6*c*C*e + 4*B*d*e + B*c*f - A*d*f) + a^2*b*(7*C*d*e + 4*c*C*f +
2*B*d*f))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d*f^2 - a
^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B
*e - 2*A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*(
c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*d*e + 7
*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*A*f)) + a*b^2
*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*(a + b*x)^(3/2)*Sqrt[
(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*
ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]
 + I*b*(b*c - a*d)*f*(d*e - c*f)*(-4*a^2*C*f + b^2*(-3*B*e + 2*A*f) + a*b*(
6*C*e + B*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e
+ f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]
], (b*d*e - a*d*f)/(b*c*f - a*d*f)])))/(3*b^4*Sqrt[-a + (b*c)/d]*(b*c - a*d
)*f*(b*e - a*f)^2*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15371 vs. $2(543) = 1086$.
time = 0.13, size = 15372, normalized size = 25.75

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2\left(b^2A-abB+Ca^2\right)\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+}}{3b^4(af-be)\left(x+\frac{a}{b}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 2420, normalized size = 4.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] -2/9*(3*(((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (5*C*a^3*b^3 - 2*B*a^2*b^
4 - A*a*b^5)*d^2)*f^3*x + (3*(C*a^3*b^3 - A*a*b^5)*c*d - (4*C*a^4*b^2 - B*a
^3*b^3 - 2*A*a^2*b^4)*d^2)*f^3 - ((3*(2*C*a*b^5 - B*b^6)*c*d - (7*C*a^2*b^4
 - 4*B*a*b^5 + A*b^6)*d^2)*f^2*x + ((5*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d -
 3*(2*C*a^3*b^3 - B*a^2*b^4)*d^2)*f^2)*e)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(
f*x + e) - (((2*C*a^2*b^4 + B*a*b^5 + 2*A*b^6)*c^2 - (11*C*a^3*b^3 - 2*B*a^
2*b^4 + 2*A*a*b^5)*c*d + (8*C*a^4*b^2 - 2*B*a^3*b^3 - A*a^2*b^4)*d^2)*f^3*x
^2 + 2*((2*C*a^3*b^3 + B*a^2*b^4 + 2*A*a*b^5)*c^2 - (11*C*a^4*b^2 - 2*B*a^3
*b^3 + 2*A*a^2*b^4)*c*d + (8*C*a^5*b - 2*B*a^4*b^2 - A*a^3*b^3)*d^2)*f^3*x
+ ((2*C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (11*C*a^5*b - 2*B*a^4*b^2
+ 2*A*a^3*b^3)*c*d + (8*C*a^6 - 2*B*a^5*b - A*a^4*b^2)*d^2)*f^3 - 3*(C*a^2*
b^4*c*d - C*a^3*b^3*d^2 + (C*b^6*c*d - C*a*b^5*d^2)*x^2 + 2*(C*a*b^5*c*d -
C*a^2*b^4*d^2)*x)*e^3 + ((6*C*b^6*c^2 - 3*(5*C*a*b^5 - 2*B*b^6)*c*d + (8*C*
a^2*b^4 - 5*B*a*b^5 - A*b^6)*d^2)*f*x^2 + 2*(6*C*a*b^5*c^2 - 3*(5*C*a^2*b^4
 - 2*B*a*b^5)*c*d + (8*C*a^3*b^3 - 5*B*a^2*b^4 - A*a*b^5)*d^2)*f*x + (6*C*a
^2*b^4*c^2 - 3*(5*C*a^3*b^3 - 2*B*a^2*b^4)*c*d + (8*C*a^4*b^2 - 5*B*a^3*b^3
 - A*a^2*b^4)*d^2)*f)*e^2 - ((3*(2*C*a*b^5 + B*b^6)*c^2 - (25*C*a^2*b^4 - 4
*B*a*b^5 - 2*A*b^6)*c*d + (17*C*a^3*b^3 - 5*B*a^2*b^4 - 4*A*a*b^5)*d^2)*f^2
*x^2 + 2*(3*(2*C*a^2*b^4 + B*a*b^5)*c^2 - (25*C*a^3*b^3 - 4*B*a^2*b^4 - 2*A
*a*b^5)*c*d + (17*C*a^4*b^2 - 5*B*a^3*b^3 - 4*A*a^2*b^4)*d^2)*f^2*x + (3*(2
*C*a^3*b^3 + B*a^2*b^4)*c^2 - (25*C*a^4*b^2 - 4*B*a^3*b^3 - 2*A*a^2*b^4)*c*
d + (17*C*a^5*b - 5*B*a^4*b^2 - 4*A*a^3*b^3)*d^2)*f^2)*e)*sqrt(b*d*f)*weier
strassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*
c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 +
 a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*
b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(((7*C*a^2*b^4 - B*a*b^5 - 2*
A*b^6)*c*d - (8*C*a^3*b^3 - 2*B*a^2*b^4 - A*a*b^5)*d^2)*f^3*x^2 + 2*((7*C*a
^3*b^3 - B*a^2*b^4 - 2*A*a*b^5)*c*d - (8*C*a^4*b^2 - 2*B*a^3*b^3 - A*a^2*b^
4)*d^2)*f^3*x + ((7*C*a^4*b^2 - B*a^3*b^3 - 2*A*a^2*b^4)*c*d - (8*C*a^5*b -
 2*B*a^4*b^2 - A*a^3*b^3)*d^2)*f^3 + 3*((C*b^6*c*d - C*a*b^5*d^2)*f*x^2 + 2
```

```
*(C*a*b^5*c*d - C*a^2*b^4*d^2)*f*x + (C*a^2*b^4*c*d - C*a^3*b^3*d^2)*f)*e^2
 - ((3*(4*C*a*b^5 - B*b^6)*c*d - (13*C*a^2*b^4 - 4*B*a*b^5 + A*b^6)*d^2)*f^
2*x^2 + 2*(3*(4*C*a^2*b^4 - B*a*b^5)*c*d - (13*C*a^3*b^3 - 4*B*a^2*b^4 + A*
a*b^5)*d^2)*f^2*x + (3*(4*C*a^3*b^3 - B*a^2*b^4)*c*d - (13*C*a^4*b^2 - 4*B*
a^3*b^3 + A*a^2*b^4)*d^2)*f^2)*e)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*
e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2
*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c
*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^
2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f
^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*
a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d
^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*
f)/(b*d*f))))/((a^2*b^7*c*d - a^3*b^6*d^2)*f^4*x^2 + 2*(a^3*b^6*c*d - a^4*b
^5*d^2)*f^4*x + (a^4*b^5*c*d - a^5*b^4*d^2)*f^4 + ((b^9*c*d - a*b^8*d^2)*f^
2*x^2 + 2*(a*b^8*c*d - a^2*b^7*d^2)*f^2*x + (a^2*b^7*c*d - a^3*b^6*d^2)*f^2
)*e^2 - 2*((a*b^8*c*d - a^2*b^7*d^2)*f^3*x^2 + 2*(a^2*b^7*c*d - a^3*b^6*d^2
)*f^3*x + (a^3*b^6*c*d - a^4*b^5*d^2)*f^3)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(5/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(5/2)),x
)
```

```
[Out] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(5/2)),
x)
```

3.72 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1034

$$\frac{2(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6Adf)-a^2b(8Cde+6cCf-Bdf))\sqrt{c}}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

```
[Out] -2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(5/2)+2/15*(4*a^3*C*d*f-b^3*(-4*A*c*f-2*A*d*e+5*B*c*e)+a*b^2*(-6*
A*d*f+B*c*f+3*B*d*e+10*C*c*e)-a^2*b*(-B*d*f+6*C*c*f+8*C*d*e))*(d*x+c)^(1/2)
*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(3/2)-2/15*(8*a^4*C*d^2*
f^2-a^3*b*d*f*(-2*B*d*f+13*C*c*f+23*C*d*e)-b^4*(2*A*d^2*e^2-c*d*e*(-3*A*f+5
*B*e)-c^2*(8*A*f^2-10*B*e*f+15*C*e^2))-a^2*b^2*(d*f*(-3*A*d*f+2*B*c*f+7*B*d
*e)-C*(3*c^2*f^2+37*c*d*e*f+23*d^2*e^2))-a*b^3*(d^2*e*(-7*A*f+3*B*e)+2*c^2*
f*(-B*f+5*C*e)+c*d*(40*C*e^2-13*f*(-A*f+B*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)
/b^2/(-a*d+b*c)^2/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/15*(8*a^4*C*d^2*f^2-a^3*b*d*
f*(-2*B*d*f+13*C*c*f+23*C*d*e)-b^4*(2*A*d^2*e^2-c*d*e*(-3*A*f+5*B*e)-c^2*(8
*A*f^2-10*B*e*f+15*C*e^2))-a^2*b^2*(d*f*(-3*A*d*f+2*B*c*f+7*B*d*e)-C*(3*c^2
*f^2+37*c*d*e*f+23*d^2*e^2))-a*b^3*(d^2*e*(-7*A*f+3*B*e)+2*c^2*f*(-B*f+5*C*
e)+c*d*(40*C*e^2-13*f*(-A*f+B*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*
c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^
(1/2)*(f*x+e)^(1/2)/b^3/(a*d-b*c)^(3/2)/(-a*f+b*e)^3/(d*x+c)^(1/2)/(b*(f*x+
e)/(-a*f+b*e))^(1/2)+2/15*(-c*f+d*e)*(4*a^3*C*d*f-b^3*(-4*A*c*f-2*A*d*e+5*B
*c*e)+a*b^2*(-6*A*d*f+B*c*f+3*B*d*e+10*C*c*e)-a^2*b*(-B*d*f+6*C*c*f+8*C*d*e
))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*
e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2
)/b^3/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]

time = 2.09, antiderivative size = 1034, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1628, 155, 157, 164, 115, 114, 122, 121}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x]),x]
```

```
[Out] (2*(4*a^3*C*d*f - b^3*(5*B*c*e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B
*d*e + B*c*f - 6*A*d*f) - a^2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[c + d*x]*
```

```
Sqrt[e + f*x])/(15*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^(3/2)) - (2*(8*a
^4*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2
 - c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(
d*f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2)
) - a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2 -
13*f*(B*e - A*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b^2*(b*c - a*d)^2*(b*e
 - a*f)^3*Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[
e + f*x])/(5*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(5/2)) + (2*Sqrt[d]*(8*a^4
*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2 -
 c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(d*
f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2))
- a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2 - 13
*f*(B*e - A*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
- a*f))])/(15*b^3*(-(b*c) + a*d)^(3/2)*(b*e - a*f)^3*Sqrt[c + d*x]*Sqrt[(b*
(e + f*x))/(b*e - a*f)]) + (2*Sqrt[d]*(d*e - c*f)*(4*a^3*C*d*f - b^3*(5*B*c
*e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B*d*e + B*c*f - 6*A*d*f) - a^
2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e
 + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c)
+ a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^3*(-(b*c) + a*d)^(3/2)*(b*
e - a*f)^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
```

```
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
```

```
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+}{}\right.}{}}{}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6.}{15b^2(bc-ad)(be-af)^2(a}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6.}{15b^2(bc-ad)(be-af)^2(a}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6.}{15b^2(bc-ad)(be-af)^2(a}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6.}{15b^2(bc-ad)(be-af)^2(a}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6.}{15b^2(bc-ad)(be-af)^2(a}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6.}{15b^2(bc-ad)(be-af)^2(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.76, size = 1449, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x])
,x]
```

```
[Out] Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5*
b^2*(b*e - a*f)*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e + A*b^3*d*e
 - 6*a*b^2*B*d*e + 11*a^2*b*C*d*e - 4*A*b^3*c*f - a*b^2*B*c*f + 6*a^2*b*c*C
*f + 3*a*A*b^2*d*f + 2*a^2*b*B*d*f - 7*a^3*C*d*f))/(15*b^2*(b*c - a*d)*(b*e
 - a*f)^2*(a + b*x)^2) - (2*(15*b^4*c^2*C*e^2 + 5*b^4*B*c*d*e^2 - 40*a*b^3*
c*C*d*e^2 - 2*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 23*a^2*b^2*C*d^2*e^2 - 10
*b^4*B*c^2*e*f - 10*a*b^3*c^2*C*e*f - 3*A*b^4*c*d*e*f + 13*a*b^3*B*c*d*e*f
+ 37*a^2*b^2*c*C*d*e*f + 7*a*A*b^3*d^2*e*f - 7*a^2*b^2*B*d^2*e*f - 23*a^3*b
*C*d^2*e*f + 8*A*b^4*c^2*f^2 + 2*a*b^3*B*c^2*f^2 + 3*a^2*b^2*c^2*C*f^2 - 13
*a*A*b^3*c*d*f^2 - 2*a^2*b^2*B*c*d*f^2 - 13*a^3*b*c*C*d*f^2 + 3*a^2*A*b^2*d
^2*f^2 + 2*a^3*b*B*d^2*f^2 + 8*a^4*C*d^2*f^2))/(15*b^2*(b*c - a*d)^2*(b*e -
 a*f)^3*(a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(8*a^4*C*d^2*f
^2 + a^3*b*d*f*(-23*C*d*e - 13*c*C*f + 2*B*d*f) + b^4*(-2*A*d^2*e^2 + c*d*e
*(5*B*e - 3*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) + a^2*b^2*(d*f*(-7*
B*d*e - 2*B*c*f + 3*A*d*f) + C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2)) + a*b
^3*(d^2*e*(-3*B*e + 7*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(-40*C*e^2 + 13*f
*(B*e - A*f))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f + (b*e)/(a + b*x
) - (a*f)/(a + b*x)) + (I*(-(b*c) + a*d)*f*(-8*a^4*C*d^2*f^2 + a^3*b*d*f*(2
3*C*d*e + 13*c*C*f - 2*B*d*f) + b^4*(2*A*d^2*e^2 + c*d*e*(-5*B*e + 3*A*f) +
 c^2*(-15*C*e^2 + 10*B*e*f - 8*A*f^2)) - a^2*b^2*(d*f*(-7*B*d*e - 2*B*c*f +
 3*A*d*f) + C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2)) + a*b^3*(d^2*e*(3*B*e
- 7*A*f) - 2*c^2*f*(-5*C*e + B*f) + c*d*(40*C*e^2 + 13*f*(-(B*e) + A*f))))*
Sqrt[1 - a/(a + b*x) + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(f
*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e
- a*d*f)/(b*c*f - a*d*f)])/Sqrt[a + b*x] - (I*b*(-(b*c) + a*d)*(d*e - c*f)*
(-4*a^3*C*d*f^2 + a^2*b*f*(11*C*d*e + 3*c*C*f - B*d*f) + b^3*(15*c*C*e^2 +
A*d*e*f + 2*c*f*(-5*B*e + 4*A*f)) + a*b^2*(-5*C*e*(3*d*e + 2*c*f) + f*(9*B*
d*e + 2*B*c*f - 9*A*d*f)))*Sqrt[1 - a/(a + b*x) + (b*c)/(d*(a + b*x))]*Sqrt
[1 - a/(a + b*x) + (b*e)/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)
/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[a + b*x]))/(15*b
^4*Sqrt[-a + (b*c)/d]*(b*c - a*d)^2*(b*e - a*f)^3*Sqrt[c + ((a + b*x)*(d -
(a*d)/(a + b*x)))/b]*Sqrt[e + ((a + b*x)*(f - (a*f)/(a + b*x)))/b])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 36157 vs. $2(972) = 1944$.
time = 0.16, size = 36158, normalized size = 34.97

method	result	size
elliptic	Expression too large to display	2330
default	Expression too large to display	36158

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 4859, normalized size = 4.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/45*(3*(((3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*c^2*d - (13*C*a^3*b^5 + 2*B*a
^2*b^6 + 13*A*a*b^7)*c*d^2 + (8*C*a^4*b^4 + 2*B*a^3*b^5 + 3*A*a^2*b^6)*d^3)
*f^3*x^2 + (5*(B*a^2*b^6 + 4*A*a*b^7)*c^2*d - (13*C*a^4*b^4 + 7*B*a^3*b^5 +
 33*A*a^2*b^6)*c*d^2 + 3*(3*C*a^5*b^3 + 2*B*a^4*b^4 + 3*A*a^3*b^5)*d^3)*f^3
*x + (15*A*a^2*b^6*c^2*d - (6*C*a^5*b^3 - B*a^4*b^4 + 26*A*a^3*b^5)*c*d^2 +
 (4*C*a^6*b^2 + B*a^5*b^3 + 9*A*a^4*b^4)*d^3)*f^3 + ((15*C*b^8*c^2*d - 5*(8
*C*a*b^7 - B*b^8)*c*d^2 + (23*C*a^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*d^3)*f*x^2 +
 (5*(4*C*a*b^7 + B*b^8)*c^2*d - (59*C*a^2*b^6 + B*a*b^7 - A*b^8)*c*d^2 + 5*
(7*C*a^3*b^5 - A*a*b^7)*d^3)*f*x + (15*C*a^4*b^4*d^3 + (8*C*a^2*b^6 + 2*B*a
*b^7 + 3*A*b^8)*c^2*d - 5*(5*C*a^3*b^5 + A*a*b^7)*c*d^2)*f)*e^2 - ((10*(C*a
*b^7 + B*b^8)*c^2*d - (37*C*a^2*b^6 + 13*B*a*b^7 - 3*A*b^8)*c*d^2 + (23*C*a
^3*b^5 + 7*B*a^2*b^6 - 7*A*a*b^7)*d^3)*f^2*x^2 + 2*((2*C*a^2*b^6 + 13*B*a*b
^7 + 2*A*b^8)*c^2*d - 20*(C*a^3*b^5 + B*a^2*b^6)*c*d^2 + (14*C*a^4*b^4 + 11
```

*B*a^3*b^5 - 6*A*a^2*b^6)*d^3)*f^2*x + (10*(B*a^2*b^6 + A*a*b^7)*c^2*d - 15
*(C*a^4*b^4 + B*a^3*b^5 + A*a^2*b^6)*c*d^2 + (11*C*a^5*b^3 + 9*B*a^4*b^4 +
A*a^3*b^5)*d^3)*f^2)*e)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) + (((3*C*
a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*c^3 + (8*C*a^3*b^5 - 3*B*a^2*b^6 - 17*A*a*b^
7)*c^2*d - (17*C*a^4*b^4 + 3*B*a^3*b^5 - 8*A*a^2*b^6)*c*d^2 + (8*C*a^5*b^3
+ 2*B*a^4*b^4 + 3*A*a^3*b^5)*d^3)*f^3*x^3 + 3*((3*C*a^3*b^5 + 2*B*a^2*b^6 +
8*A*a*b^7)*c^3 + (8*C*a^4*b^4 - 3*B*a^3*b^5 - 17*A*a^2*b^6)*c^2*d - (17*C*
a^5*b^3 + 3*B*a^4*b^4 - 8*A*a^3*b^5)*c*d^2 + (8*C*a^6*b^2 + 2*B*a^5*b^3 + 3
*A*a^4*b^4)*d^3)*f^3*x^2 + 3*((3*C*a^4*b^4 + 2*B*a^3*b^5 + 8*A*a^2*b^6)*c^3
+ (8*C*a^5*b^3 - 3*B*a^4*b^4 - 17*A*a^3*b^5)*c^2*d - (17*C*a^6*b^2 + 3*B*a
^5*b^3 - 8*A*a^4*b^4)*c*d^2 + (8*C*a^7*b + 2*B*a^6*b^2 + 3*A*a^5*b^3)*d^3)*
f^3*x + ((3*C*a^5*b^3 + 2*B*a^4*b^4 + 8*A*a^3*b^5)*c^3 + (8*C*a^6*b^2 - 3*B
*a^5*b^3 - 17*A*a^4*b^4)*c^2*d - (17*C*a^7*b + 3*B*a^6*b^2 - 8*A*a^5*b^3)*c
*d^2 + (8*C*a^8 + 2*B*a^7*b + 3*A*a^6*b^2)*d^3)*f^3 - (30*C*a^3*b^5*c^2*d -
5*(10*C*a^4*b^4 + B*a^3*b^5)*c*d^2 + (22*C*a^5*b^3 + 3*B*a^4*b^4 + 2*A*a^3
*b^5)*d^3 + (30*C*b^8*c^2*d - 5*(10*C*a*b^7 + B*b^8)*c*d^2 + (22*C*a^2*b^6
+ 3*B*a*b^7 + 2*A*b^8)*d^3)*x^3 + 3*(30*C*a*b^7*c^2*d - 5*(10*C*a^2*b^6 + B
*a*b^7)*c*d^2 + (22*C*a^3*b^5 + 3*B*a^2*b^6 + 2*A*a*b^7)*d^3)*x^2 + 3*(30*C
*a^2*b^6*c^2*d - 5*(10*C*a^3*b^5 + B*a^2*b^6)*c*d^2 + (22*C*a^4*b^4 + 3*B*a
^3*b^5 + 2*A*a^2*b^6)*d^3)*x)*e^3 + ((15*C*b^8*c^3 + 5*(5*C*a*b^7 + 2*B*b^8
)*c^2*d - (67*C*a^2*b^6 + 33*B*a*b^7 + 2*A*b^8)*c*d^2 + (33*C*a^3*b^5 + 17*
B*a^2*b^6 + 8*A*a*b^7)*d^3)*f*x^3 + 3*(15*C*a*b^7*c^3 + 5*(5*C*a^2*b^6 + 2*
B*a*b^7)*c^2*d - (67*C*a^3*b^5 + 33*B*a^2*b^6 + 2*A*a*b^7)*c*d^2 + (33*C*a^
4*b^4 + 17*B*a^3*b^5 + 8*A*a^2*b^6)*d^3)*f*x^2 + 3*(15*C*a^2*b^6*c^3 + 5*(5
*C*a^3*b^5 + 2*B*a^2*b^6)*c^2*d - (67*C*a^4*b^4 + 33*B*a^3*b^5 + 2*A*a^2*b^
6)*c*d^2 + (33*C*a^5*b^3 + 17*B*a^4*b^4 + 8*A*a^3*b^5)*d^3)*f*x + (15*C*a^3
*b^5*c^3 + 5*(5*C*a^4*b^4 + 2*B*a^3*b^5)*c^2*d - (67*C*a^5*b^3 + 33*B*a^4*b
^4 + 2*A*a^3*b^5)*c*d^2 + (33*C*a^6*b^2 + 17*B*a^5*b^3 + 8*A*a^4*b^4)*d^3)*
f)*e^2 - ((10*(C*a*b^7 + B*b^8)*c^3 + (27*C*a^2*b^6 - 17*B*a*b^7 + 7*A*b^8)
*c^2*d - (58*C*a^3*b^5 + 7*B*a^2*b^6 + 18*A*a*b^7)*c*d^2 + (27*C*a^4*b^4 +
8*B*a^3*b^5 + 17*A*a^2*b^6)*d^3)*f^2*x^3 + 3*(10*(C*a^2*b^6 + B*a*b^7)*c^3
+ (27*C*a^3*b^5 - 17*B*a^2*b^6 + 7*A*a*b^7)*c^2*d - (58*C*a^4*b^4 + 7*B*a^3
*b^5 + 18*A*a^2*b^6)*c*d^2 + (27*C*a^5*b^3 + 8*B*a^4*b^4 + 17*A*a^3*b^5)*d^
3)*f^2*x^2 + 3*(10*(C*a^3*b^5 + B*a^2*b^6)*c^3 + (27*C*a^4*b^4 - 17*B*a^3*b
^5 + 7*A*a^2*b^6)*c^2*d - (58*C*a^5*b^3 + 7*B*a^4*b^4 + 18*A*a^3*b^5)*c*d^2
+ (27*C*a^6*b^2 + 8*B*a^5*b^3 + 17*A*a^4*b^4)*d^3)*f^2*x + (10*(C*a^4*b^4
+ B*a^3*b^5)*c^3 + (27*C*a^5*b^3 - 17*B*a^4*b^4 + 7*A*a^3*b^5)*c^2*d - (58*
C*a^6*b^2 + 7*B*a^5*b^3 + 18*A*a^4*b^4)*c*d^2 + (27*C*a^7*b + 8*B*a^6*b^2 +
17*A*a^5*b^3)*d^3)*f^2)*e)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^
2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f
^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*
a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d
^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*
f)/(b*d*f)) + 3*(((3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*c^2*d - (13*C*a^3*b^5
+ 2*B*a^2*b^6 + 13*A*a*b^7)*c*d^2 + (8*C*a^4*b^4 + 2*B*a^3*b^5 + 3*A*a^2*b

```
^6)*d^3)*f^3*x^3 + 3*((3*C*a^3*b^5 + 2*B*a^2*b^6 + 8*A*a*b^7)*c^2*d - (13*C
*a^4*b^4 + 2*B*a^3*b^5 + 13*A*a^2*b^6)*c*d^2 + (8*C*a^5*b^3 + 2*B*a^4*b^4 +
 3*A*a^3*b^5)*d^3)*f^3*x^2 + 3*((3*C*a^4*b^4 + 2*B*a^3*b^5 + 8*A*a^2*b^6)*c
^2*d - (13*C*a^5*b^3 + 2*B*a^4*b^4 + 13*A*a^3*b^5)*c*d^2 + (8*C*a^6*b^2 + 2
*B*a^5*b^3 + 3*A*a^4*b^4)*d^3)*f^3*x + ((3*C*a^5*b^3 + 2*B*a^4*b^4 + 8*A*a^
3*b^5)*c^2*d - (13*C*a^6*b^2 + 2*B*a^5*b^3 + 13*A*a^4*b^4)*c*d^2 + (8*C*a^7
*b + 2*B*a^6*b^2 + 3*A*a^5*b^3)*d^3)*f^3 + ((15*C*b^8*c^2*d - 5*(8*C*a*b^7
- B*b^8)*c*d^2 + (23*C*a^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*d^3)*f*x^3 + 3*(15*C*
a*b^7*c^2*d - 5*(8*C*a^2*b^6 - B*a*b^7)*c*d^2 +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(7/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(7/2)),x
)
```

```
[Out] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(7/2)),
x)
```

3.73 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=838

$$-\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)+(3adf-4b(de+cf))(2aCdf-b(7Bdf-6C(de+cf))))\sqrt{a+}}{105bd^3f^3}$$

```
[Out] -2/35*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e)))*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*
x+e)^(1/2)/b/d^2/f^2+2/7*C*(b*x+a)^(5/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-
2/105*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*b*c*e)+(3*a*d*f-4*b*(c*f+d*e
))*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/b/d^3/f^3-2/105*(3*b*d*f*(5*a*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*
b*c*e)-(a*c*f+a*d*e+3*b*c*e)*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e))))+2*(1/2*
a*d*f-b*(c*f+d*e))*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*b*c*e)+(3*a*d*f
-4*b*(c*f+d*e))*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e)))))*EllipticE(d^(1/2)*(
b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(
1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/d^(7/2)/f^4/(d*x+c)^(1/
2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/105*(-a*f+b*e)*(3*a^2*C*d^2*f^2*(-c*f+d*e
)-3*a*b*d*f*(7*d*f*(-5*A*d*f+2*B*c*f+3*B*d*e)-C*(11*c^2*f^2+8*c*d*e*f+16*d^
2*e^2))-b^2*(C*(24*c^3*f^3+17*c^2*d*e*f^2+16*c*d^2*e^2*f+48*d^3*e^3)+7*d*f*
(5*A*d*f*(c*f+2*d*e)-B*(4*c^2*f^2+3*c*d*e*f+8*d^2*e^2))))*EllipticF(d^(1/2)
*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)
^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/d^(7/2
)/f^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 1.37, antiderivative size = 831, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$,
Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$\frac{2C\sqrt{c+dx}\,\sqrt{e+fx}\,(a+bx)^{5/2}}{7bdf}+\frac{2(7bBdf-2aCdf-6bC(de+cf))\sqrt{c+dx}\,\sqrt{e+fx}\,(a+bx)^{3/2}}{35bd^2f^2}-\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf)))\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{a+bx}}{105bd^3f^3}-\text{[illegible]}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) - (3*a*d*f - 4*b*(
d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f)))*Sqrt[a + b*x]*Sqrt
[c + d*x]*Sqrt[e + f*x])/(105*b*d^3*f^3) + (2*(7*b*B*d*f - 2*a*C*d*f - 6*b*
C*(d*e + c*f))*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(35*b*d^2*f^2)
+ (2*C*(a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(7*b*d*f) - (2*Sqrt[-(b
```

```
*c) + a*d]*(3*b*d*f*(5*a*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) +
(3*b*c*e + a*d*e + a*c*f)*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))) + 2*
((a*d*f)/2 - b*(d*e + c*f))*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b
*d*f) - (3*a*d*f - 4*b*(d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c
*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
105*b^2*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqr
t[-(b*c) + a*d]*(b*e - a*f)*(3*a^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f
*(3*B*d*e + 2*B*c*f - 5*A*d*f) - C*(16*d^2*e^2 + 8*c*d*e*f + 11*c^2*f^2)) -
 b^2*(C*(48*d^3*e^3 + 16*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 24*c^3*f^3) + 7*d*f
*(5*A*d*f*(2*d*e + c*f) - B*(8*d^2*e^2 + 3*c*d*e*f + 4*c^2*f^2))))*Sqrt[(b*
(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(S
qrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]
)/(105*b^2*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
```

```
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = \frac{2C(a+bx)^{5/2}\sqrt{c+dx}\,\sqrt{e+fx}}{7bdf} + \frac{2\int \frac{(a+bx)^{3/2}\left(-\frac{1}{2}b(5bcCe+aCde+acCf-\cdots\right.}{\sqrt{c+d\cdots}}}{7\cdots}$$

$$= \frac{2(7bBdf-2aCdf-6bC(de+cf))(a+bx)^{3/2}\sqrt{c+dx}\,\sqrt{e+fx}}{35bd^2f^2} + \cdots$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7b\cdots}{105bd^3f\cdots}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7b\cdots}{105bd^3f\cdots}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7b\cdots}{105bd^3f\cdots}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7b\cdots}{105bd^3f\cdots}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7b\cdots}{105bd^3f\cdots}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 29.66, size = 1000, normalized size = 1.19

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x])
,x]
```

```
[Out] (2*(-(b^2*Sqrt[-a + (b*c)/d]*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(-7*B*d*f +
 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72*c^2*f^2) + 7
*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5*c*d^2*e^2*f +
 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) - B*(8*d^2*e^2 +
7*c*d*e*f + 8*c^2*f^2))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f
*(a + b*x)*(c + d*x)*(e + f*x)*(3*a^2*C*d^2*f^2 + 3*a*b*d*f*(14*B*d*f + C*(
-11*d*e - 11*c*f + 8*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e - 4*c*f + 3*
d*f*x)) + C*(24*c^2*f^2 + c*d*f*(23*e - 18*f*x) + 3*d^2*(8*e^2 - 6*e*f*x +
5*f^2*x^2)))) - I*(b*c - a*d)*f*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(-7*B*d*
f + 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72*c^2*f^2)
+ 7*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5*c*d^2*e^2*
f + 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) - B*(8*d^2*e^2
 + 7*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x
))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d
]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(3*a
^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f*(-2*B*d*e - 3*B*c*f + 5*A*d*f)
+ C*(11*d^2*e^2 + 8*c*d*e*f + 16*c^2*f^2)) + b^2*(C*(24*d^3*e^3 + 17*c*d^2*
e^2*f + 16*c^2*d*e*f^2 + 48*c^3*f^3) + 7*d*f*(5*A*d*f*(d*e + 2*c*f) - B*(4*
d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(
a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a +
(b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(105*b^3*Sqrt[-
a + (b*c)/d]*d^4*f^4*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9579 vs. $2(774) = 1548$.
time = 0.11, size = 9580, normalized size = 11.43

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cb\,x^2\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+}}{7df}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 1387, normalized size = 1.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/315*(3*(15*C*b^4*d^4*f^4*x^2 + 24*C*b^4*d^4*f^2*e^2 - 3*(6*C*b^4*c*d^3 -
(8*C*a*b^3 + 7*B*b^4)*d^4)*f^4*x + (24*C*b^4*c^2*d^2 - (33*C*a*b^3 + 28*B*b
^4)*c*d^3 + (3*C*a^2*b^2 + 42*B*a*b^3 + 35*A*b^4)*d^4)*f^4 - (18*C*b^4*d^4*
f^3*x - (23*C*b^4*c*d^3 - (33*C*a*b^3 + 28*B*b^4)*d^4)*f^3)*e)*sqrt(b*x + a
)*sqrt(d*x + c)*sqrt(f*x + e) + (48*C*b^4*d^4*e^4 + (48*C*b^4*c^4 - 8*(12*C
*a*b^3 + 7*B*b^4)*c^3*d + (39*C*a^2*b^2 + 119*B*a*b^3 + 70*A*b^4)*c^2*d^2 +
 (9*C*a^3*b - 56*B*a^2*b^2 - 175*A*a*b^3)*c*d^3 + (6*C*a^4 - 21*B*a^3*b + 1
75*A*a^2*b^2)*d^4)*f^4 + (16*C*b^4*c^3*d - 7*(4*C*a*b^3 + 3*B*b^4)*c^2*d^2
+ 7*(C*a^2*b^2 + 7*B*a*b^3 + 5*A*b^4)*c*d^3 + (9*C*a^3*b - 56*B*a^2*b^2 - 1
75*A*a*b^3)*d^4)*f^3*e + (11*C*b^4*c^2*d^2 - 7*(4*C*a*b^3 + 3*B*b^4)*c*d^3
+ (39*C*a^2*b^2 + 119*B*a*b^3 + 70*A*b^4)*d^4)*f^2*e^2 + 8*(2*C*b^4*c*d^3 -
 (12*C*a*b^3 + 7*B*b^4)*d^4)*f*e^3)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^
2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b
^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*
(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c
 + a*d)*f)/(b*d*f)) + 3*(48*C*b^4*d^4*f*e^3 + (48*C*b^4*c^3*d - 8*(9*C*a*b^
3 + 7*B*b^4)*c^2*d^2 + (12*C*a^2*b^2 + 91*B*a*b^3 + 70*A*b^4)*c*d^3 + (6*C*
a^3*b - 21*B*a^2*b^2 - 140*A*a*b^3)*d^4)*f^4 + (40*C*b^4*c^2*d^2 - (62*C*a*
b^3 + 49*B*b^4)*c*d^3 + (12*C*a^2*b^2 + 91*B*a*b^3 + 70*A*b^4)*d^4)*f^3*e +
 8*(5*C*b^4*c*d^3 - (9*C*a*b^3 + 7*B*b^4)*d^4)*f^2*e^2)*sqrt(b*d*f)*weierst
rassZeta(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d +
a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b
```

```
*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstrassPI
nverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*
b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d
^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x
+ b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^3*d^5*f^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x
)
```

```
[Out] int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
x)
```

3.74 $\int \frac{\sqrt{a+bx}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=528

$$-\frac{2(2aCdf-b(5Bdf-4C(de+cf)))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15bd^2f^2}+\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\,\sqrt{e+fx}}{5bdf}-\frac{2\cdots}{}$$

```
[Out] 2/5*C*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/15*(2*a*C*d*f-b*(5*
B*d*f-4*C*(c*f+d*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^2/f^2-2
/15*(3*b*d*f*(-5*A*b*d*f+C*a*c*f+C*a*d*e+3*C*b*c*e)+(a*d*f-2*b*(c*f+d*e))*(
2*a*C*d*f-b*(5*B*d*f-4*C*(c*f+d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-
b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-
a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/d^(5/2)/f^3/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*
f+b*e))^(1/2)-2/15*(-a*f+b*e)*(a*C*d*f*(-c*f+d*e)-b*(5*d*f*(-3*A*d*f+B*c*f+
2*B*d*e)-C*(4*c^2*f^2+3*c*d*e*f+8*d^2*e^2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2
)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*
x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/d^(5/2)/f^3/(d*x+c)
^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 524, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$-\frac{2\sqrt{ad-bc}\,(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sqrt{\frac{b(e+fx)}{be-af}}\,\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC(4c^2f^2+3cdef+8d^2e^2)\right)F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{15b^2d^{5/2}f^3\sqrt{c+dx}\,\sqrt{e+fx}}-\frac{2\sqrt{e+fx}\,\sqrt{ad-bc}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,(3bdf(acCf+aCde-5Abdf+3bcCe)-(adf-2b(cf+de))(-2aCdf+5bBdf-4bC(cf+de)))E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{15b^2d^{5/2}f^3\sqrt{c+dx}\,\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{2\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,(-2aCdf+5bBdf-4bC(cf+de))}{15bd^2f^2}+\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\,\sqrt{e+fx}}{5bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*(5*b*B*d*f - 2*a*C*d*f - 4*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*
Sqrt[e + f*x])/(15*b*d^2*f^2) + (2*C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e +
 f*x])/(5*b*d*f) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(3*b*c*C*e + a*C*d*e + a*
c*C*f - 5*A*b*d*f) - (a*d*f - 2*b*(d*e + c*f))*(5*b*B*d*f - 2*a*C*d*f - 4*b
*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[Ar
cSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e -
 a*f))])/(15*b^2*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)])
 - (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a*C*d*f*(d*e - c*f) + b*C*(8*d^2*e^2
+ 3*c*d*e*f + 4*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(2*d*e + c*f)))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
15*b^2*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\,\sqrt{e+fx}}{5bdf} + \frac{2\int \frac{\sqrt{a+bx}\,(-\frac{1}{2}b(3bcCe+aCde+acCf-5A}{\sqrt{c+dx}\,\sqrt{}}}{5b^2d}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a}{}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a}{}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a}{}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a}{}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 25.60, size = 615, normalized size = 1.16

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}\,(2a^2Cd^2f^2+abdf(-5Bdf+3C(de+cf))-b^2(C(8d^2e^2+7cdef+8c^2f^2)+5df(3Adf-2B(de+cf))))(c+dx)(e+fx)-b^2\sqrt{-a+\frac{bc}{d}}\,df(a+bx)(c+dx)(e+fx)(5bBdf+aCdf+bC(-4de-4cf+3dfx))+i(bc-ad)f(2a^2Cd^2f^2+abdf(-5Bdf+3C(de+cf))-b^2(C(8d^2e^2+7cdef+8c^2f^2)+5df(3Adf-2B(de+cf))))(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)\middle|\frac{bde-adf}{bcf-adf}\right)+ib(bc-ad)f(aCdf(-de+cf)+bC(4d^2e^2+3cdef+8c^2f^2)+5bdf(3Adf-B(de+2cf)))(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)\middle|\frac{bde-adf}{bcf-adf}\right)\right)}{15b^3\sqrt{-a+\frac{bc}{d}}\,d^3f^3\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*e
 + c*f)) - b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f - 2*
```

```
B*(d*e + c*f))))*(c + d*x)*(e + f*x) - b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x)
*(c + d*x)*(e + f*x)*(5*b*B*d*f + a*C*d*f + b*C*(-4*d*e - 4*c*f + 3*d*f*x))
 + I*(b*c - a*d)*f*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*e + c*f))
- b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f - 2*B*(d*e +
c*f))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x)
)/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*
d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(a*C*d*f*(-(d*e) + c*f) +
 b*C*(4*d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(d*e + 2*c*
f)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(
f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e
 - a*d*f)/(b*c*f - a*d*f)]))/(15*b^3*Sqrt[-a + (b*c)/d]*d^3*f^3*Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5220 vs. $2(474) = 948$.
time = 0.12, size = 5221, normalized size = 9.89

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cx\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+\cdots}}{5df}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 1036, normalized size = 1.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/45*(3*(3*C*b^3*d^3*f^3*x - 4*C*b^3*d^3*f^2*e - (4*C*b^3*c*d^2 - (C*a*b^2
+ 5*B*b^3)*d^3)*f^3)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (8*C*b^3*d
^3*e^3 + (8*C*b^3*c^3 - (7*C*a*b^2 + 10*B*b^3)*c^2*d - (2*C*a^2*b - 10*B*a*
b^2 - 15*A*b^3)*c*d^2 - (2*C*a^3 - 5*B*a^2*b + 30*A*a*b^2)*d^3)*f^3 + (3*C*
b^3*c^2*d - (2*C*a*b^2 + 5*B*b^3)*c*d^2 - (2*C*a^2*b - 10*B*a*b^2 - 15*A*b^
3)*d^3)*f^2*e + (3*C*b^3*c*d^2 - (7*C*a*b^2 + 10*B*b^3)*d^3)*f*e^2)*sqrt(b*
d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f
^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*
c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b
^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3
), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(8*C*b^3*d^3*f*e^2
+ (8*C*b^3*c^2*d - (3*C*a*b^2 + 10*B*b^3)*c*d^2 - (2*C*a^2*b - 5*B*a*b^2 -
15*A*b^3)*d^3)*f^3 + (7*C*b^3*c*d^2 - (3*C*a*b^2 + 10*B*b^3)*d^3)*f^2*e)*sq
rt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*
f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3
*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*
b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^
3), weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^
2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c
^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^
2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3)
, 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^3*d^4*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x
)
```

```
[Out] int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
x)
```

3.75 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=387

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{2\sqrt{-bc+ad}\,(2aCdf-b(3Bdf-2C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}\,E}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

```
[Out] 2/3*C*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/3*(2*a*C*d*f-b*(3*B
*d*f-2*C*(c*f+d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d
+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(
f*x+e)^(1/2)/b^2/d^(3/2)/f^2/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/3
*(a*C*f*(-c*f+d*e)-b*(3*d*f*(-A*f+B*e)-C*e*(c*f+2*d*e)))*EllipticF(d^(1/2)*
(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^
(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/d^(3/2)
/f^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 384, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1629, 164, 115, 114, 122, 121}

$$-\frac{2\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))F\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\,(-2aCdf+3bBdf-2bC(cf+de))E\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) + (2*Sqrt[-(b*c)
+ a*d]*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[(b*(c + d*x))/(b*c
- a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c)
+ a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*d^(3/2)*f^2*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(B*e - A
*f) - a*C*f*(d*e - c*f) - b*C*e*(2*d*e + c*f))*Sqrt[(b*(c + d*x))/(b*c - a*
d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x]
)/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*d^(3/2)*f^2
*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
```

```
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&& !LtQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
```

```
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\int \frac{-\frac{1}{2}b(bcCe+aCde+acCf-3Abdf)+\frac{1}{2}b(3b\cdots}{\sqrt{a+bx}\sqrt{c+dx}\cdots}}{3b^2df}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{(3bBdf-2aCdf-2bC(de+cf))\cdots}{3bdf^2\cdots}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\Big((3bdf(Be-Af)-aCf(de-cf\cdots}{\cdots}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\sqrt{-bc+ad}\,(3bBdf-2aCdf-2\cdots}{\cdots}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\sqrt{-bc+ad}\,(3bBdf-2aCdf-2\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.17, size = 418, normalized size = 1.08

$$\frac{\sqrt{a+bx}\left(2b^2Cdf(c+dx)(e+fx)-\frac{2b^2(-3bBdf+2aCdf+2bC(de+cf))(c+dx)(e+fx)}{a+bx}+2i\sqrt{-a+\frac{bc}{d}}\,df(3bBdf-2aCdf-2bC(de+cf))\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)\middle|\frac{bde-adf}{bcf-adf}\right)+\frac{2ibf(aCd(-de+cf)+b(2c^2Cf+3Ad^2f+cd(Ce-3Bf)))\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)\middle|\frac{bde-adf}{bcf-adf}\right)}{\sqrt{-a+\frac{bc}{d}}}\right)}{3b^3d^2f^2\sqrt{c+dx}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[a + b*x]*(2*b^2*C*d*f*(c + d*x)*(e + f*x) - (2*b^2*(-3*b*B*d*f + 2*a*
C*d*f + 2*b*C*(d*e + c*f))*(c + d*x)*(e + f*x))/(a + b*x) + (2*I)*Sqrt[-a +
 (b*c)/d]*d*f*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqr
t[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[
I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f
)] + ((2*I)*b*f*(a*C*d*(-(d*e) + c*f) + b*(2*c^2*C*f + 3*A*d^2*f + c*d*(C*e
 - 3*B*f)))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*
x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (
b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[-a + (b*c)/d]))/(3*b^3*d^2*f^2*Sqrt[c
 + d*x]*Sqrt[e + f*x])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2504 vs. $2(341) = 682$.
time = 0.11, size = 2505, normalized size = 6.47

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2C\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+a}}{3bdf}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2/3*(-C*b^2*c*d*f^3*x^2-C*b^2*d^2*e*f^2*x^2-C*a*b*d^2*f^3*x^2+C*(-(f*x+e)*
d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*
EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*b^2
*c*d*e^2*f-C*b^2*d^2*f^3*x^3-3*B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a
*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))
^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b*d^2*e*f^2-3*B*(-(f*x+e)*d/(c*f-
d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*Ellipti
cE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*b^2*c*d*e*
f^2-C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(
c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-
b*e))^(1/2))*a*b*d^2*e^2*f-2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f
```

```
-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(
1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*d^2*e*f^2+C*(-(f*x+e)*d/(c*f-d*e)
)^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((
-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b*c^2*f^3-C*
(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*
e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^
(1/2))*b^2*c^2*e*f^2-3*B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))
^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),(
(c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b*c*d*f^3+3*B*(-(f*x+e)*d/(c*f-d*e))^(1/2
)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+
e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b*d^2*e*f^2+3*B*(-
(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e)
)^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1
/2))*a*b*c*d*f^3+3*B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/
2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f
-d*e)*b/d/(a*f-b*e))^(1/2))*b^2*d^2*e^2*f+2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*
((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)
*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*c*d*f^3+3*A*(-(f*x
+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1
/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))
*b^2*c*d*f^3-3*A*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*(
(d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e
)*b/d/(a*f-b*e))^(1/2))*b^2*d^2*e*f^2-2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*
x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(
c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*c*d*f^3+2*C*(-(f*x+e)*
d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*
EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2
*d^2*e*f^2-2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d
*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*
b/d/(a*f-b*e))^(1/2))*a*b*c^2*f^3+2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)
*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-
d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*b^2*c^2*e*f^2-C*a*b*c*d*f^3*x-
C*a*b*d^2*e*f^2*x-C*b^2*c*d*e*f^2*x-C*a*b*c*d*e*f^2+2*C*(-(f*x+e)*d/(c*f-d*
e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE
((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b*c*d*e*f^
2-2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(
c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-
b*e))^(1/2))*b^2*d^2*e^3)*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/f^3/d^2
/b^2/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 808, normalized size = 2.09

$$\frac{2\left(3\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\,Cb^2d^2f^2+\left(2Cb^2d^2e^2+\left(2Cb^2c^2+(Cab-3Bb^2)cd+(2Ca^2-3Bab+9Ab^2)d^2\right)f^2+\left(Cb^2cd+(Cab-3Bb^2)d^2\right)fe\right)\sqrt{bdf}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(b^2d^2e^2+(b^2c^2-abcd+a^2d^2)f^2-(b^2cd+abd^2)fe\right)}{3b^2d^2f^2},-\frac{4\left(2b^3d^3e^3+(2b^3c^3-3ab^2c^2d-3a^2bcd^2+2a^3d^3)f^3-3(b^3c^2d-4ab^2cd^2+a^2bd^3)f^2e-3(b^3cd^2+ab^2d^3)fe^2\right)}{27b^3d^3f^3},\frac{3bdfx+bde+(bc+ad)f}{3bdf}\right)+3\left(2Cb^2d^2fe+\left(2Cb^2cd+(2Cab-3Bb^2)d^2\right)f^2\right)\sqrt{bdf}\,\mathrm{weierstrassZeta}\left(\frac{4\left(b^2d^2e^2+(b^2c^2-abcd+a^2d^2)f^2-(b^2cd+abd^2)fe\right)}{3b^2d^2f^2},-\frac{4\left(2b^3d^3e^3+(2b^3c^3-3ab^2c^2d-3a^2bcd^2+2a^3d^3)f^3-3(b^3c^2d-4ab^2cd^2+a^2bd^3)f^2e-3(b^3cd^2+ab^2d^3)fe^2\right)}{27b^3d^3f^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(b^2d^2e^2+(b^2c^2-abcd+a^2d^2)f^2-(b^2cd+abd^2)fe\right)}{3b^2d^2f^2},-\frac{4\left(2b^3d^3e^3+(2b^3c^3-3ab^2c^2d-3a^2bcd^2+2a^3d^3)f^3-3(b^3c^2d-4ab^2cd^2+a^2bd^3)f^2e-3(b^3cd^2+ab^2d^3)fe^2\right)}{27b^3d^3f^3},\frac{3bdfx+bde+(bc+ad)f}{3bdf}\right)\right)\right)}{9b^3d^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/9*(3*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*C*b^2*d^2*f^2 + (2*C*b^2*d
^2*e^2 + (2*C*b^2*c^2 + (C*a*b - 3*B*b^2)*c*d + (2*C*a^2 - 3*B*a*b + 9*A*b^
2)*d^2)*f^2 + (C*b^2*c*d + (C*a*b - 3*B*b^2)*d^2)*f*e)*sqrt(b*d*f)*weierstr
assPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d
 + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^
2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d
*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(2*C*b^2*d^2*f*e + (2*C*b^2*c*d
+ (2*C*a*b - 3*B*b^2)*d^2)*f^2)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^
2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f
^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*
a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d
^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2
+ (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2
), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^
3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2
 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)
/(b*d*f))))/(b^3*d^3*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.76 $\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=422

$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}-\frac{2(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{\frac{b(c+}{bc-}}}{b^2\sqrt{d}\,\sqrt{-bc+ad}\,f(be-af)}$$

```
[Out] -2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/
(b*x+a)^(1/2)-2*(2*a^2*C*d*f+b^2*(A*d*f+C*c*e)-a*b*(B*d*f+C*c*f+C*d*e))*Ell
ipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1
/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/f/(-a*f+b*e)/d^(1/2)/(a
*d-b*c)^(1/2)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2*(a*C*(-c*f+d*e)-
b*(A*d*f-B*c*f+C*c*e))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a
*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*
f+b*e))^(1/2)/b^2/f/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1628, 164, 115, 114, 122, 121}

$$-\frac{2\sqrt{e+fx}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe))\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\,f\sqrt{c+dx}\,\sqrt{ad-bc}\,(be-af)\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{2\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sqrt{\frac{b(e+fx)}{be-af}}\,(aC(de-cf)-b(Adf-Bcf+cCe))F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\,f\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{ad-bc}}-\frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,(Ab^2-a(bB-aC))}{b\sqrt{a+bx}\,(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*
e - a*f)*Sqrt[a + b*x]) - (2*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*
e + c*C*f + B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE
[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*
e - a*f))])/(b^2*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[c + d*x]*Sqr
t[(b*(e + f*x))/(b*e - a*f)]) - (2*(a*C*(d*e - c*f) - b*(c*C*e - B*c*f + A*
d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*Ellip
ticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(b^2*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*Sqrt[c + d*x]*Sqrt[e + f*
x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
```

```
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
```

```
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int -\frac{b^2Bce+a^2C(de+cf)-\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} + \frac{(aC(de-cf)-b(cC\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} + \frac{\left((aC(de-cf)-b(\cdots\right.}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2(2a^2Cdf+b^2(cCe\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2(2a^2Cdf+b^2(cCe\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.71, size = 477, normalized size = 1.13

$$\frac{2\left(-b^2(Ab^2+a(-bB+aC))(c+dx)(e+fx)+\frac{b^2(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))(c+dx)(e+fx)}{df}+\frac{i(bc-ad)(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)|\frac{bde-adf}{bcf-adf}\right)}{\sqrt{-a+\frac{bc}{d}}\,d}+\frac{ib(-bc+ad)(aC(de-cf)+b(cCe-Bde+Adf))(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-a+\frac{bc}{d}}}{\sqrt{a+bx}}\right)|\frac{bde-adf}{bcf-adf}\right)}{\sqrt{-a+\frac{bc}{d}}\,d}\right)}{b^3(bc-ad)(be-af)\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (2*(-(b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)*(e + f*x)) + (b^2*(2*a^2*C*d
*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*(c + d*x)*(e + f*x)
)/(d*f) + (I*(b*c - a*d)*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e +
c*C*f + B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/(Sqrt[-a + (b*c)/d]*d) + (I*b*(-(b*c
) + a*d)*(a*C*(d*e - c*f) + b*(c*C*e - B*d*e + A*d*f))*(a + b*x)^(3/2)*Sqrt
[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I
*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)
])/(Sqrt[-a + (b*c)/d]*d)))/(b^3*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]*Sqrt
[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3762 vs. $2(382) = 764$.

time = 0.12, size = 3763, normalized size = 8.92

method	result
elliptic	$\dfrac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(-\dfrac{2\left(bdf\,x^2+bcfx+bdex+bce\right)\left(b^2A-abB+C\,a^2\right)}{\left(a^2df-abcf-abde+b^2ce\right)b^2\sqrt{\left(x+\frac{a}{b}\right)\left(bdf\,x^2+bcfx+bdex+bce\right)}}+\dfrac{2\left(B\right.}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2*(A*b^3*d^2*f^3*x^2+A*b^3*c*d*e*f^2+A*b^3*d^2*e*f^2*x-B*a*b^2*d^2*f^3*x^2
+C*a^2*b*d^2*f^3*x^2+A*b^3*c*d*f^3*x+A*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a
)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f
-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b^2*c*d*f^3-A*(-(f*x+e)*d/(
c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*Ell
ipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b^2*
d^2*e*f^2-A*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+
c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d
/(a*f-b*e))^(1/2))*b^3*c*d*e*f^2+B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/
```

(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e
))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*b*c*d*f^3-B*(-(f*x+e)*d/(c*f-
d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*Ellipti
cF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*b*d^2*
e*f^2+B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f
/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*
f-b*e))^(1/2))*a*b^2*d^2*e^2*f-B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a
*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))
^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*b^3*c*d*e^2*f-B*(-(f*x+e)*d/(c*f-d*
e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE
((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*b*c*d*f^
3+B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*
f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*
e))^(1/2))*a^2*b*d^2*e*f^2-B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b
e))^(1/2)((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/
2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b^2*d^2*e^2*f-2*C*(-(f*x+e)*d/(c*f-d*
e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF
((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*b*d^2*e^
2*f-2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f
/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*
f-b*e))^(1/2))*a*b^2*c^2*e*f^2+3*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/
(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e
))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*b*d^2*e^2*f+2*C*(-(f*x+e)*d/(
c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*Ell
ipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b^2*
c^2*e*f^2-B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+
c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d
/(a*f-b*e))^(1/2))*a*b^2*c^2*f^3+B*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/
(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e
))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*b^3*c^2*e*f^2-2*C*(-(f*x+e)*d/(c*
f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*Ellip
ticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^3*c*d*
f^3+2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f
/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*
f-b*e))^(1/2))*a^3*d^2*e*f^2+2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a
*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)*d/(c*f-d*e))
^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*b*c^2*f^3+2*C*(-(f*x+e)*d/(c*f-
d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*Ellipti
cE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^3*c*d*f^
3-2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(
c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-
b*e))^(1/2))*a^3*d^2*e*f^2-C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b
e))^(1/2)((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/
2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a^2*b*c^2*f^3-C*(-(f*x+e)*d/(c*f-d*e))^
(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(

```
f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*b^2*d^2*e^3-C*
(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*
e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^
(1/2))*b^3*c^2*e^2*f+C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(
1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c
*f-d*e)*b/d/(a*f-b*e))^(1/2))*b^3*c*d*e^3+C*a^2*b*c*d*f^3*x+C*a^2*b*d^2*e*f
^2*x-B*a*b^2*c*d*e*f^2+C*a^2*b*c*d*e*f^2+A*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b
*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticE((-(f*x+e)*d/
(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*b^3*d^2*e^2*f+B*(-(f*x+e)
*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)
*EllipticE((-(f*x+e)*d/(c*f-d*e))^(1/2),((c*f-d*e)*b/d/(a*f-b*e))^(1/2))*a*
b^2*c*d*e*f^2+2*C*(-(f*x+e)*d/(c*f-d*e))^(1/2)*((b*x+a)*f/(a*f-b*e))^(1/2)*
((d*x+c)*f/(c*f-d*e))^(1/2)*EllipticF((-(f*x+e)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 1239, normalized size = 2.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/3*(3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x +
 e)*d^2*f^2 - ((C*a*b^3*c^2 + (2*C*a^2*b^2 - 2*B*a*b^3 - A*b^4)*c*d - (2*C*
a^3*b - B*a^2*b^2 - 2*A*a*b^3)*d^2)*f^2*x + (C*a^2*b^2*c^2 + (2*C*a^3*b - 2
*B*a^2*b^2 - A*a*b^3)*c*d - (2*C*a^4 - B*a^3*b - 2*A*a^2*b^2)*d^2)*f^2 - (C
*a*b^3*c*d - C*a^2*b^2*d^2 + (C*b^4*c*d - C*a*b^3*d^2)*x)*e^2 - ((C*b^4*c^2
 + (2*C*a*b^3 - 3*B*b^4)*c*d - (2*C*a^2*b^2 - 2*B*a*b^3 - A*b^4)*d^2)*f*x +
 (C*a*b^3*c^2 + (2*C*a^2*b^2 - 3*B*a*b^3)*c*d - (2*C*a^3*b - 2*B*a^2*b^2 -
A*a*b^3)*d^2)*f)*e)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2
*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/
27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)
*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b
```

```
^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*
f)) - 3*((C*a*b^3*c*d - (2*C*a^2*b^2 - B*a*b^3 + A*b^4)*d^2)*f^2*x + (C*a^2
*b^2*c*d - (2*C*a^3*b - B*a^2*b^2 + A*a*b^3)*d^2)*f^2 - ((C*b^4*c*d - C*a*b
^3*d^2)*f*x + (C*a*b^3*c*d - C*a^2*b^2*d^2)*f)*e)*sqrt(b*d*f)*weierstrassZe
ta(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^
2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d -
3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*
f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weierstrassPInverse
(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)
*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^
2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*
e + (b*c + a*d)*f)/(b*d*f))))/((a*b^5*c*d^2 - a^2*b^4*d^3)*f^3*x + (a^2*b^4
*c*d^2 - a^3*b^3*d^3)*f^3 - ((b^6*c*d^2 - a*b^5*d^3)*f^2*x + (a*b^5*c*d^2 -
 a^2*b^4*d^3)*f^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(a + bx)^{\frac{3}{2}} \sqrt{c + dx} \sqrt{e + fx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2 + B\,x + A}{\sqrt{e + f\,x}\,{\left(a + b\,x\right)}^{3/2}\,\sqrt{c + d\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

3.77 $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=642

$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4Adf)-b^3(3Bce-2A(}{3b(bc-ad)^2(be-}$$

```
[Out] -2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(3/2)+2/3*(2*a^3*C*d*f+a*b^2*(-4*A*d*f+B*c*f+B*d*e+6*C*c*e)-b^3*(
3*B*c*e-2*A*(c*f+d*e))+a^2*b*(B*d*f-4*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f*x+e)^(
1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(1/2)-2/3*(2*a^3*C*d*f+a*b^2*(-4*A
*d*f+B*c*f+B*d*e+6*C*c*e)-b^3*(3*B*c*e-2*A*(c*f+d*e))+a^2*b*(B*d*f-4*C*(c*f
+d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a
*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/(a*d
-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/3*(a^
2*C*d*(-c*f+d*e)-b^2*(A*c*d*f+2*A*d^2*e-3*B*c*d*e+3*C*c^2*e)+a*b*(3*(A*d^2+
C*c^2)*f-B*d*(2*c*f+d*e)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),
((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/
(-a*f+b*e))^(1/2)/b^2/(a*d-b*c)^(3/2)/(-a*f+b*e)/d^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)
```

Rubi [A]
time = 0.99, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 157, 164, 115, 114, 122, 121}

$$-\frac{2\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(a^2Cd(de-cf)+ab(3f(Ad^2+c^2C)-Bd(2cf+de))-b^2(Acdf+2Ad^2e-3Bcde+3c^2Ce)\right)F\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2\sqrt{d}\sqrt{c+dx}\sqrt{e+fx}(ad-bc)^{3/2}(be-af)}-\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)E\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2\sqrt{c+dx}(ad-bc)^{3/2}(be-af)^2\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{3b\sqrt{a+bx}(bc-ad)^2(be-af)^2}-\frac{2\sqrt{c+dx}\sqrt{e+fx}(Ab^2-a(bB-aC))}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^(3/2)) + (2*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*
c*f - 4*A*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e
+ c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)^2*(b*e - a*f)^2*Sqrt
[a + b*x]) - (2*Sqrt[d]*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A
*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqr
t[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(3*b^2*(
-(b*c) + a*d)^(3/2)*(b*e - a*f)^2*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a
*f)]) - (2*(a^2*C*d*(d*e - c*f) - b^2*(3*c^2*C*e - 3*B*c*d*e + 2*A*d^2*e +
```

```
A*c*d*f) + a*b*(3*(c^2*C + A*d^2)*f - B*d*(d*e + 2*c*f)))*Sqrt[(b*(c + d*x)
)/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sq
rt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*
Sqrt[d]*(-(b*c) + a*d)^(3/2)*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{2\int -\frac{a^2C(de+cf)-ab(3cCe}{}}{}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf+ab^2(6cC}{}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf+ab^2(6cC}{}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf+ab^2(6cC}{}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf+ab^2(6cC}{}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf+ab^2(6cC}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.87, size = 699, normalized size = 1.09

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*C))
*(b*c - a*d)*(b*e - a*f) + (-2*a^3*C*d*f - a*b^2*(6*c*C*e + B*d*e + B*c*f -
```

```
4*A*d*f) + b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(-(B*d*f) + 4*C*(d*e +
c*f)))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(2*a^3*C*d*f + a*b^2*
(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e + 2*A*(d*e + c*f)) + a^
2*b*(B*d*f - 4*C*(d*e + c*f)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(2*a^3
*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e + 2*A*(d
*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*(a + b*x)^(3/2)*Sqrt[(b*(c +
d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[
Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(
b*c - a*d)*(a^2*C*f*(d*e - c*f) + b^2*(3*c*C*e^2 + A*d*e*f + c*f*(-3*B*e +
2*A*f)) + a*b*(-3*C*d*e^2 + f*(2*B*d*e + B*c*f - 3*A*d*f)))*(a + b*x)^(3/2)
*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipt
icF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a
*d*f)])))/(3*b^3*Sqrt[-a + (b*c)/d]*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^(
3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 13109 vs. $2(588) = 1176$.
time = 0.13, size = 13110, normalized size = 20.42

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(-\frac{2\left(b^2A-abB+C\,a^2\right)\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex}}{3\left(a^2df-abcf-abde+b^2ce\right)b^3\left(x+\frac{a}{b}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 2343, normalized size = 3.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] -2/9*(3*(((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (2*C*a^3*b^3 + B*a^2*b^4
- 4*A*a*b^5)*d^2)*f^2*x + (3*(C*a^3*b^3 - A*a*b^5)*c*d - (C*a^4*b^2 + 2*B*a
^3*b^3 - 5*A*a^2*b^4)*d^2)*f^2 - ((3*(2*C*a*b^5 - B*b^6)*c*d - (4*C*a^2*b^4
 - B*a*b^5 - 2*A*b^6)*d^2)*f*x + ((5*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d - 3
*(C*a^3*b^3 - A*a*b^5)*d^2)*f)*e)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)
 - (((5*C*a^2*b^4 + B*a*b^5 + 2*A*b^6)*c^2 - (5*C*a^3*b^3 + 4*B*a^2*b^4 + 5
*A*a*b^5)*c*d + (2*C*a^4*b^2 + B*a^3*b^3 + 5*A*a^2*b^4)*d^2)*f^2*x^2 + 2*((
5*C*a^3*b^3 + B*a^2*b^4 + 2*A*a*b^5)*c^2 - (5*C*a^4*b^2 + 4*B*a^3*b^3 + 5*A
*a^2*b^4)*c*d + (2*C*a^5*b + B*a^4*b^2 + 5*A*a^3*b^3)*d^2)*f^2*x + ((5*C*a^
4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (5*C*a^5*b + 4*B*a^4*b^2 + 5*A*a^3*b
^3)*c*d + (2*C*a^6 + B*a^5*b + 5*A*a^4*b^2)*d^2)*f^2 + (9*C*a^2*b^4*c^2 - 3
*(4*C*a^3*b^3 + B*a^2*b^4)*c*d + (5*C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*d^
2 + (9*C*b^6*c^2 - 3*(4*C*a*b^5 + B*b^6)*c*d + (5*C*a^2*b^4 + B*a*b^5 + 2*A
*b^6)*d^2)*x^2 + 2*(9*C*a*b^5*c^2 - 3*(4*C*a^2*b^4 + B*a*b^5)*c*d + (5*C*a^
3*b^3 + B*a^2*b^4 + 2*A*a*b^5)*d^2)*x)*e^2 - ((3*(4*C*a*b^5 + B*b^6)*c^2 -
(13*C*a^2*b^4 + 11*B*a*b^5 + A*b^6)*c*d + (5*C*a^3*b^3 + 4*B*a^2*b^4 + 5*A*
a*b^5)*d^2)*f*x^2 + 2*(3*(4*C*a^2*b^4 + B*a*b^5)*c^2 - (13*C*a^3*b^3 + 11*B
*a^2*b^4 + A*a*b^5)*c*d + (5*C*a^4*b^2 + 4*B*a^3*b^3 + 5*A*a^2*b^4)*d^2)*f*
x + (3*(4*C*a^3*b^3 + B*a^2*b^4)*c^2 - (13*C*a^4*b^2 + 11*B*a^3*b^3 + A*a^2
*b^4)*c*d + (5*C*a^5*b + 4*B*a^4*b^2 + 5*A*a^3*b^3)*d^2)*f)*e)*sqrt(b*d*f)*
weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 -
(b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 -
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*
d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/
3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(((4*C*a^2*b^4 - B*a*b^5
 - 2*A*b^6)*c*d - (2*C*a^3*b^3 + B*a^2*b^4 - 4*A*a*b^5)*d^2)*f^2*x^2 + 2*((
4*C*a^3*b^3 - B*a^2*b^4 - 2*A*a*b^5)*c*d - (2*C*a^4*b^2 + B*a^3*b^3 - 4*A*a
^2*b^4)*d^2)*f^2*x + ((4*C*a^4*b^2 - B*a^3*b^3 - 2*A*a^2*b^4)*c*d - (2*C*a^
5*b + B*a^4*b^2 - 4*A*a^3*b^3)*d^2)*f^2 - ((3*(2*C*a*b^5 - B*b^6)*c*d - (4*
C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*d^2)*f*x^2 + 2*(3*(2*C*a^2*b^4 - B*a*b^5)*c*
d - (4*C*a^3*b^3 - B*a^2*b^4 - 2*A*a*b^5)*d^2)*f*x + (3*(2*C*a^3*b^3 - B*a^
```

```
2*b^4)*c*d - (4*C*a^4*b^2 - B*a^3*b^3 - 2*A*a^2*b^4)*d^2)*f)*e)*sqrt(b*d*f)
*weierstrassZeta(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^
2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2
 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), weier
strassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*
c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 +
 a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*
b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/((a^2*b^7*c^2*d - 2*a^3*b^6*c*d
^2 + a^4*b^5*d^3)*f^3*x^2 + 2*(a^3*b^6*c^2*d - 2*a^4*b^5*c*d^2 + a^5*b^4*d^
3)*f^3*x + (a^4*b^5*c^2*d - 2*a^5*b^4*c*d^2 + a^6*b^3*d^3)*f^3 + ((b^9*c^2*
d - 2*a*b^8*c*d^2 + a^2*b^7*d^3)*f*x^2 + 2*(a*b^8*c^2*d - 2*a^2*b^7*c*d^2 +
 a^3*b^6*d^3)*f*x + (a^2*b^7*c^2*d - 2*a^3*b^6*c*d^2 + a^4*b^5*d^3)*f)*e^2
- 2*((a*b^8*c^2*d - 2*a^2*b^7*c*d^2 + a^3*b^6*d^3)*f^2*x^2 + 2*(a^2*b^7*c^2
*d - 2*a^3*b^6*c*d^2 + a^4*b^5*d^3)*f^2*x + (a^3*b^6*c^2*d - 2*a^4*b^5*c*d^
2 + a^5*b^4*d^3)*f^2)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1/2)), x)
```

3.78 $\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1116

$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}+\frac{2(2a^3Cdf+ab^2(10cCe+Bde+Bcf-8Adf)-b^3(5Bce-4}{15b(bc-ad)^2(be}$$

```
[Out] -2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(5/2)+2/15*(2*a^3*C*d*f+a*b^2*(-8*A*d*f+B*c*f+B*d*e+10*C*c*e)-b^3
*(5*B*c*e-4*A*(c*f+d*e))+3*a^2*b*(B*d*f-2*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/15*(2*a^4*C*d^2*f^2+a^
3*b*d*f*(3*B*d*f-7*C*(c*f+d*e))-b^4*(8*A*d^2*e^2-c*d*e*(-7*A*f+10*B*e)+c^2*
(8*A*f^2-10*B*e*f+15*C*e^2))-a*b^3*(d^2*e*(-23*A*f+2*B*e)-2*c^2*f*(-B*f+5*C
*e)-c*d*(23*A*f^2-33*B*e*f+10*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-13*c*d*e*f+3*d^
2*e^2)+d*f*(23*A*d*f-7*B*(c*f+d*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b
*c)^3/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/15*(2*a^4*C*d^2*f^2+a^3*b*d*f*(3*B*d*f-7
*C*(c*f+d*e))-b^4*(8*A*d^2*e^2-c*d*e*(-7*A*f+10*B*e)+c^2*(8*A*f^2-10*B*e*f+
15*C*e^2))-a*b^3*(d^2*e*(-23*A*f+2*B*e)-2*c^2*f*(-B*f+5*C*e)-c*d*(23*A*f^2-
33*B*e*f+10*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-13*c*d*e*f+3*d^2*e^2)+d*f*(23*A*d
*f-7*B*(c*f+d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+
b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1
/2)/b^2/(a*d-b*c)^(5/2)/(-a*f+b*e)^3/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(
1/2)+2/15*(a^3*C*d*f*(-c*f+d*e)+b^3*(8*A*d^2*e^2-c*d*e*(-3*A*f+10*B*e)+c^2*
(4*A*f^2-5*B*e*f+15*C*e^2))+a*b^2*(d^2*e*(-19*A*f+2*B*e)-c^2*f*(-B*f+20*C*e
)-c*d*(11*A*f^2-27*B*e*f+10*C*e^2))-3*a^2*b*(d*f*(-5*A*d*f+3*B*c*f+2*B*d*e)
-C*(3*c^2*f^2+c*d*e*f+d^2*e^2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^
(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/
2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/(a*d-b*c)^(5/2)/(-a*f+b*e)^2/(d*x+c)^(1
/2)/(f*x+e)^(1/2)
```

Rubi [A]

time = 2.16, antiderivative size = 1116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 157, 164, 115, 114, 122, 121}

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(5*b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^(5/2)) + (2*(2*a^3*C*d*f + a*b^2*(10*c*C*e + B*d*e + B
```

```
*c*f - 8*A*d*f) - b^3*(5*B*c*e - 4*A*(d*e + c*f)) + 3*a^2*b*(B*d*f - 2*C*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b*(b*c - a*d)^2*(b*e - a*f)^2*
(a + b*x)^(3/2)) + (2*(2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*
f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f
+ 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e - 23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(
10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c
^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(
15*b*(b*c - a*d)^3*(b*e - a*f)^3*Sqrt[a + b*x]) + (2*Sqrt[d]*(2*a^4*C*d^2*f
^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B
*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e -
23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a
^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e +
 c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sq
rt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])
/(15*b^2*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]) + (2*Sqrt[d]*(a^3*C*d*f*(d*e - c*f) + b^3*(8*A*d^2*e^2 - c*
d*e*(10*B*e - 3*A*f) + c^2*(15*C*e^2 - 5*B*e*f + 4*A*f^2)) + a*b^2*(d^2*e*(
2*B*e - 19*A*f) - c^2*f*(20*C*e - B*f) - c*d*(10*C*e^2 - 27*B*e*f + 11*A*f^
2)) - 3*a^2*b*(d*f*(2*B*d*e + 3*B*c*f - 5*A*d*f) - C*(d^2*e^2 + c*d*e*f + 3
*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]
*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(15*b^2*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^2*Sqrt[c + d
*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
```

```
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \frac{-a^2C(de+cf)-ab(5cCe+\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2(2a^3Cdf+ab^2(10cC\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2(2a^3Cdf+ab^2(10cC\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2(2a^3Cdf+ab^2(10cC\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2(2a^3Cdf+ab^2(10cC\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2(2a^3Cdf+ab^2(10cC\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\,\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2(2a^3Cdf+ab^2(10cC\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.37, size = 1520, normalized size = 1.36

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5*
b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e -
 4*A*b^3*d*e - a*b^2*B*d*e + 6*a^2*b*C*d*e - 4*A*b^3*c*f - a*b^2*B*c*f + 6*
a^2*b*c*C*f + 8*a*A*b^2*d*f - 3*a^2*b*B*d*f - 2*a^3*C*d*f))/(15*b*(b*c - a*
d)^2*(b*e - a*f)^2*(a + b*x)^2) - (2*(15*b^4*c^2*C*e^2 - 10*b^4*B*c*d*e^2 -
 10*a*b^3*c*C*d*e^2 + 8*A*b^4*d^2*e^2 + 2*a*b^3*B*d^2*e^2 + 3*a^2*b^2*C*d^2
*e^2 - 10*b^4*B*c^2*e*f - 10*a*b^3*c^2*C*e*f + 7*A*b^4*c*d*e*f + 33*a*b^3*B
*c*d*e*f - 13*a^2*b^2*c*C*d*e*f - 23*a*A*b^3*d^2*e*f - 7*a^2*b^2*B*d^2*e*f
+ 7*a^3*b*C*d^2*e*f + 8*A*b^4*c^2*f^2 + 2*a*b^3*B*c^2*f^2 + 3*a^2*b^2*c^2*C
*f^2 - 23*a*A*b^3*c*d*f^2 - 7*a^2*b^2*B*c*d*f^2 + 7*a^3*b*c*C*d*f^2 + 23*a^
2*A*b^2*d^2*f^2 - 3*a^3*b*B*d^2*f^2 - 2*a^4*C*d^2*f^2))/(15*b*(b*c - a*d)^3
*(b*e - a*f)^3*(a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(-2*a^4
*C*d^2*f^2 + a^3*b*d*f*(-3*B*d*f + 7*C*(d*e + c*f)) + a*b^3*(d^2*e*(2*B*e -
 23*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(-10*C*e^2 + 33*B*e*f - 23*A*f^2))
+ b^4*(8*A*d^2*e^2 + c*d*e*(-10*B*e + 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8
*A*f^2)) + a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + d*f*(23*A*d*f
- 7*B*(d*e + c*f))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f + (b*e)/(a
+ b*x) - (a*f)/(a + b*x)) - (I*(-(b*c) + a*d)*f*(-2*a^4*C*d^2*f^2 + a^3*b*d
*f*(-3*B*d*f + 7*C*(d*e + c*f)) + a*b^3*(d^2*e*(2*B*e - 23*A*f) + 2*c^2*f*(
-5*C*e + B*f) + c*d*(-10*C*e^2 + 33*B*e*f - 23*A*f^2)) + b^4*(8*A*d^2*e^2 +
 c*d*e*(-10*B*e + 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) + a^2*b^2*(
C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e + c*f))))
*Sqrt[1 - a/(a + b*x) + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(
f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e
 - a*d*f)/(b*c*f - a*d*f)])/Sqrt[a + b*x] + (I*b*(-(b*c) + a*d)*f*(a^3*C*d*
f*(-(d*e) + c*f) + a*b^2*(d^2*e*(B*e - 11*A*f) + 2*c^2*f*(-5*C*e + B*f) + c
*d*(-20*C*e^2 + 27*B*e*f - 19*A*f^2)) + b^3*(4*A*d^2*e^2 + c*d*e*(-5*B*e +
3*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) + 3*a^2*b*(d*f*(-3*B*d*e - 2*
B*c*f + 5*A*d*f) + C*(3*d^2*e^2 + c*d*e*f + c^2*f^2)))*Sqrt[1 - a/(a + b*x)
 + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(f*(a + b*x))]*Ellipti
cF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*
d*f)])/Sqrt[a + b*x]))/(15*b^3*Sqrt[-a + (b*c)/d]*(b*c - a*d)^3*(b*e - a*f)
^3*Sqrt[c + ((a + b*x)*(d - (a*d)/(a + b*x)))/b]*Sqrt[e + ((a + b*x)*(f - (
a*f)/(a + b*x)))/b])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32151 vs. $2(1054) = 2108$.
time = 0.21, size = 32152, normalized size = 28.81

method	result	size
elliptic	Expression too large to display	2283
default	Expression too large to display	32152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 5104, normalized size = 4.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/45*(3*(((3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*c^2*d + (7*C*a^3*b^5 - 7*B*a^
2*b^6 - 23*A*a*b^7)*c*d^2 - (2*C*a^4*b^4 + 3*B*a^3*b^5 - 23*A*a^2*b^6)*d^3)
*f^3*x^2 + (5*(B*a^2*b^6 + 4*A*a*b^7)*c^2*d + 2*(11*C*a^4*b^4 - 6*B*a^3*b^5
 - 29*A*a^2*b^6)*c*d^2 - 3*(2*C*a^5*b^3 + 3*B*a^4*b^4 - 18*A*a^3*b^5)*d^3)*
f^3*x + (15*A*a^2*b^6*c^2*d + (9*C*a^5*b^3 + B*a^4*b^4 - 41*A*a^3*b^5)*c*d^
2 - (C*a^6*b^2 + 9*B*a^5*b^3 - 34*A*a^4*b^4)*d^3)*f^3 + ((15*C*b^8*c^2*d -
10*(C*a*b^7 + B*b^8)*c*d^2 + (3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*d^3)*f*x^2
 + (5*(4*C*a*b^7 + B*b^8)*c^2*d - 2*(2*C*a^2*b^6 + 13*B*a*b^7 + 2*A*b^8)*c*
d^2 + 5*(B*a^2*b^6 + 4*A*a*b^7)*d^3)*f*x + (15*A*a^2*b^6*d^3 + (8*C*a^2*b^6
 + 2*B*a*b^7 + 3*A*b^8)*c^2*d - 10*(B*a^2*b^6 + A*a*b^7)*c*d^2)*f)*e^2 - ((
10*(C*a*b^7 + B*b^8)*c^2*d + (13*C*a^2*b^6 - 33*B*a*b^7 - 7*A*b^8)*c*d^2 -
(7*C*a^3*b^5 - 7*B*a^2*b^6 - 23*A*a*b^7)*d^3)*f^2*x^2 + 2*((2*C*a^2*b^6 + 1
3*B*a*b^7 + 2*A*b^8)*c^2*d + 5*(5*C*a^3*b^5 - 7*B*a^2*b^6 - 3*A*a*b^7)*c*d^
2 - (11*C*a^4*b^4 - 6*B*a^3*b^5 - 29*A*a^2*b^6)*d^3)*f^2*x + (10*(B*a^2*b^6
 + A*a*b^7)*c^2*d + 5*(5*C*a^4*b^4 - 5*B*a^3*b^5 - 7*A*a^2*b^6)*c*d^2 - (9*
C*a^5*b^3 + B*a^4*b^4 - 41*A*a^3*b^5)*d^3)*f^2)*e)*sqrt(b*x + a)*sqrt(d*x +
```

c)*sqrt(f*x + e) + (((3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*c^3 - (17*C*a^3*b^5 + 8*B*a^2*b^6 + 27*A*a*b^7)*c^2*d + (8*C*a^4*b^4 + 17*B*a^3*b^5 + 33*A*a^2*b^6)*c*d^2 - (2*C*a^5*b^3 + 3*B*a^4*b^4 + 22*A*a^3*b^5)*d^3)*f^3*x^3 + 3*((3*C*a^3*b^5 + 2*B*a^2*b^6 + 8*A*a*b^7)*c^3 - (17*C*a^4*b^4 + 8*B*a^3*b^5 + 27*A*a^2*b^6)*c^2*d + (8*C*a^5*b^3 + 17*B*a^4*b^4 + 33*A*a^3*b^5)*c*d^2 - (2*C*a^6*b^2 + 3*B*a^5*b^3 + 22*A*a^4*b^4)*d^3)*f^3*x^2 + 3*((3*C*a^4*b^4 + 2*B*a^3*b^5 + 8*A*a^2*b^6)*c^3 - (17*C*a^5*b^3 + 8*B*a^4*b^4 + 27*A*a^3*b^5)*c^2*d + (8*C*a^6*b^2 + 17*B*a^5*b^3 + 33*A*a^4*b^4)*c*d^2 - (2*C*a^7*b + 3*B*a^6*b^2 + 22*A*a^5*b^3)*d^3)*f^3*x + ((3*C*a^5*b^3 + 2*B*a^4*b^4 + 8*A*a^3*b^5)*c^3 - (17*C*a^6*b^2 + 8*B*a^5*b^3 + 27*A*a^4*b^4)*c^2*d + (8*C*a^7*b + 17*B*a^6*b^2 + 33*A*a^5*b^3)*c*d^2 - (2*C*a^8 + 3*B*a^7*b + 22*A*a^6*b^2)*d^3)*f^3 + (15*C*a^3*b^5*c^2*d - 10*(C*a^4*b^4 + B*a^3*b^5)*c*d^2 + (3*C*a^5*b^3 + 2*B*a^4*b^4 + 8*A*a^3*b^5)*d^3 + (15*C*b^8*c^2*d - 10*(C*a*b^7 + B*b^8)*c*d^2 + (3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*d^3)*x^3 + 3*(15*C*a*b^7*c^2*d - 10*(C*a^2*b^6 + B*a*b^7)*c*d^2 + (3*C*a^3*b^5 + 2*B*a^2*b^6 + 8*A*a*b^7)*d^3)*x^2 + 3*(15*C*a^2*b^6*c^2*d - 10*(C*a^3*b^5 + B*a^2*b^6)*c*d^2 + (3*C*a^4*b^4 + 2*B*a^3*b^5 + 8*A*a^2*b^6)*d^3)*x)*e^3 + ((15*C*b^8*c^3 - 5*(16*C*a*b^7 + B*b^8)*c^2*d + (58*C*a^2*b^6 + 37*B*a*b^7 + 3*A*b^8)*c*d^2 - (17*C*a^3*b^5 + 8*B*a^2*b^6 + 27*A*a*b^7)*d^3)*f*x^3 + 3*(15*C*a*b^7*c^3 - 5*(16*C*a^2*b^6 + B*a*b^7)*c^2*d + (58*C*a^3*b^5 + 37*B*a^2*b^6 + 3*A*a*b^7)*c*d^2 - (17*C*a^4*b^4 + 8*B*a^3*b^5 + 27*A*a^2*b^6)*d^3)*f*x^2 + 3*(15*C*a^2*b^6*c^3 - 5*(16*C*a^3*b^5 + B*a^2*b^6)*c^2*d + (58*C*a^4*b^4 + 37*B*a^3*b^5 + 3*A*a^2*b^6)*c*d^2 - (17*C*a^5*b^3 + 8*B*a^4*b^4 + 27*A*a^3*b^5)*d^3)*f*x + (15*C*a^3*b^5*c^3 - 5*(16*C*a^4*b^4 + B*a^3*b^5)*c^2*d + (58*C*a^5*b^3 + 37*B*a^4*b^4 + 3*A*a^3*b^5)*c*d^2 - (17*C*a^6*b^2 + 8*B*a^5*b^3 + 27*A*a^4*b^4)*d^3)*f)*e^2 - ((10*(C*a*b^7 + B*b^8)*c^3 - (58*C*a^2*b^6 + 37*B*a*b^7 + 3*A*b^8)*c^2*d + 4*(8*C*a^3*b^5 + 17*B*a^2*b^6 + 3*A*a*b^7)*c*d^2 - (8*C*a^4*b^4 + 17*B*a^3*b^5 + 33*A*a^2*b^6)*d^3)*f^2*x^3 + 3*(10*(C*a^2*b^6 + B*a*b^7)*c^3 - (58*C*a^3*b^5 + 37*B*a^2*b^6 + 3*A*a*b^7)*c^2*d + 4*(8*C*a^4*b^4 + 17*B*a^3*b^5 + 3*A*a^2*b^6)*c*d^2 - (8*C*a^5*b^3 + 17*B*a^4*b^4 + 33*A*a^3*b^5)*d^3)*f^2*x^2 + 3*(10*(C*a^3*b^5 + B*a^2*b^6)*c^3 - (58*C*a^4*b^4 + 37*B*a^3*b^5 + 3*A*a^2*b^6)*c^2*d + 4*(8*C*a^5*b^3 + 17*B*a^4*b^4 + 3*A*a^3*b^5)*c*d^2 - (8*C*a^6*b^2 + 17*B*a^5*b^3 + 33*A*a^4*b^4)*d^3)*f^2*x + (10*(C*a^4*b^4 + B*a^3*b^5)*c^3 - (58*C*a^5*b^3 + 37*B*a^4*b^4 + 3*A*a^3*b^5)*c^2*d + 4*(8*C*a^6*b^2 + 17*B*a^5*b^3 + 3*A*a^4*b^4)*c*d^2 - (8*C*a^7*b + 17*B*a^6*b^2 + 33*A*a^5*b^3)*d^3)*f^2)*e)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2 - (b^2*c*d + a*b*d^2)*f*e)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3 - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*f^2*e - 3*(b^3*c*d^2 + a*b^2*d^3)*f*e^2)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(((3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*c^2*d + (7*C*a^3*b^5 - 7*B*a^2*b^6 - 23*A*a*b^7)*c*d^2 - (2*C*a^4*b^4 + 3*B*a^3*b^5 - 23*A*a^2*b^6)*d^3)*f^3*x^3 + 3*((3*C*a^3*b^5 + 2*B*a^2*b^6 + 8*A*a*b^7)*c^2*d + (7*C*a^4*b^4 - 7*B*a^3*b^5 - 23*A*a^2*b^6)*c*d^2 - (2*C*a^5*b^3 + 3*B*a^4*b^4 - 23*A*a^3*b^5)*d^3)*f^3*x^2 + 3*((3*C*a^4*b^4 + 2

```
*B*a^3*b^5 + 8*A*a^2*b^6)*c^2*d + (7*C*a^5*b^3 - 7*B*a^4*b^4 - 23*A*a^3*b^5
)*c*d^2 - (2*C*a^6*b^2 + 3*B*a^5*b^3 - 23*A*a^4*b^4)*d^3)*f^3*x + ((3*C*a^5
*b^3 + 2*B*a^4*b^4 + 8*A*a^3*b^5)*c^2*d + (7*C*a^6*b^2 - 7*B*a^5*b^3 - 23*A
*a^4*b^4)*c*d^2 - (2*C*a^7*b + 3*B*a^6*b^2 - 23*A*a^5*b^3)*d^3)*f^3 + ((15*
C*b^8*c^2*d - 10*(C*a*b^7 + B*b^8)*c*d^2 + (3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b
^8)*d^3)*f*x^3 + 3*(15*C*a*b^7*c^2*d - 10*(C*a^...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(7/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{7/2}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)^(1/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```